Structure of *p-sd* shell Λ hypernuclei with antisymmetrized molecular dynamics

Masahiro Isaka (RIKEN)

Grand challenges of hypernuclear physics

Interaction: To understand baryon-baryon interaction

- **2 body interaction between baryons (nucleon, hyperon)**
  - hyperon-nucleon (YN)
  - hyperon-hyperon (YY)

A major issue in hypernuclear physics

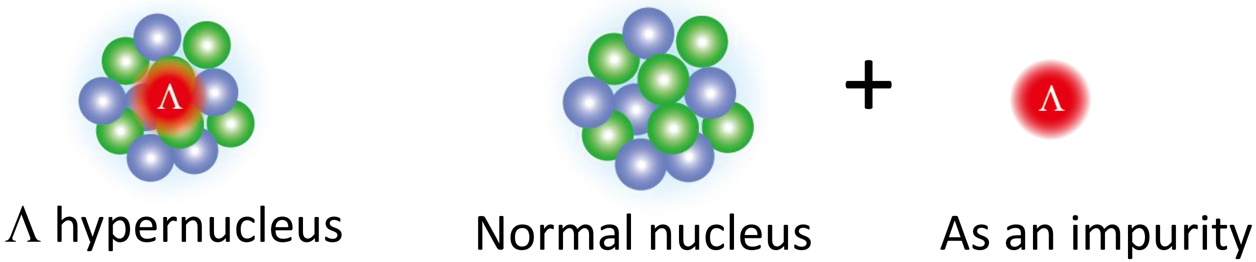
Structure: To understand many-body system of nucleons and hyperon

- **Addition of hyperon(s) shows us new features of nuclear structure**

Ex.) Structure change by hyperon(s) "impurity effect"

- No Pauli exclusion between N and Y
- YN interaction is different from NN

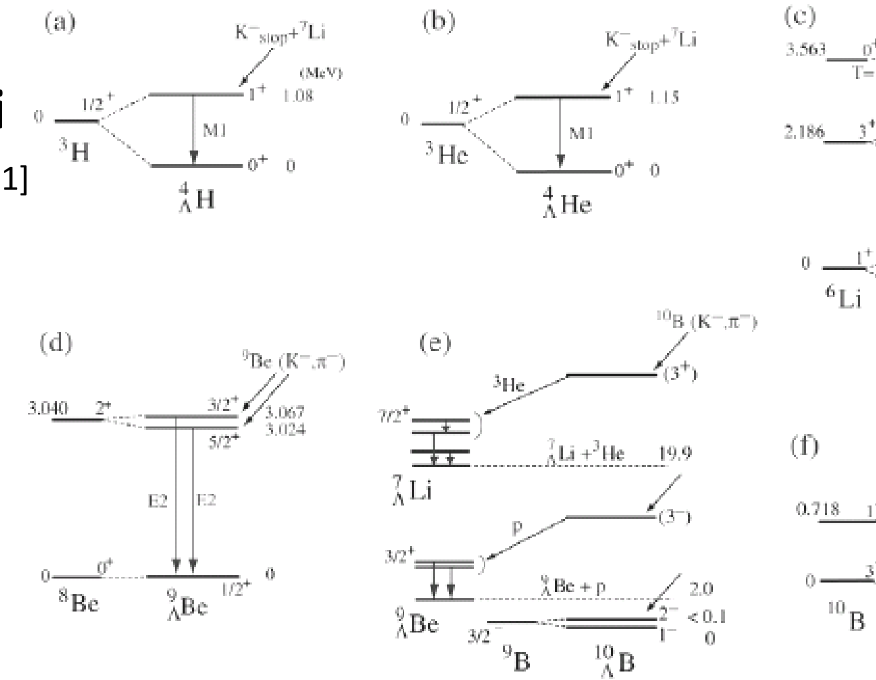
"Hyperon as an impurity in nuclei"



Recent achievements in (hyper)nuclear physics

Knowledge of ΛN interaction

- **Study of light (*s*, *p*-shell) Λ hypernuclei**
  - Accurate calculations of few-body system [1]
  - ΛN effective interactions
  - Increases of experimental information [2]



Development of theoretical models

- **Through the study of unstable nuclei**

  Ex.: Antisymmetrized Molecular Dynamics (AMD)[3]

  - AMD can describe dynamical changes of various structure
  - No assumption on clustering and deformation

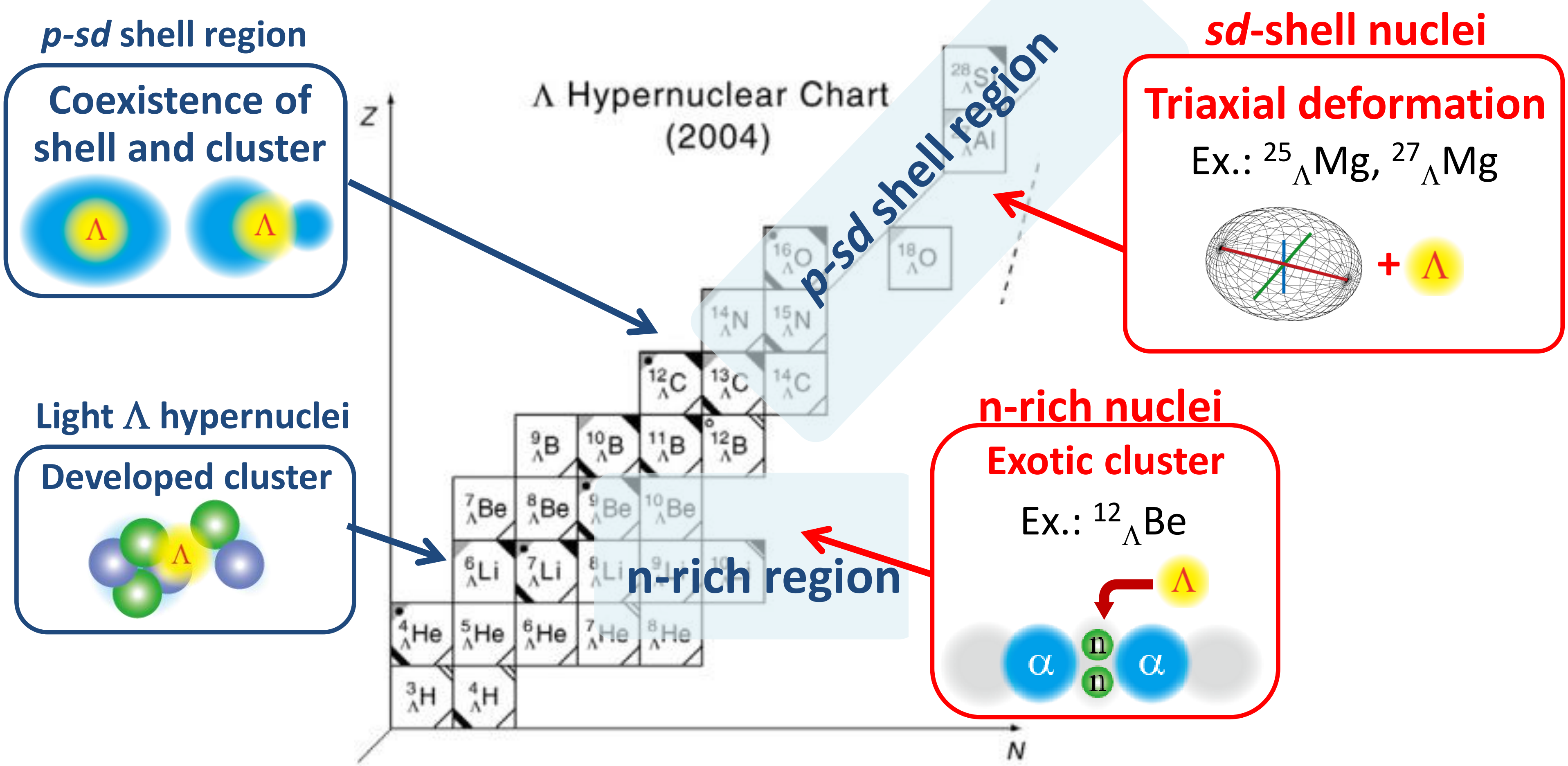
Recent developments enable us to study structure of Λ hypernuclei

[1] E. Hiyama, NPA **805** (2008), 190c,
[2] O. Hashimoto and H. Tamura, PPNP **57** (2006), 564., [3] Y. Kanada-En'yo *et al.*, PTP **93** (1995), 115.

Toward heavier and exotic Λ hypernuclei

Experiments at J-PARC, JLab and Mainz *etc.*

- **Heavier** and **neutron-rich** **Λ hypernuclei can be produced**
- **Various structures will appear in hypernuclei**

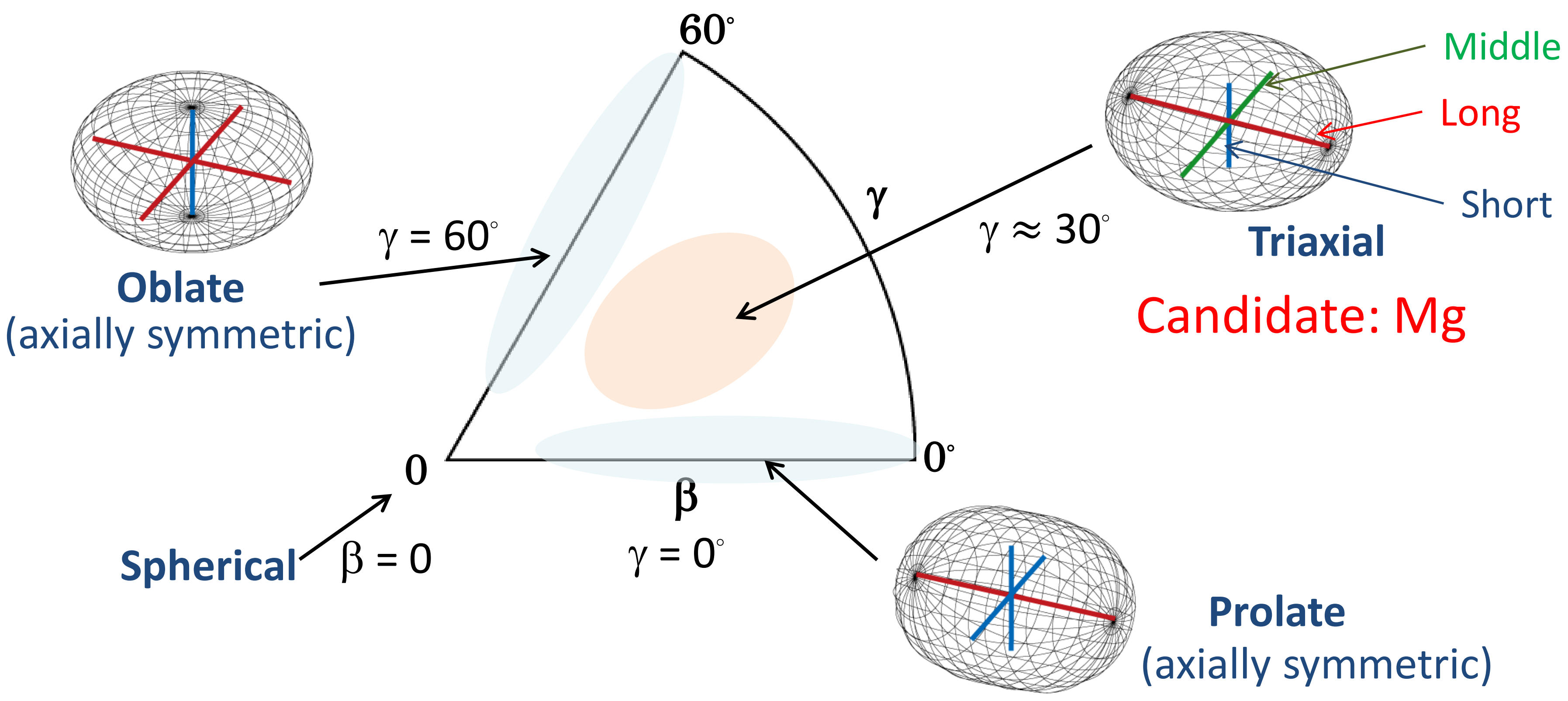


Taken from O. Hashimoto and H. Tamura, PPNP **57**(2006),564.

Topic 1: Triaxial deformation of nuclei

- **Many nuclei manifests various quadrupole deformation**
  (parameterized by quadrupole deformation β and γ)

- **Most of them are prolate or oblate deformed (axially symmetric)**

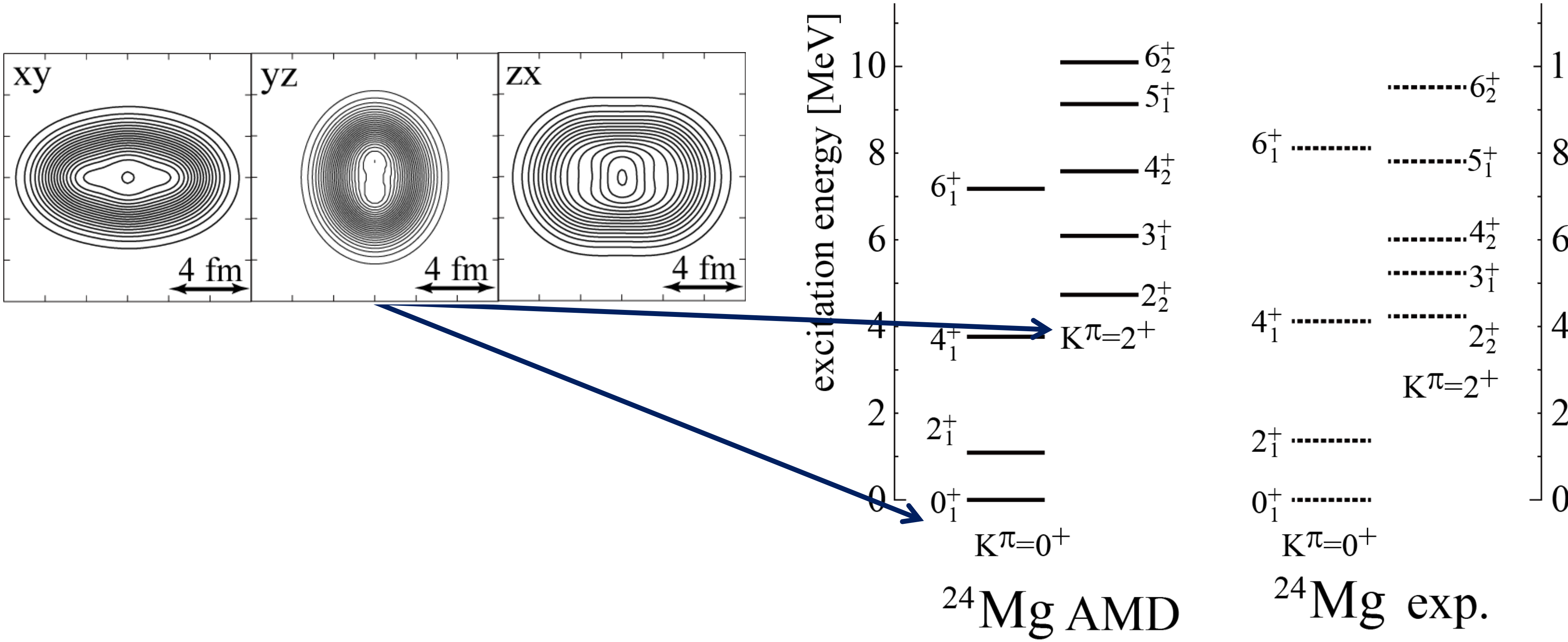


Topic 1: Triaxial deformation of nuclei

Triaxial deformed nuclei are not many, Mg isotopes are the candidates

Ex.) ^{24}Mg

- Largely deformed
- Low-lying 2nd 2^+ indicates having the triaxial deformation



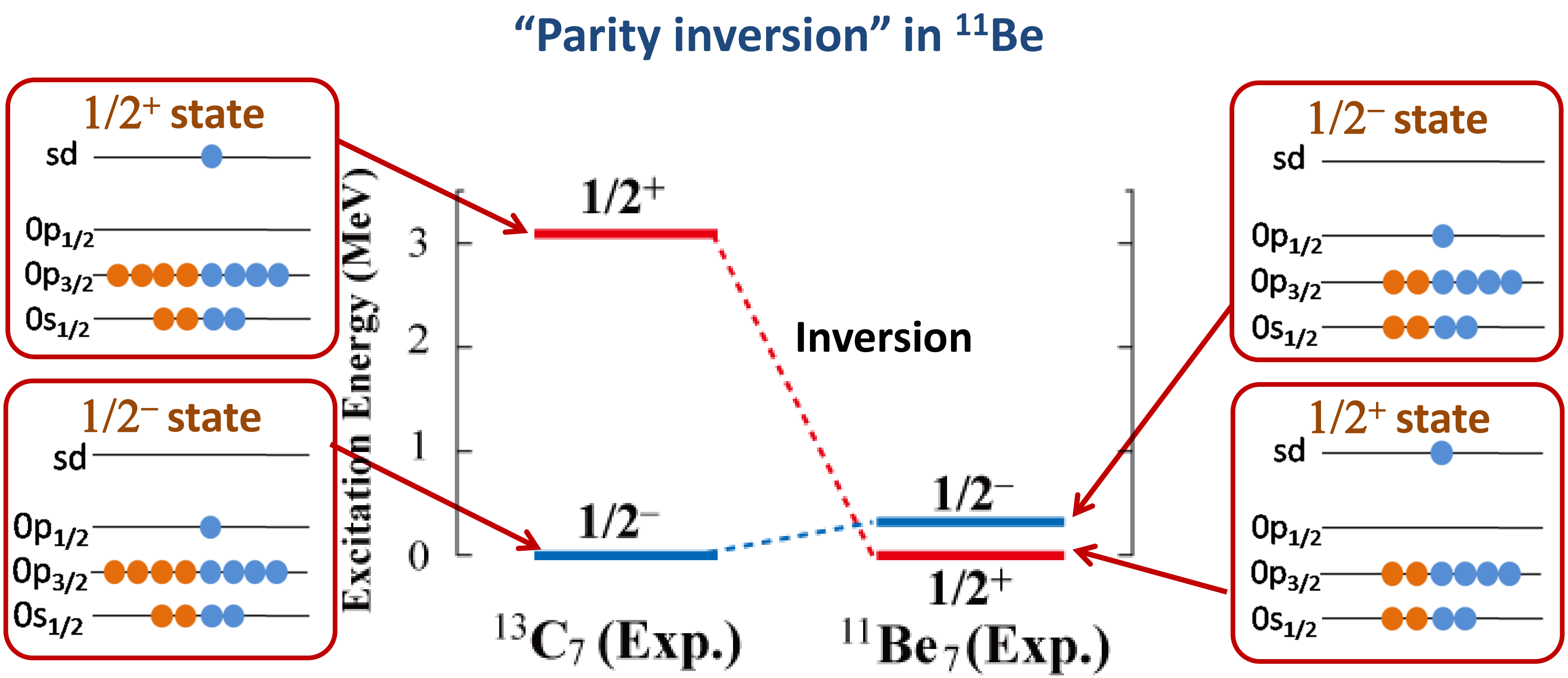
Identification of triaxial deformation is not easy

Our task: to identify triaxial deformation of Mg by using Λ

Topic 2: Structure of neutron-rich nuclei

Ex.) Be isotopes

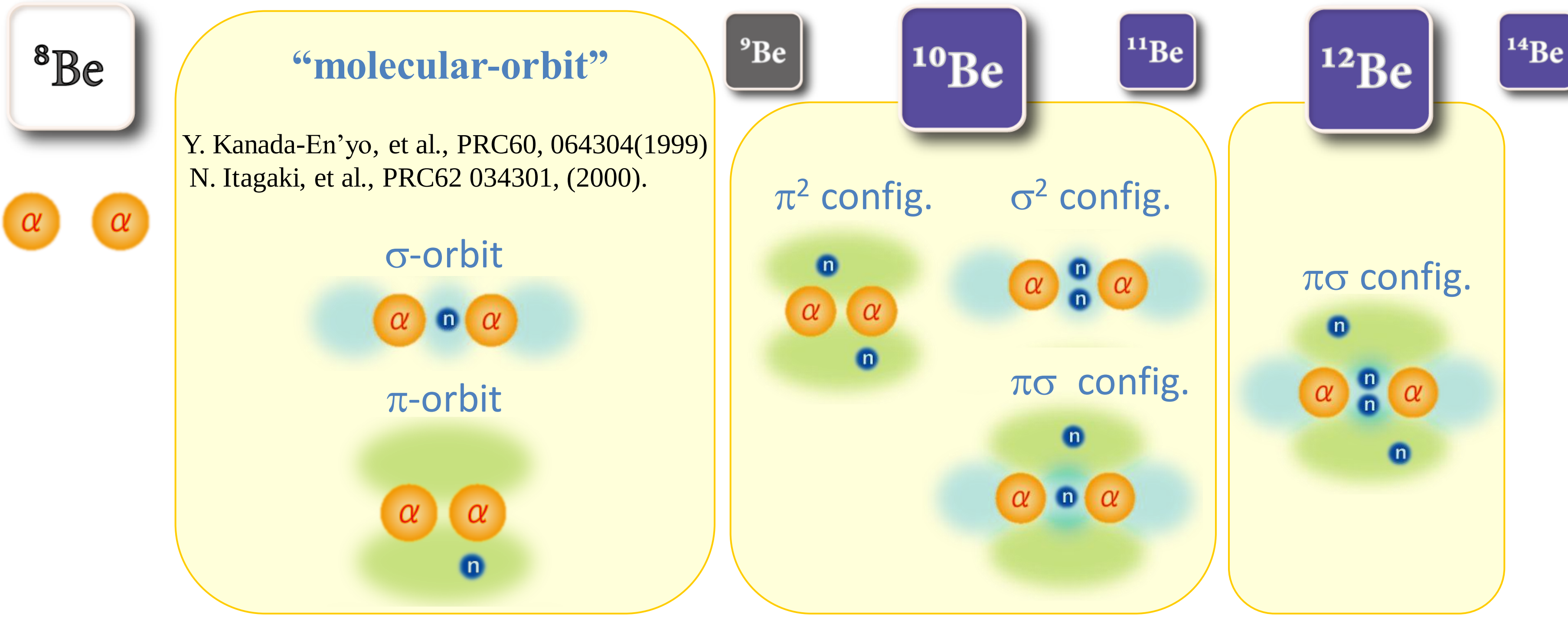
- **Exotic structure exists in the ground state regions**



Topic 2: Structure of neutron-rich nuclei

Ex.) Be isotopes

- **Exotic structure exists in the ground state regions**
- **Be isotopes have a 2α cluster structure**
  - 2α cluster structure is changed depending on the neutron number



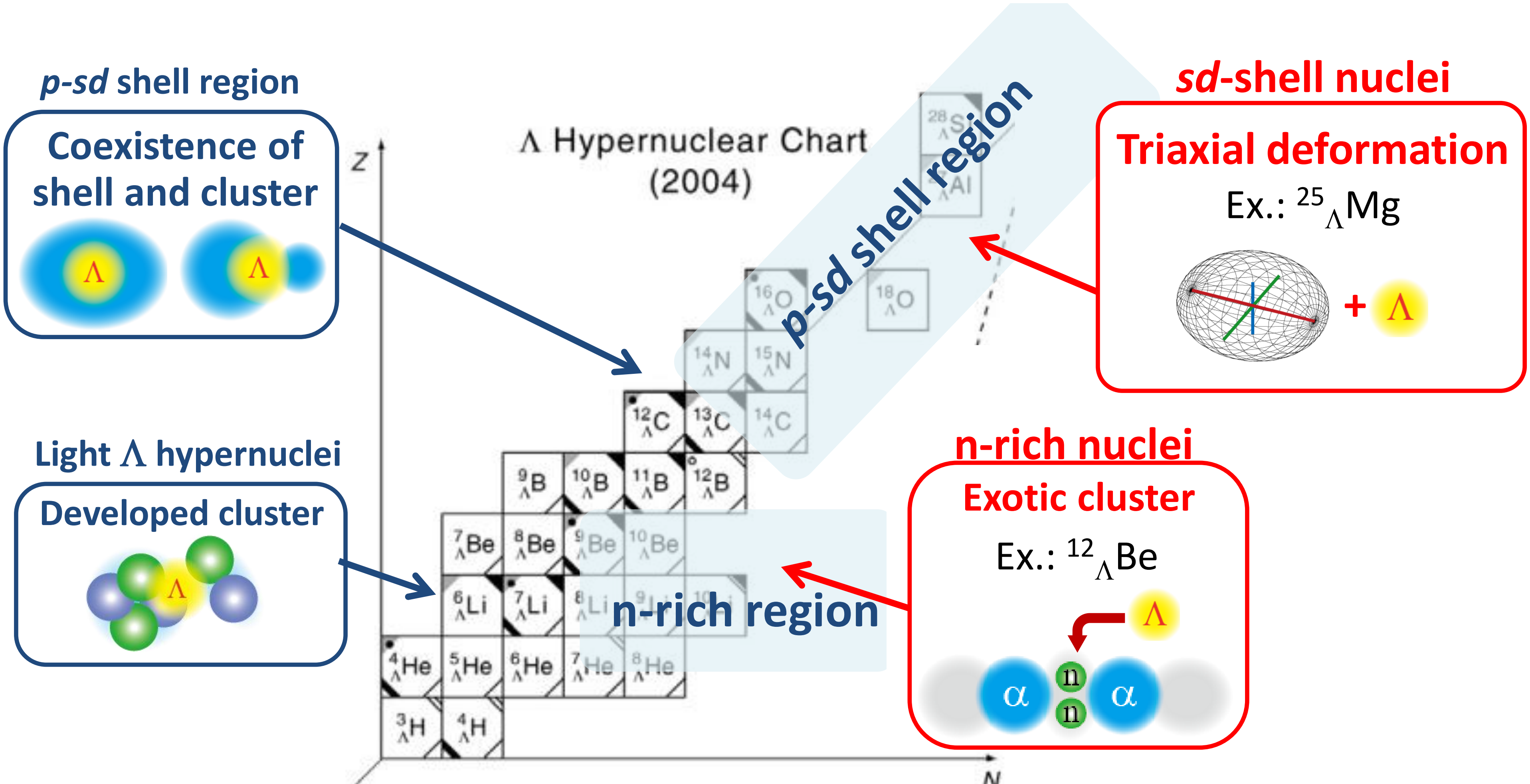
What is happen by adding a Λ to these exotic cluster structure ?

Our task: to reveal structure changes in n-rich $^{12}{}_{\Lambda}$Be

Λ Hypernuclei chart will be extended

“Structure of Λ hypernuclei”

How does a Λ particle modify structures of *p-sd* shell/n-rich nuclei ?



Our method: antisymmetrized molecular dynamics (AMD)

Hypernuclear chart: taken from O. Hashimoto and H. Tamura, PPNP **57**(2006),564.

Theoretical Framework: HyperAMD

M.Isaka, *et al.*, PRC**83**(2011) 044323
M. Isaka, *et al.*, PRC**83**(2011) 054304

We extended the AMD to hypernuclei

HyperAMD (Antisymmetrized Molecular Dynamics for hypernuclei)

◆Hamiltonian

$$\hat{H} = \hat{T}_N + \hat{V}_{NN} + \hat{T}_\Lambda + \hat{V}_{\Lambda N} - \hat{T}_g$$

NN : Gogny D1S

ΛN : YNG interactions (NF[1], NSC97f[2])

◆Wave function

- **Nucleon part : Slater determinant**

  Spatial part of single particle w.f. is described as Gaussian packet

$$\varphi_N(\vec{r}) = \frac{1}{\sqrt{A!}} \det\left[\varphi_i(\vec{r}_j)\right]$$

$$\varphi_i(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \nu_\sigma (r - Z_i)_\sigma^2\right] \chi_i \eta_i \qquad \chi_i = \alpha_i \chi_\uparrow + \beta_i \chi_\downarrow$$

- **Single-particle w.f. of Λ hyperon:**

  Superposition of Gaussian packets

$$\varphi_\Lambda(r) = \sum_m c_m \varphi_m(r)$$

$$\varphi_m(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \mu\nu_\sigma (r - z_m)_\sigma^2\right] \chi_m \qquad \chi_m = a_m \chi_\uparrow + b_m \chi_\downarrow$$

- **Total w.f. :**

$$\psi(\vec{r}) = \sum_m c_m \varphi_m(r_\Lambda) \otimes \frac{1}{\sqrt{A!}} \det\left[\varphi_i(\vec{r}_j)\right]$$

[1] Y. Yamamoto, *et al.*, PTP Suppl. **117** (1994), 361., [2] Y. Yamamoto, private communication

Theoretical Framework: HyperAMD

M.Isaka, *et al.*, PRC**83**(2011) 044323
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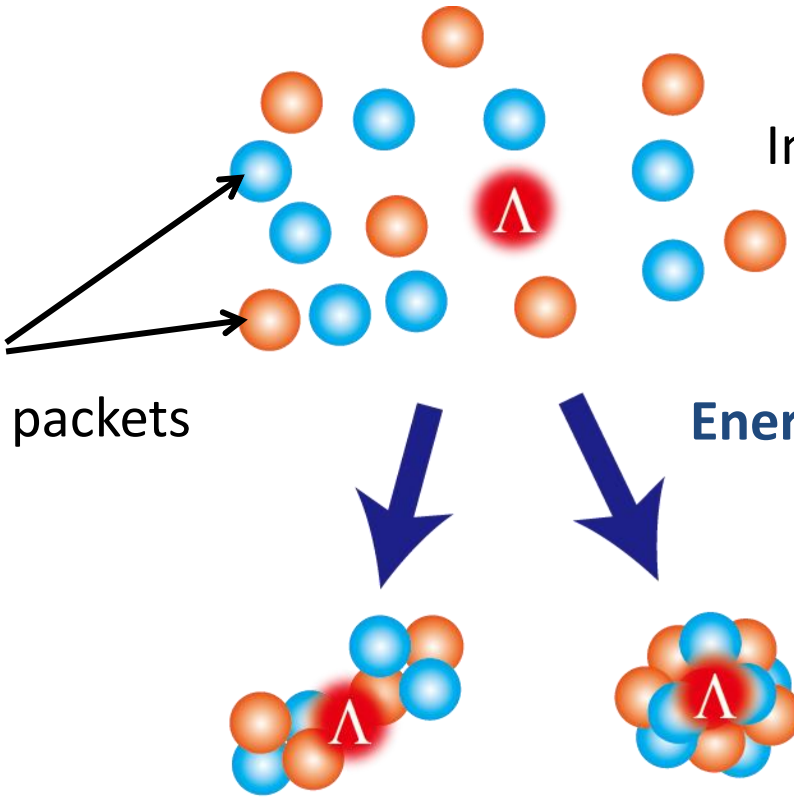
◆ Procedure of the calculation

Variational Calculation

- Imaginary time development method $\frac{dX_i}{dt} = \frac{\kappa}{\hbar} \frac{\partial H^{\pm}}{\partial X_i^*}$ $\kappa < 0$
- Variational parameters: $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, \nu_i, c_i$

Initial w.f.: randomly generated

Described by
Gaussian wave packets



Energy variation

Cluster or deformed states

Theoretical Framework: HyperAMD

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↓

Angular Momentum Projection

$$\left|\Phi_K^s; JM\right\rangle = \int d\Omega D_{MK}^{J*}(\Omega) R(\Omega) \left|\Phi^{s+}\right\rangle$$

↓

Generator Coordinate Method(GCM)

- Superposition of the w.f. with different configuration
- Diagonalization of $H_{sK,s'K'}^{J\pm}$ and $N_{sK,s'K'}^{J\pm}$

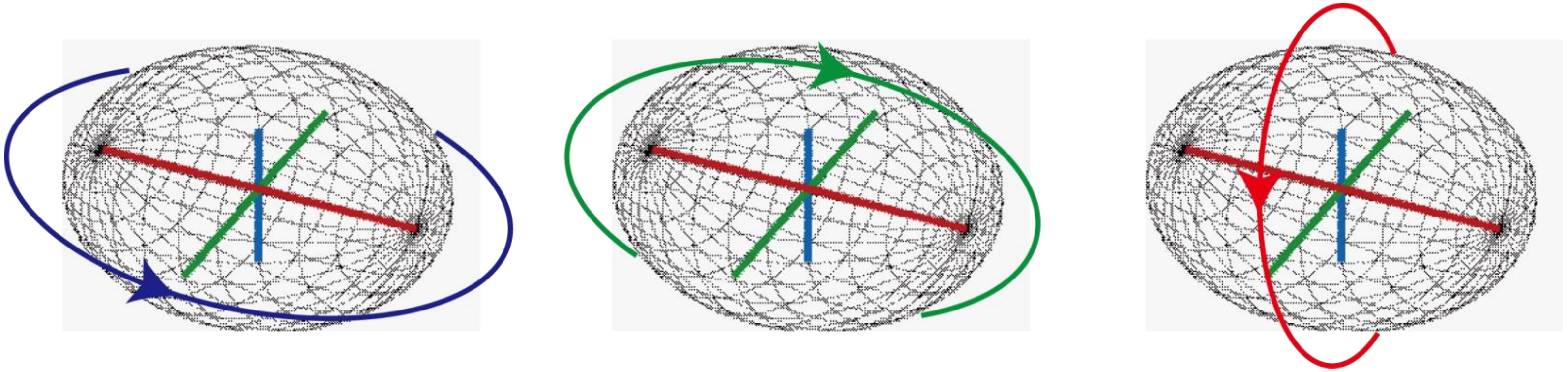
$$H_{sK,s'K'}^{J\pm} = \left\langle \Phi_K^s; J^{\pm}M \middle| \hat{H} \middle| \Phi_{K'}^{s'}; J^{\pm}M \right\rangle$$

$$N_{sK,s'K'}^{J\pm} = \left\langle \Phi_K^s; J^{\pm}M \middle| \Phi_{K'}^{s'}; J^{\pm}M \right\rangle$$

$$\left|\Psi^{J\pm M}\right\rangle = \sum_{sK} g_{sK} \left|\Phi_K^s; J^{\pm}M\right\rangle$$

1. Triaxial deformation

To identify triaxial deformation by using Λ in p orbit



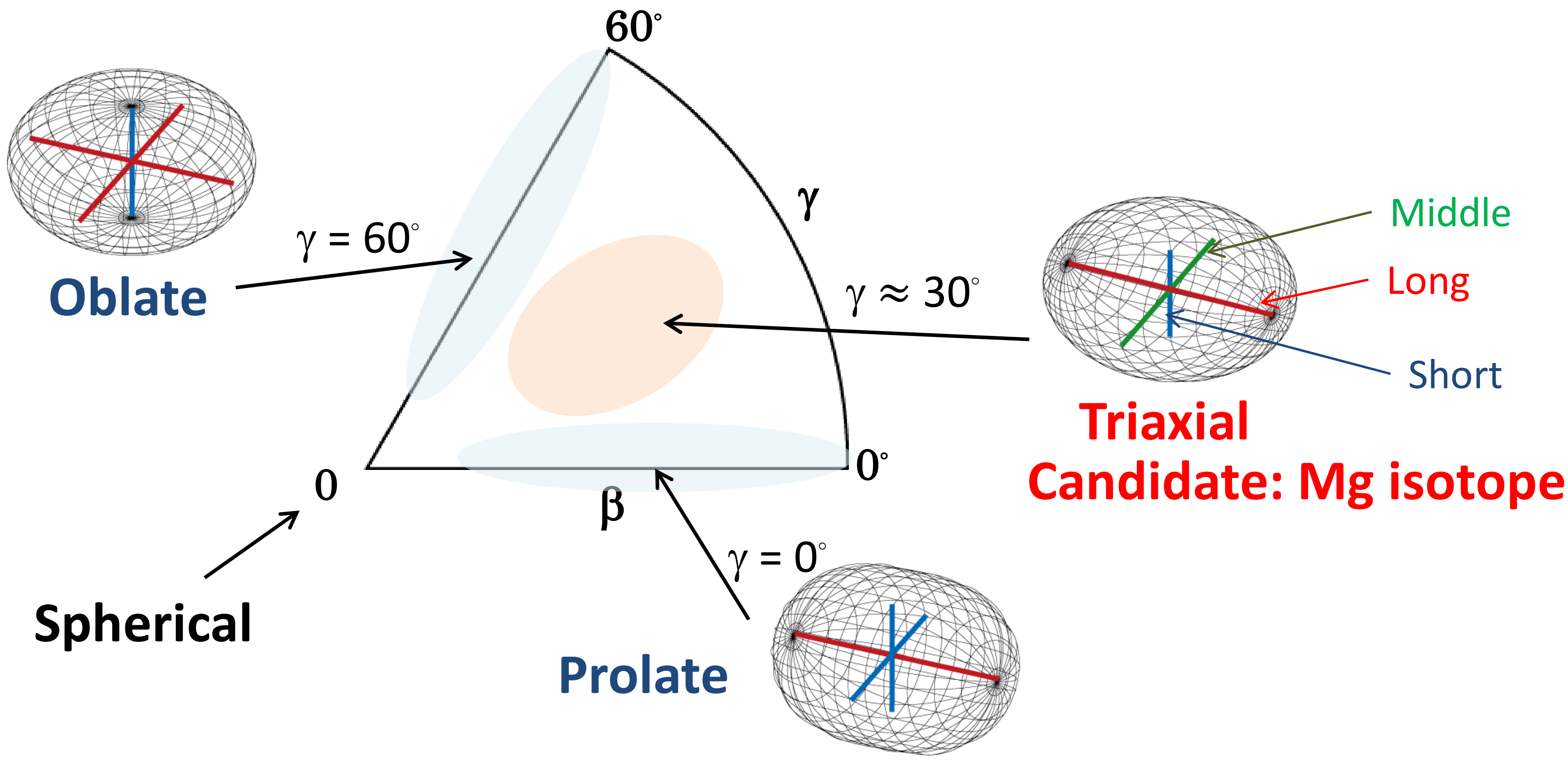
Example: ^{24}Mg (,^{26}Mg)

Based on

M. Isaka, M. Kimura, A. Dote, and A. Ohnishi, PRC**87**, 021304(R) (2013)

Deformation of nuclei

- **Nuclear deformations are described by quadrupole deformation parameters (β, γ)**



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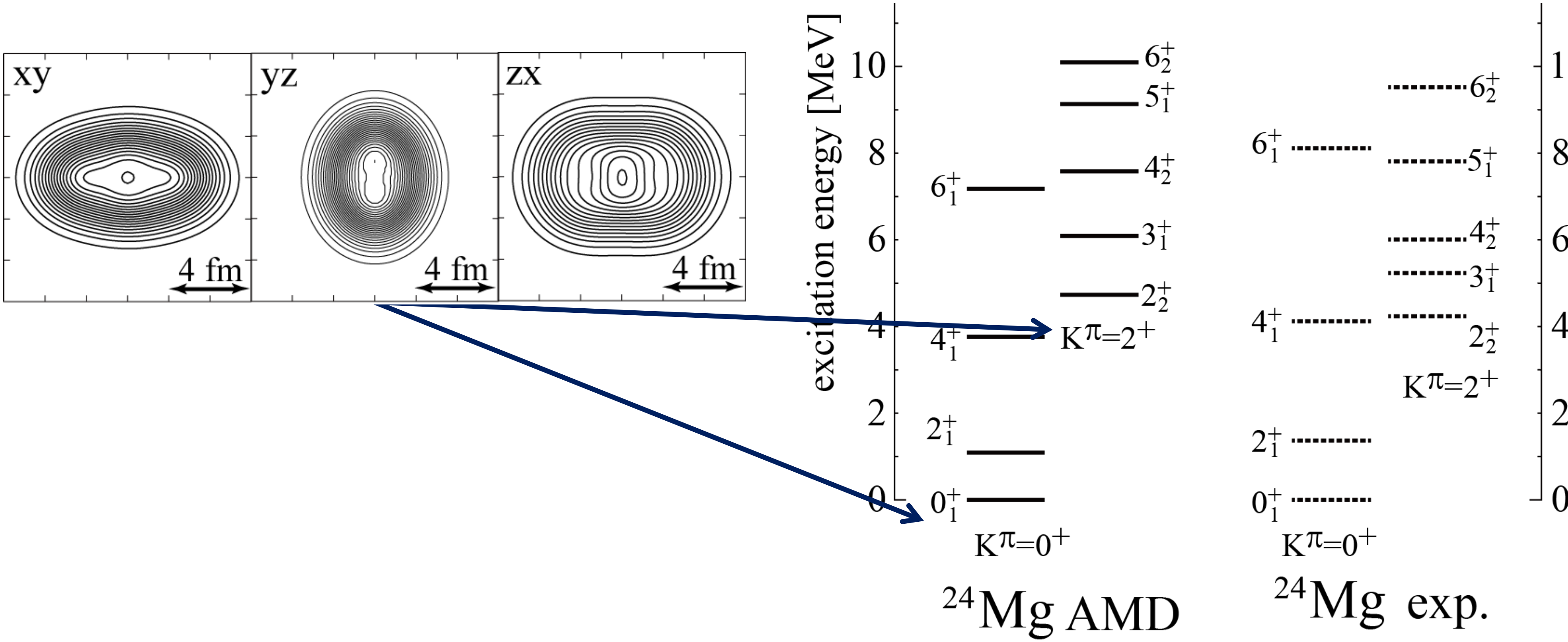
Triaxial deformed nuclei are not many, Mg isotopes are the candidates

Ex.) ^{24}Mg

- Largely deformed
- Low-lying 2nd 2^+ indicates having the triaxial deformation



"<u>Λ in *p* orbit</u> can be a probe to study nuclear (triaxial) deformation"

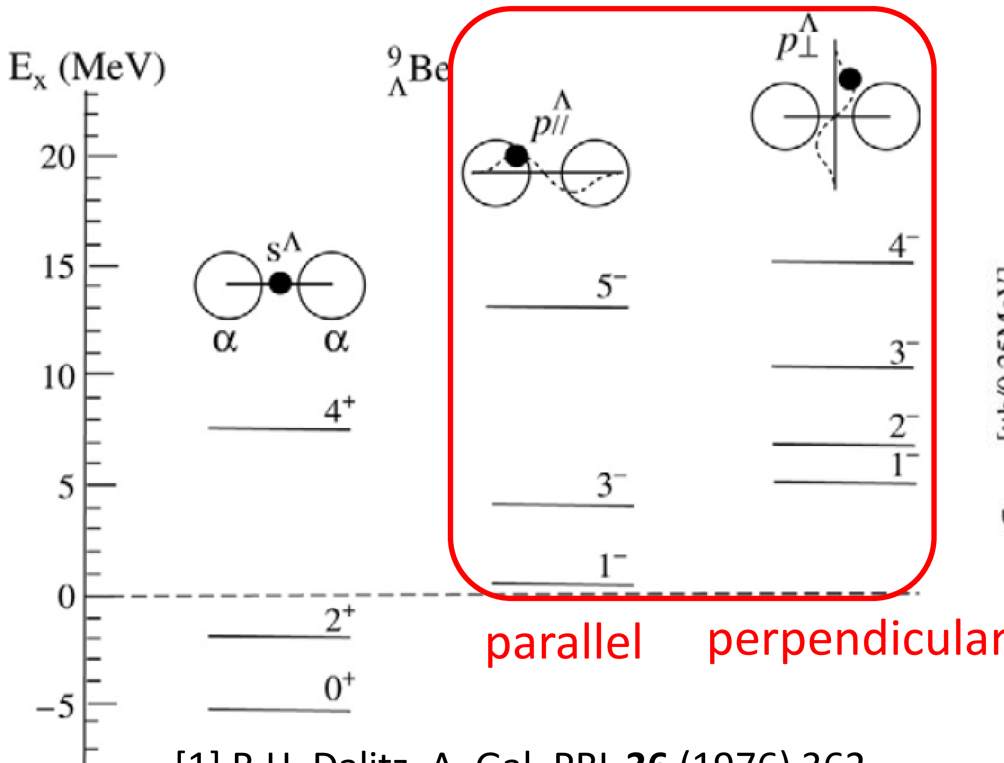
Example: *p*-states of $^{9}_{\Lambda}$Be

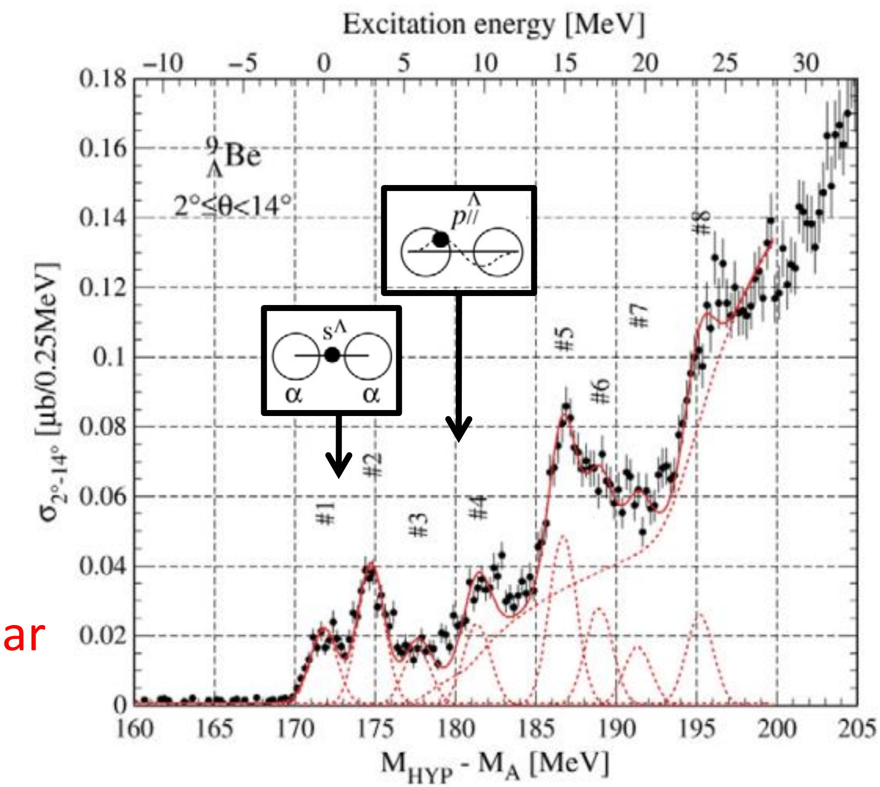
$^{9}_{\Lambda}$Be: axially symmetric 2α clustering

Two bands will be generated as *p*-states [1,2]

- **Anisotropic *p* orbit of Λ hyperon**
- **Axial symmetry of 2α clustering**

→ ***p*-orbit parallel to/perpendicular to the 2α clustering**



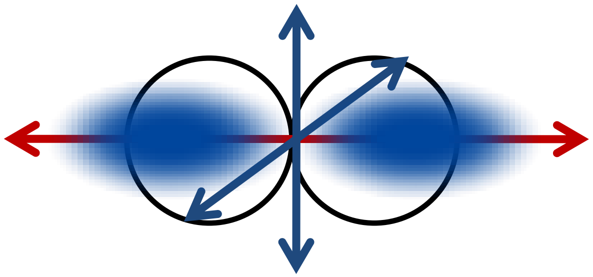


[1] R.H. Dalitz, A. Gal, PRL **36** (1976) 362.
[2] H. Bando, et al., PTP **66** (1981) 2118.; T. Motoba, et al., PTP **70** (1983) 189
[3] O. Hashimoto et al., NPA **639** (1998) 93c.

Split of p-state in $^{9}_{\Lambda}Be$

◆ $^{9}_{\Lambda}Be$ with 2α cluster structure

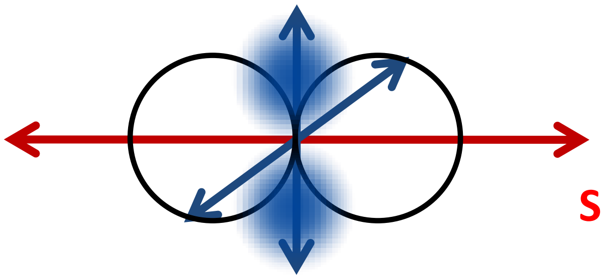
<u>*p* orbit parallel to 2α (long axis)</u>



Large overlap

Deeply bound

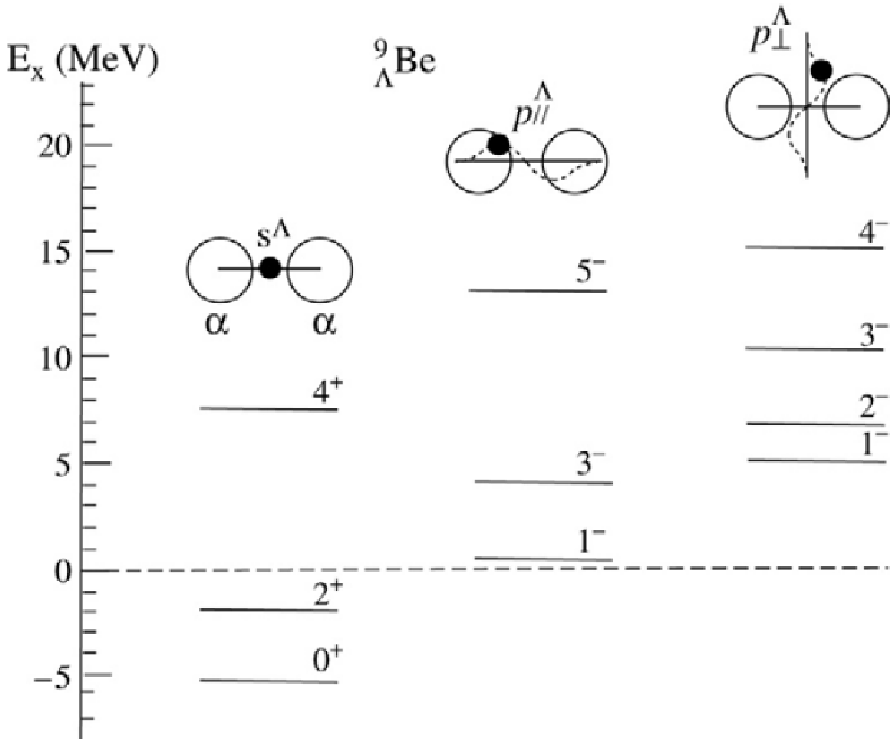
<u>*p* orbit perpendicular to 2α (short axes)</u>



Small overlap

Shallowly bound

Split corresponding to long/short axes



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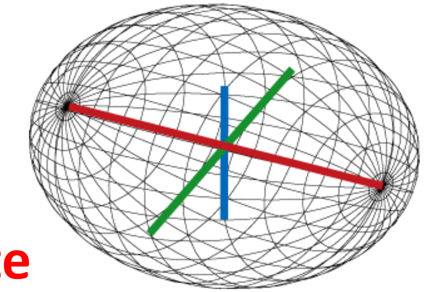
***p*-states splits into 2 bands corresponding on directions of *p*-orbits**

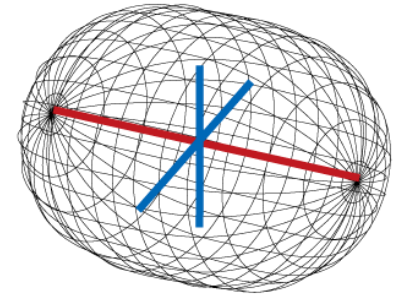
Triaxial deformation

If ^{24}Mg is triaxially deformed nuclei

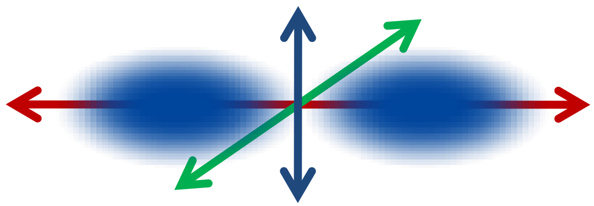
→ *p*-states split into 3 different state



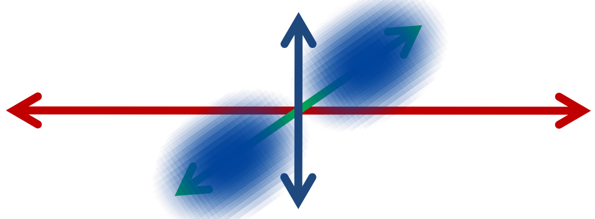
Triaxial deformation



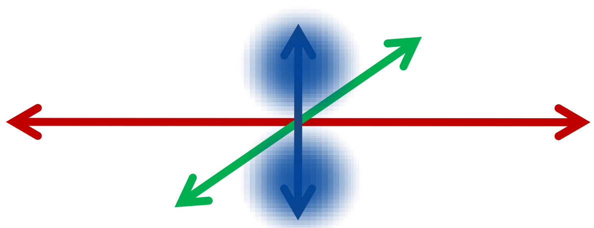
Prolate deformation



Large overlap leads to deep binding



Middle

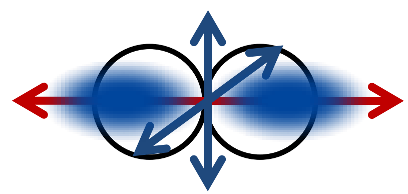


Small overlap leads to shallow binding

cf. prolate deformation

Ex.) $^{9}_{\Lambda}$Be

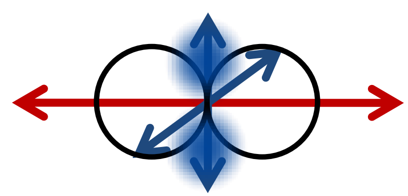
***p* orbit parallel to 2α (long axis)**



Large overlap

Deeply bound

***p* orbit perpendicular to 2α (short axes)**



Small overlap

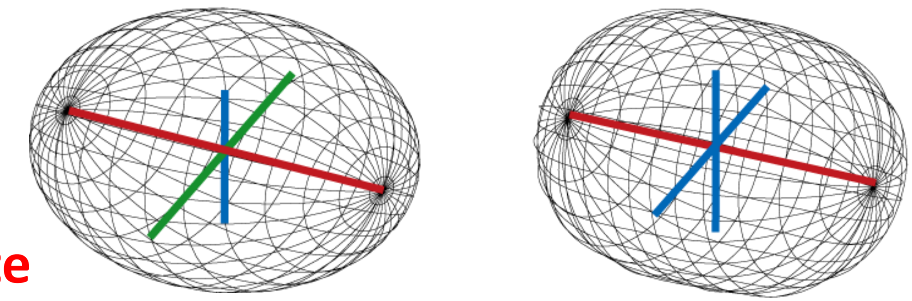
Shallowly bound

Split corresponding to long/short axes

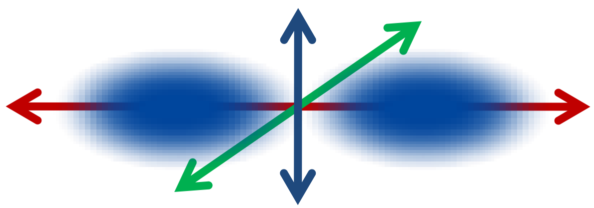
Triaxial deformation

If ^{24}Mg is triaxially deformed nuclei

→ *p*-states split into 3 different state



Triaxial deformation Prolate deformation



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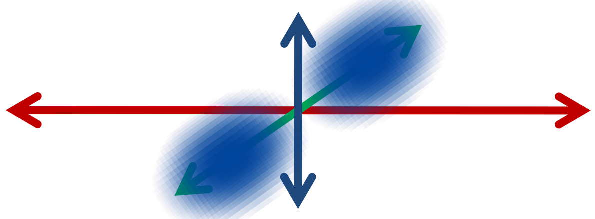

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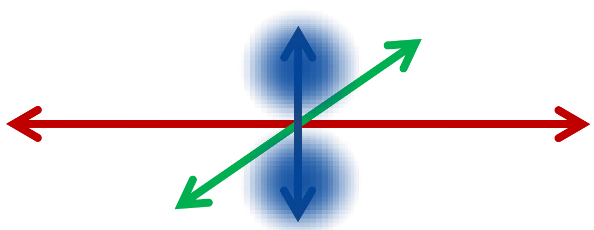
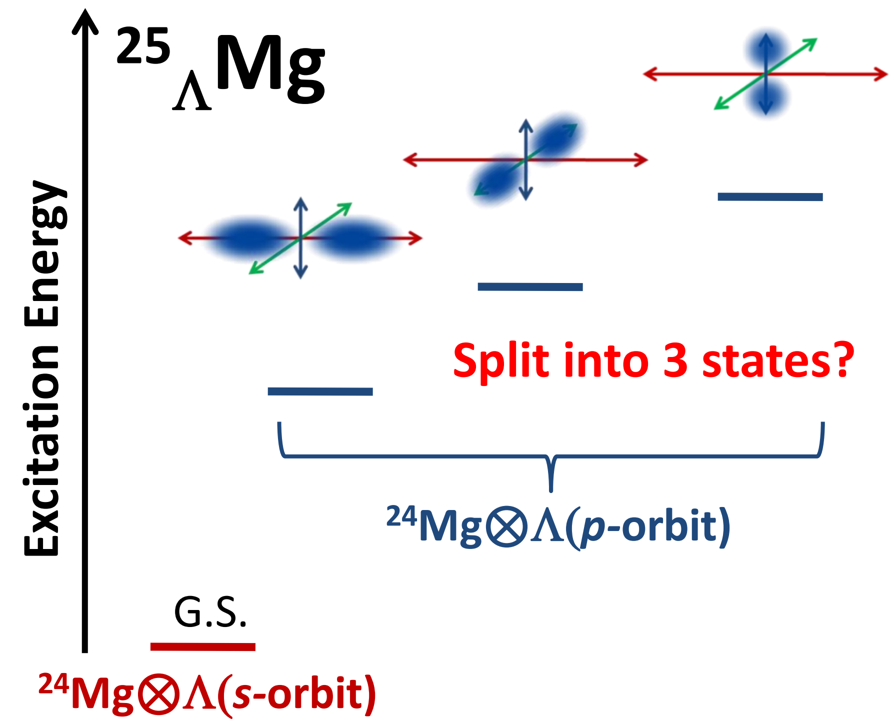


Small overlap leads to shallow binding



Observing the 3 different *p*-states is strong evidence of triaxial deformation

Our (first) task: To predict the level structure of the *p*-states in $^{25}_{\Lambda}$Mg

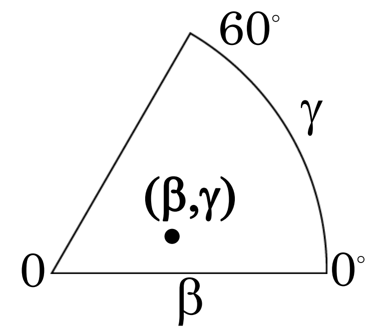
AMD calculation for $^{25}{}_{\Lambda}$Mg

◆ Procedure of the calculation

Variational Calculation

- Imaginary time development method: $\frac{dX_i}{dt} = \frac{\kappa}{\hbar}\frac{\partial H^{\pm}}{\partial X_i^*}$ $\quad \kappa < 0$
- Variational parameters: $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, \nu_i, c_i$

With constraints on (β,γ) and Λ single particle w.f.



Energy surface on (β,γ) plane

Single particle states of Λ

Angular Momentum Projection

$$\left|\Phi_K^s; JM\right\rangle = \int d\Omega D_{MK}^{J*}(\Omega) R(\Omega) \left|\Phi^{s+}\right\rangle$$

Generator Coordinate Method(GCM)

- Superposition of the w.f. with different configuration
- Diagonalization of $H_{sK,s'K'}^{J\pm}$ and $N_{sK,s'K'}^{J\pm}$

$$H_{sK,s'K'}^{J\pm} = \left\langle \Phi_K^s; J^{\pm}M \middle| \hat{H} \middle| \Phi_{K'}^{s'}; J^{\pm}M \right\rangle$$

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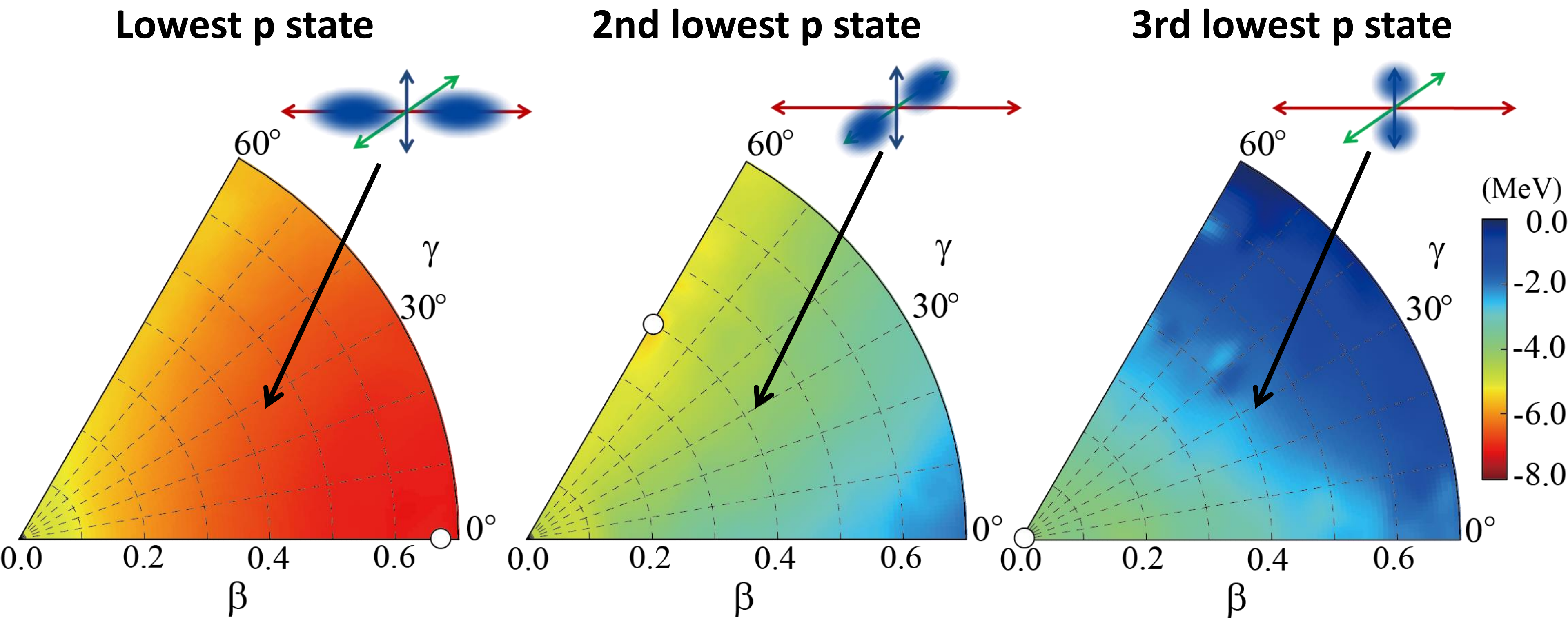
Excitation spectra of $^{25}{}_{\Lambda}$Mg

Results： Single particle energy of Λ hyperon

◆ **Λ single particle energy on (β, γ) plane**

$$\varepsilon_{\Lambda}(\beta,\gamma)=E_{hyp}(\beta,\gamma)-E_{N}(\beta,\gamma)$$

${}^{25}_{\Lambda}$Mg (AMD, Λ in *p* orbit)



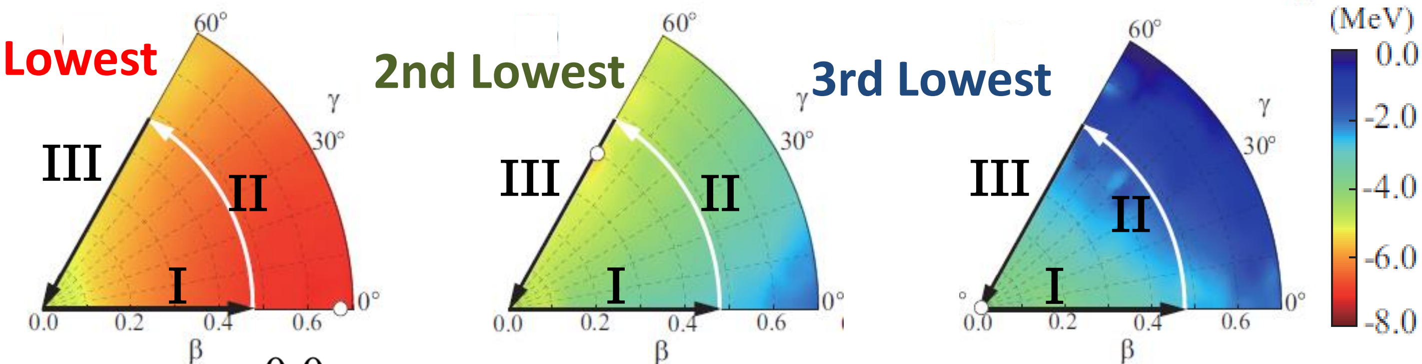
- **Single particle energy of Λ hyperon is different from each p state**
  - **This is due to the difference of overlap between Λ and nucleons**

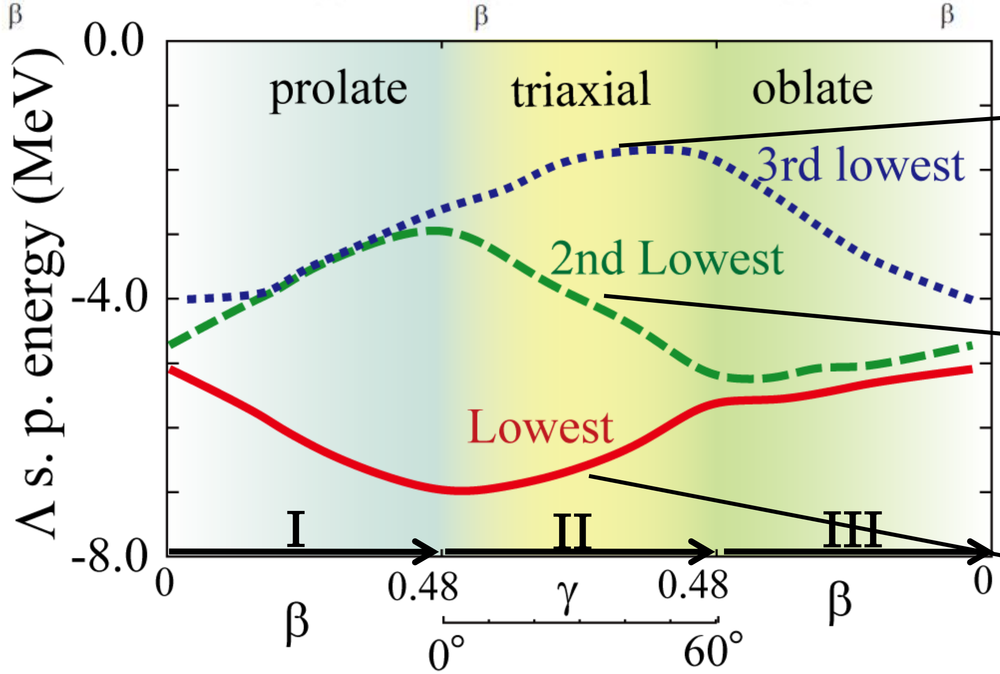
Results: Single particle energy of Λ hyperon ε_Λ

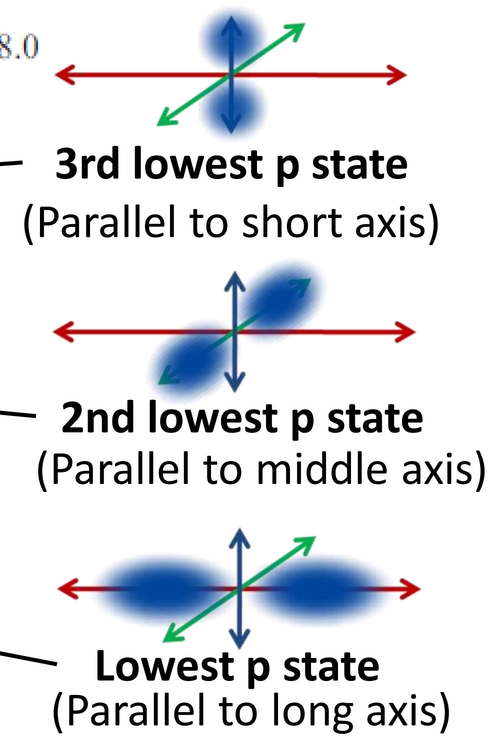
$^{25}_{\Lambda}$Mg (AMD)

$$\varepsilon_\Lambda(\beta,\gamma) = E_{hyp}(\beta,\gamma) - E_N(\beta,\gamma)$$

3 *p*-states with different spatial distributions of Λ in *p*-orbit



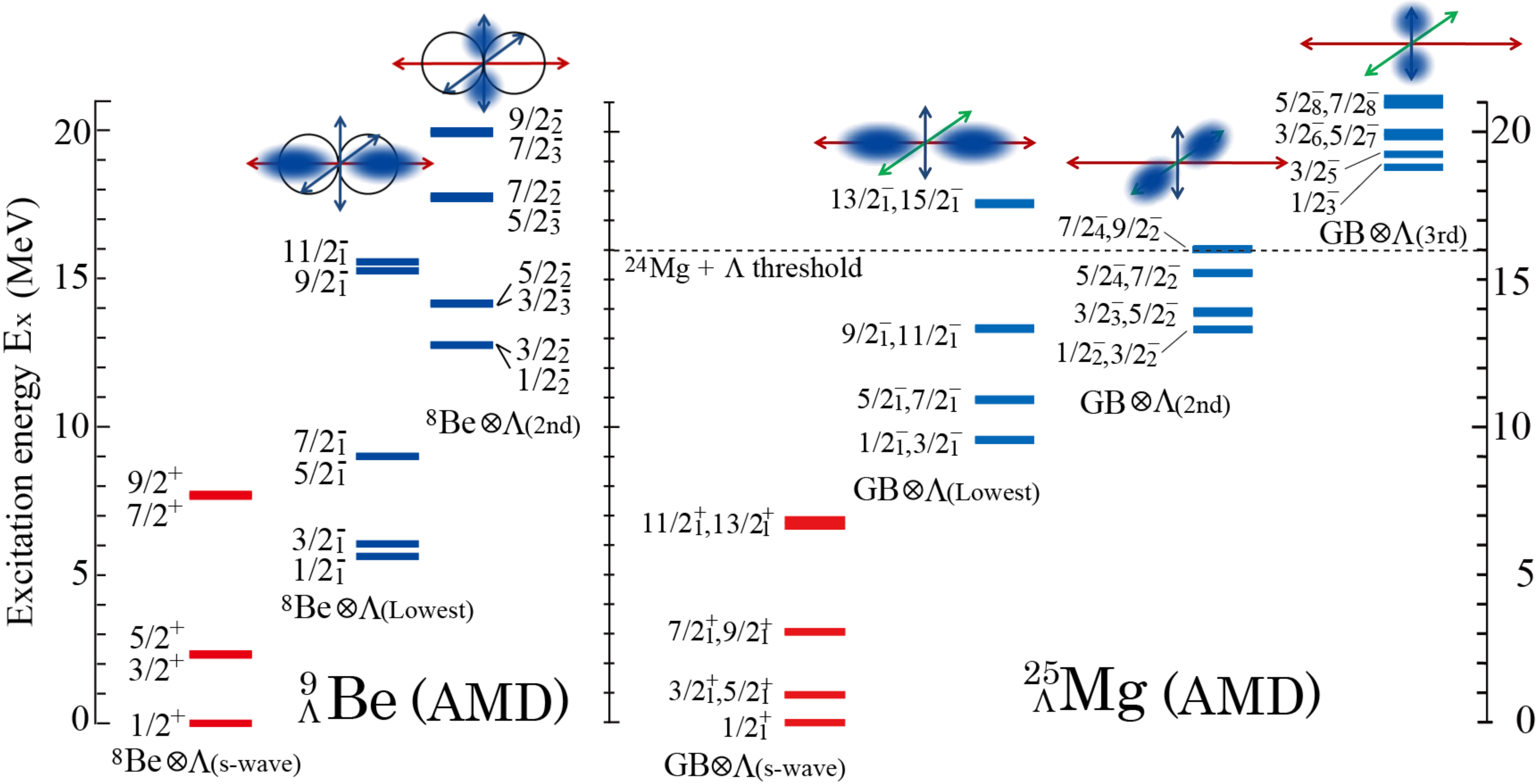




Λ s. p. energy is different from each other with triaxial deformation

Results: Excitation spectra

- **3 bands are obtained by Λ hyperon in *p*-orbit** → **Splitting of the *p* states**
  - **^{24}Mg⊗Λp(lowest), ^{24}Mg⊗Λp(2nd lowest), ^{24}Mg⊗Λp(3rd lowest)**



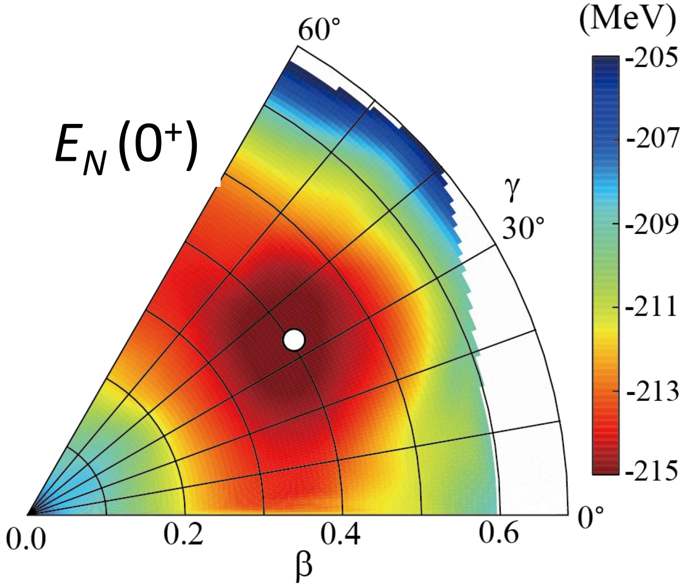
Lowest threshold $^{21}_{\Lambda}\mathrm{Ne}+\Lambda$: in between 8.3 and 12.5 MeV

Triaxial deformation of ^{26}Mg ($^{27}{}_{\Lambda}$Mg)

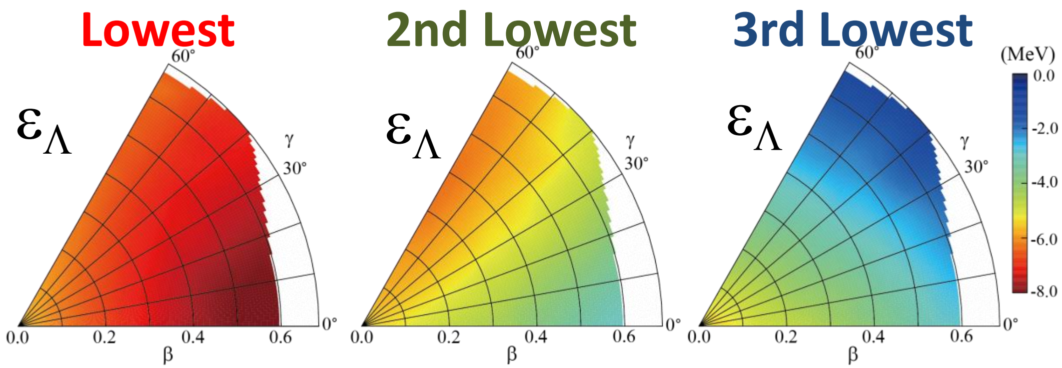
Similar discussions can be possible in $^{27}{}_{\Lambda}$Mg

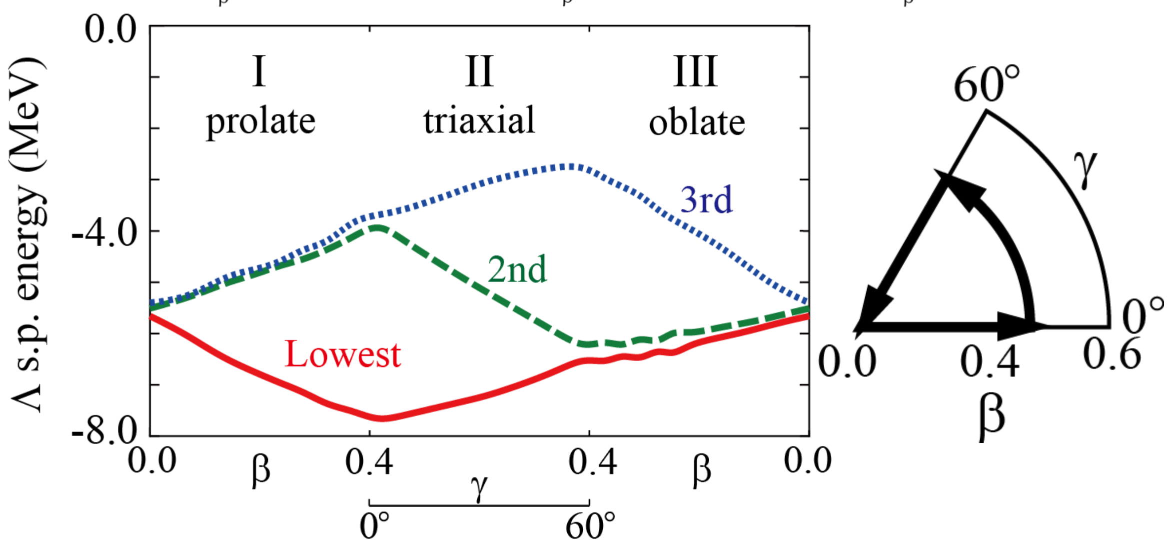
^{26}Mg (AMD)

Energy surface (0^+ state)



$^{27}{}_{\Lambda}$Mg (AMD) $\varepsilon_\Lambda(\beta,\gamma) = E_{hyp}(\beta,\gamma) - E_N(\beta,\gamma)$

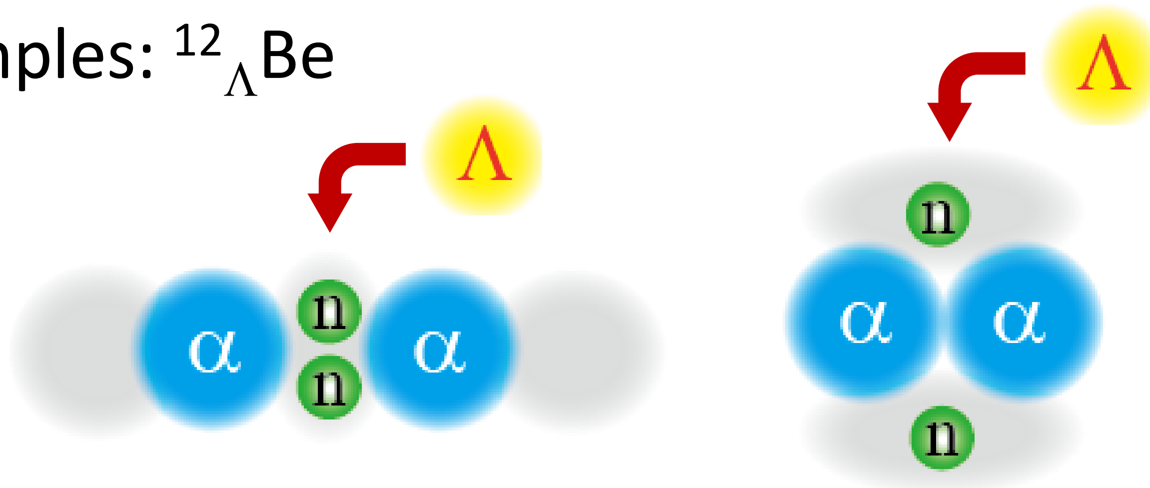




2. neutron-rich Λ hypernuclei

How does Λ modify exotic cluster structure ?

Examples: $^{12}_{\Lambda}\mathrm{Be}$

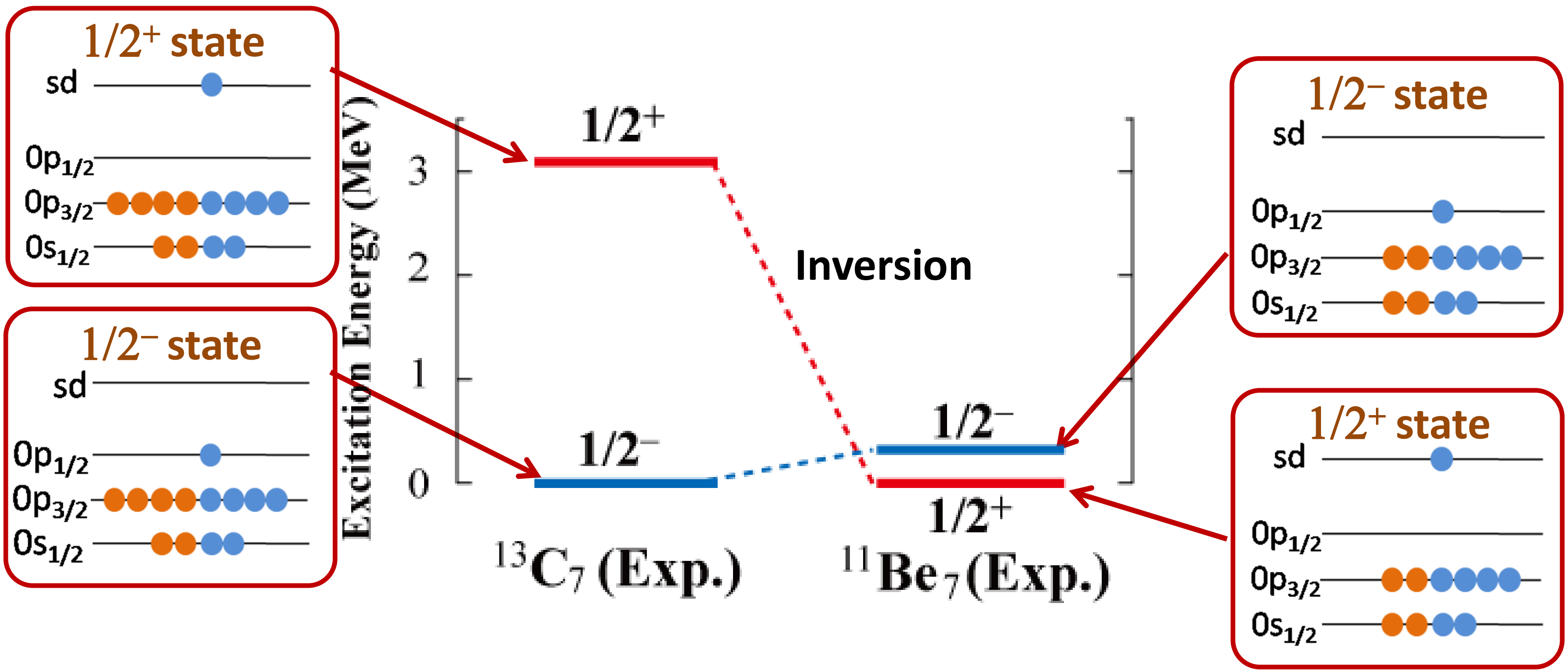


Molecular orbit structure of Be isotopes

H. Homma, M. Isaka and M. Kimura

Exotic structure of ^{11}Be

- **Parity inverted ground state of the $^{11}_{4}Be_7$**
  - **Ground state of ^{11}Be is $1/2^+$,**
    while ordinary nuclei have a $1/2^-$ state as the ground state
    → **Vanishing of the magic number N=8**



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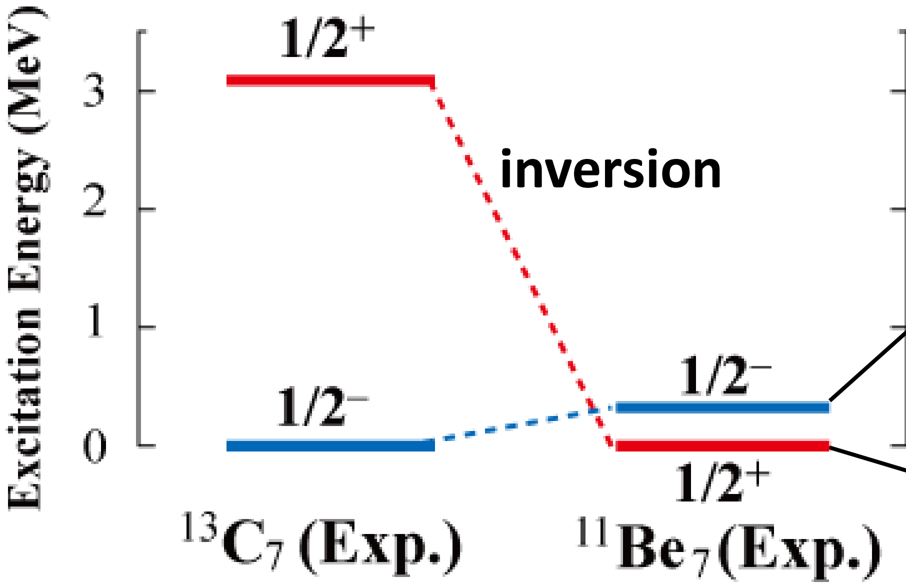
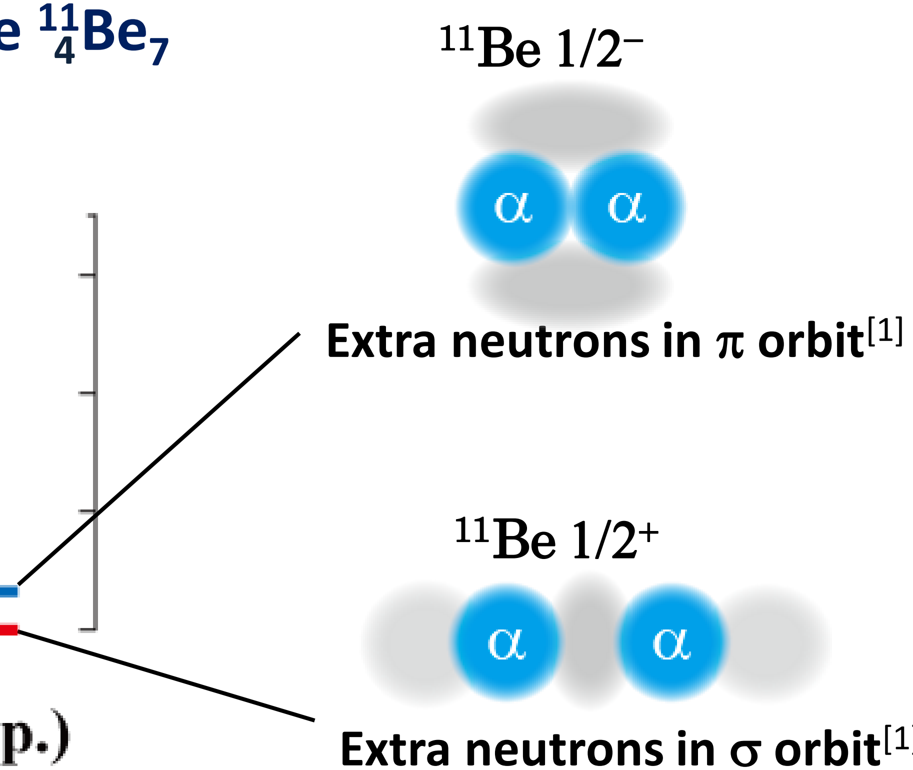
  - **Ground state of ^{11}Be is 1/2$^+$**





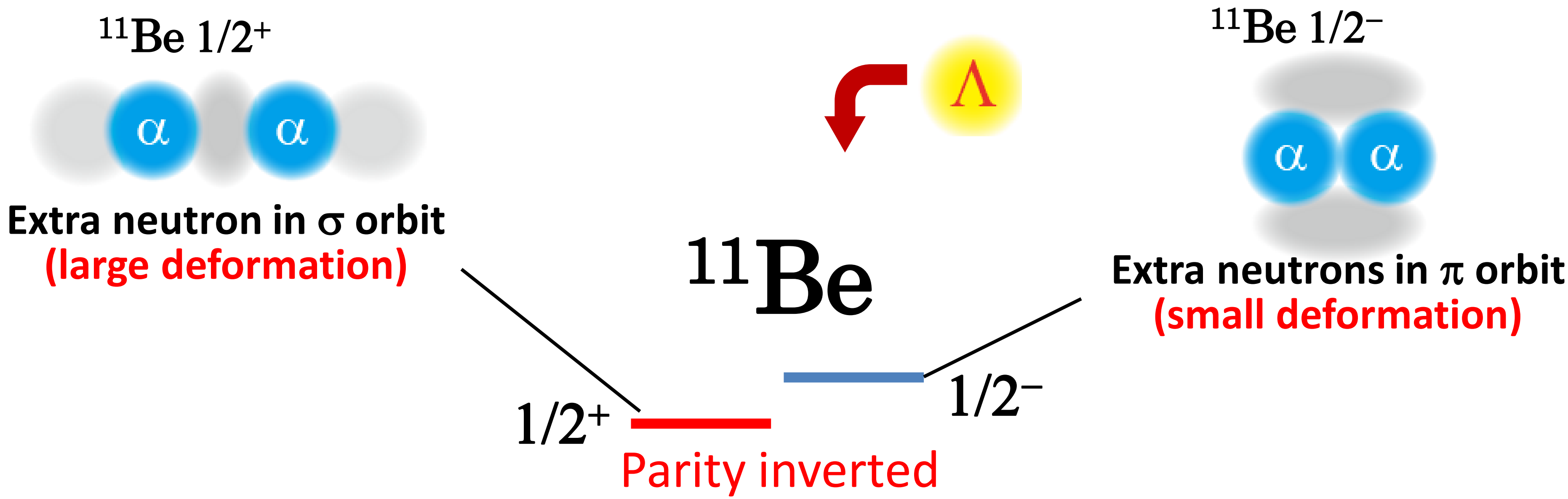
- **Main reason of the parity inversion: molecular orbit structure**
- **^{11}Be has 2α clusters with 3 surrounding neutrons**

⟶ Extra neutrons occupy molecular orbits around the 2α cluster

However, no one observed molecular orbit structure in ^{11}Be

[1] Y. Kanada-En'yo and H. Horiuchi, PRC **66** (2002), 024305.

Structure change in $^{12}_{\Lambda}$Be



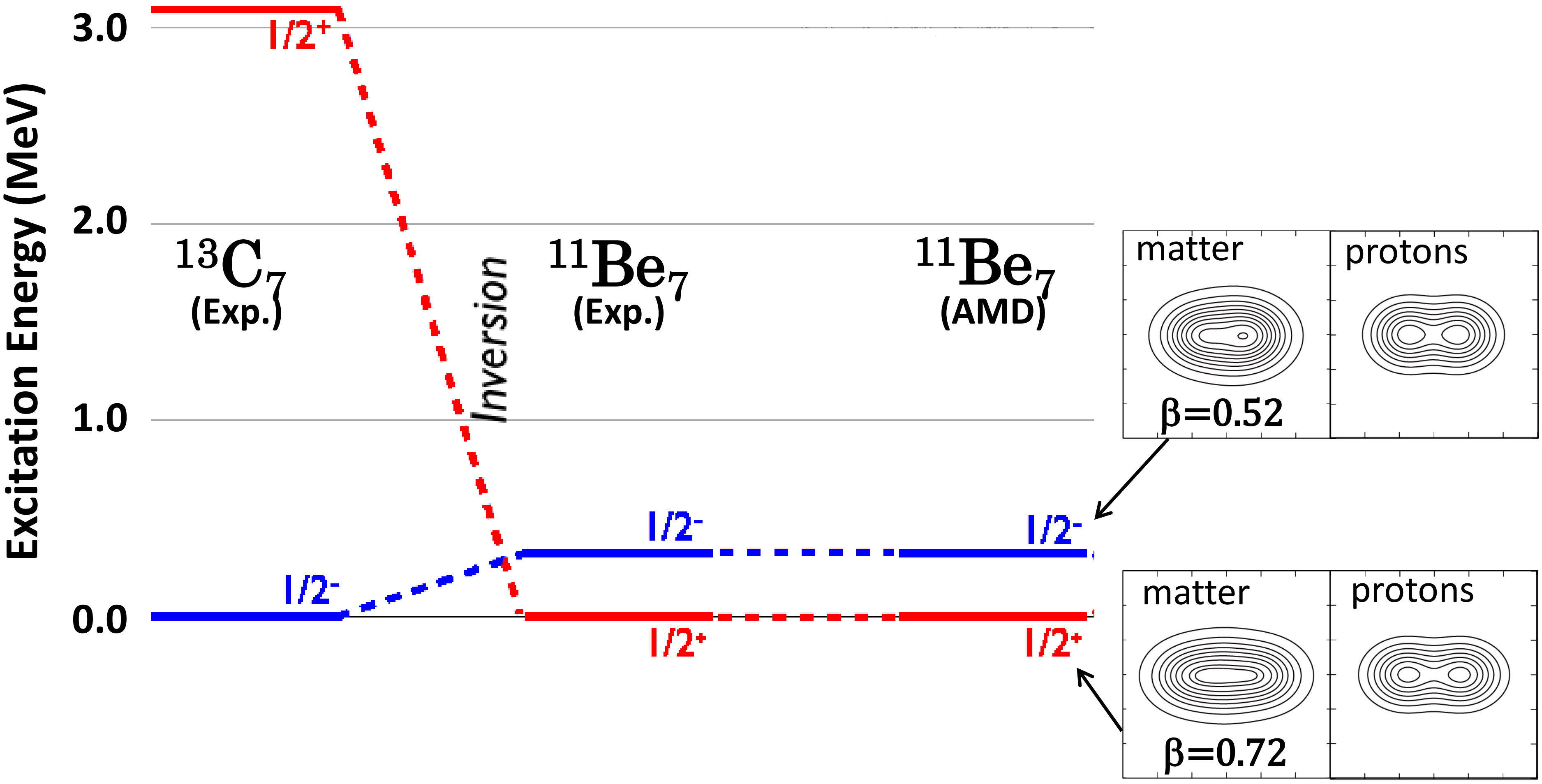
What is happen by Λ in these states with different deformations?

Λ reduces deformations ?

Λ affects parity-inverted ground state ?

AMD results for $^{12}{}_{\Lambda}$Be

- **Ground state of $^{12}{}_{\Lambda}$Be**

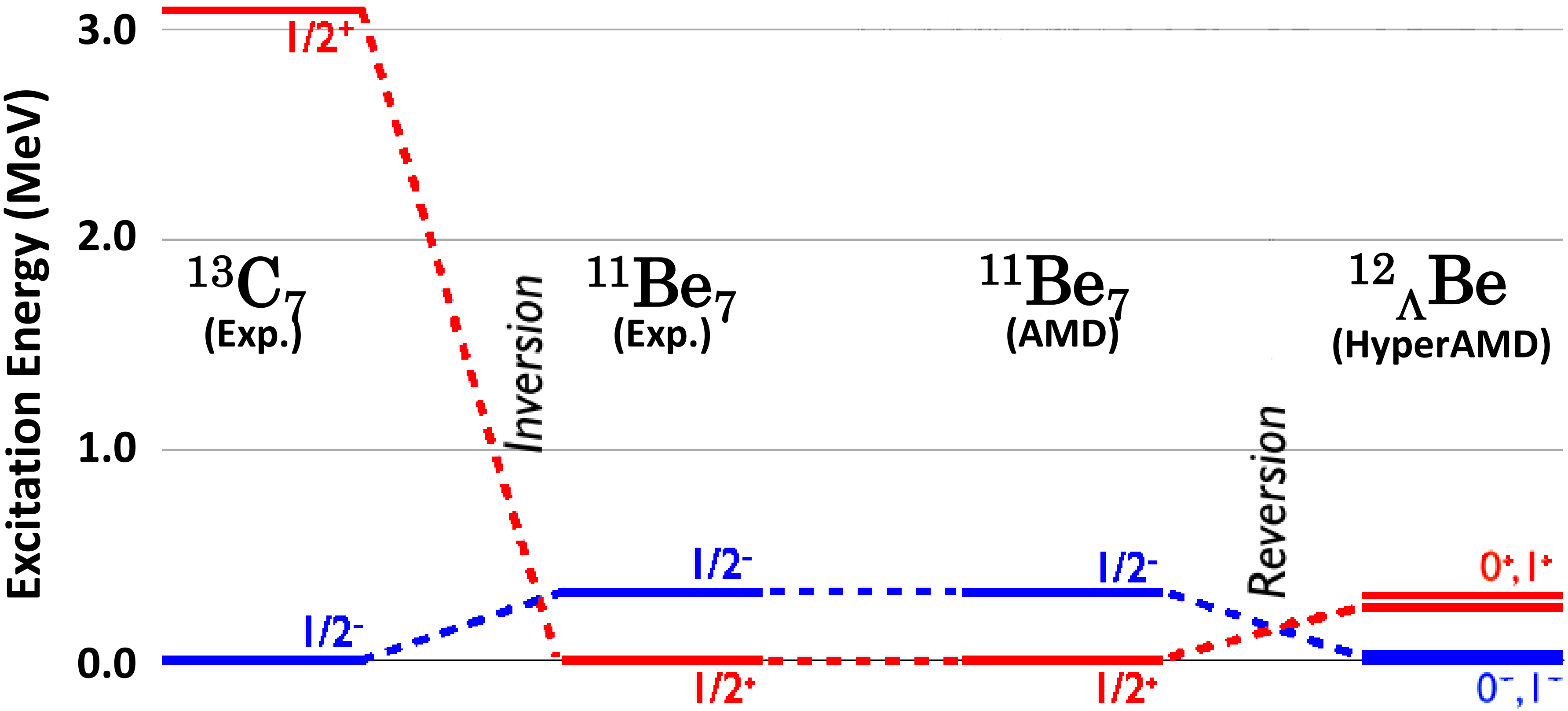


Difference of deformation between 1/2+ and 1/2- states

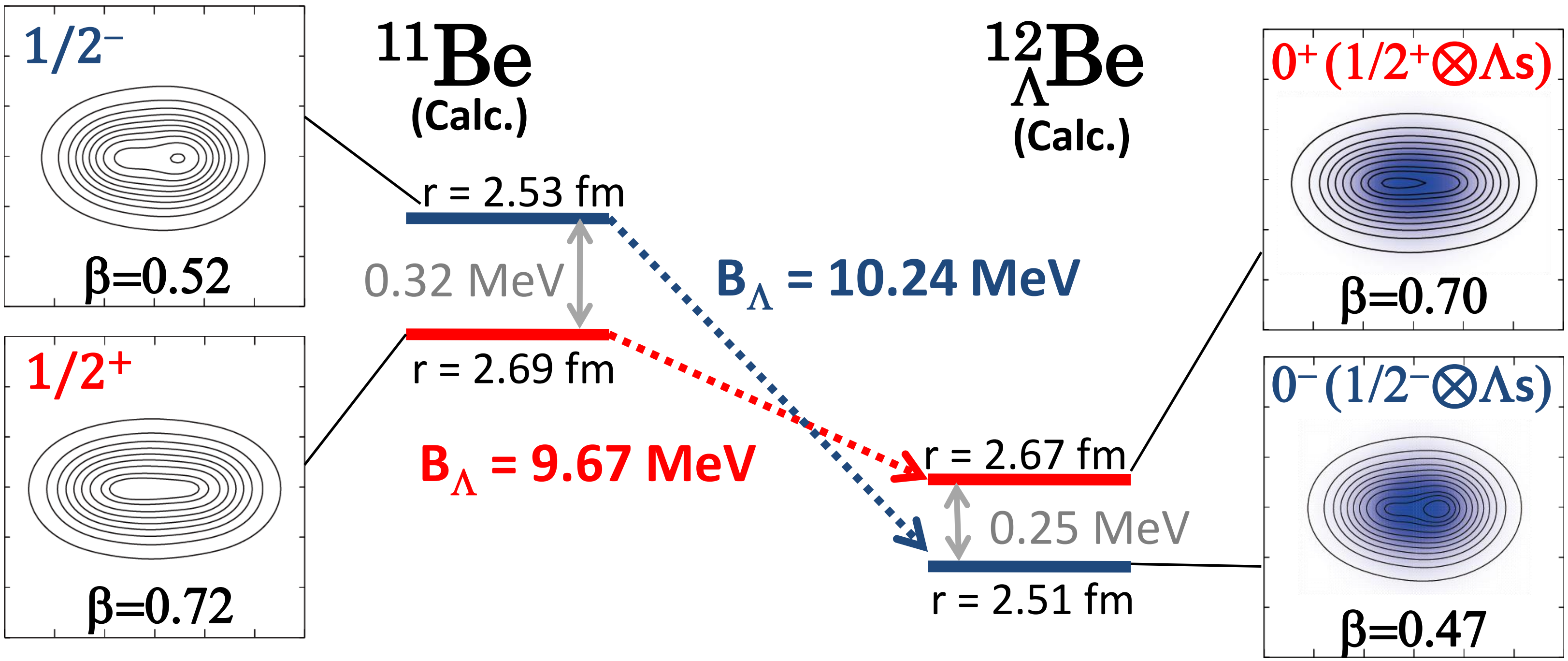
AMD results for $^{12}{}_{\Lambda}Be$

◆ Ground state of $^{12}{}_{\Lambda}Be$

- **The parity reversion of the $^{12}{}_{\Lambda}Be$ g.s. occurs by the Λ hyperon**



Deformation and Λ binding energy



- **Λ slightly reduces deformations, but the deformation is still different**
- **Parity reversion is due to difference of B_Λ associated with difference of deformations**

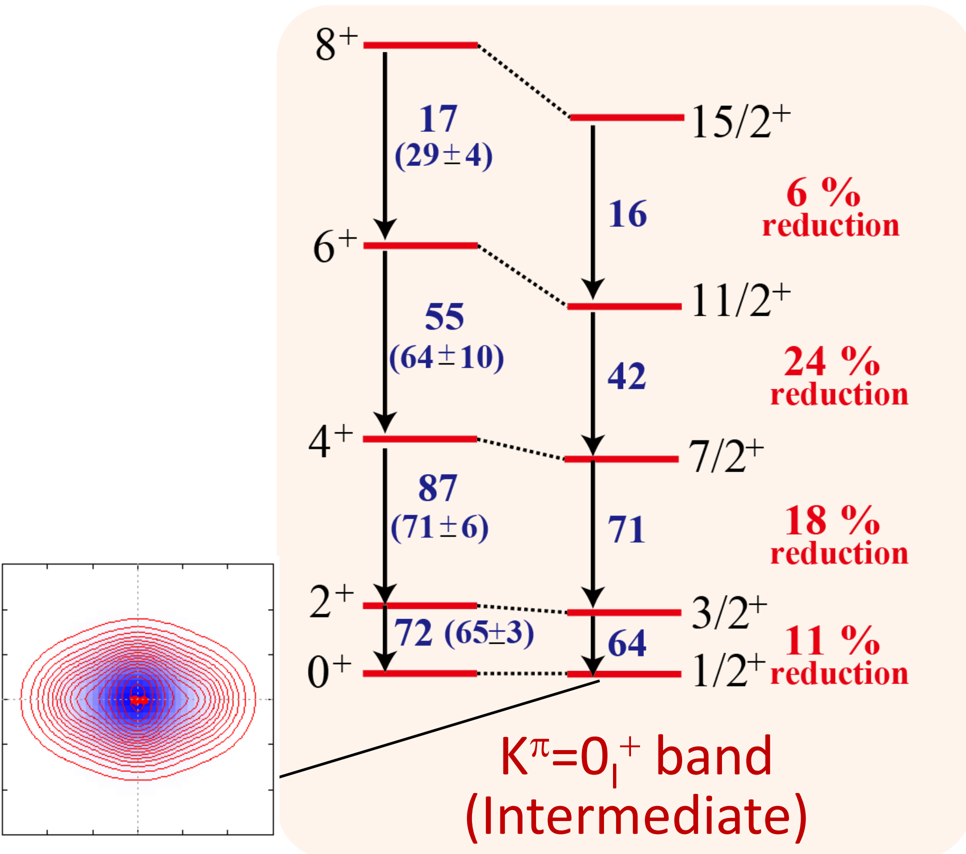
If the parity reversion is observed, we can confirm the difference of the deformations between the 1/2+ and 1/2- states by Λ

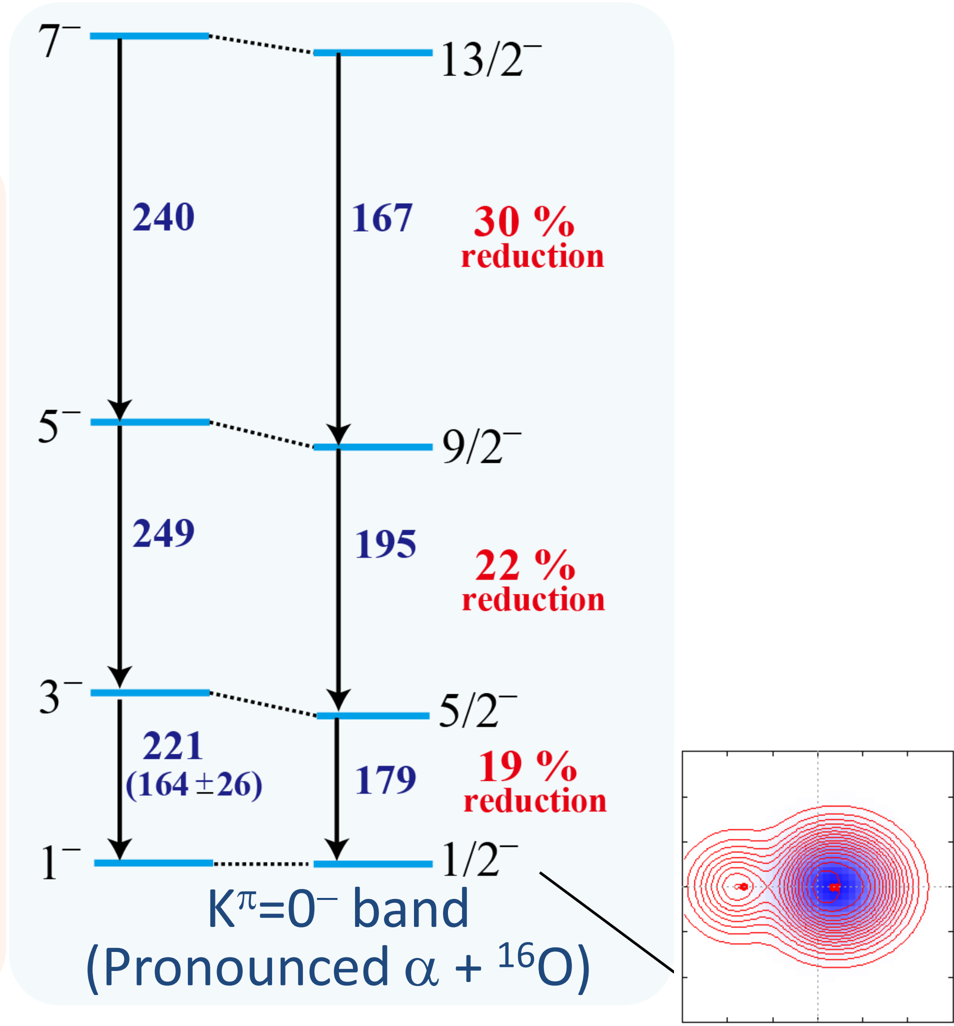
Other example of “impurity effects”

Structure dependence of “impurity effects”

Ex.) $^{21}_{\Lambda}$Ne   Shell-model like and α + ^{16}O + Λ structures coexist

M. Isaka, *et al.*, PRC**83**, 054304(2011)

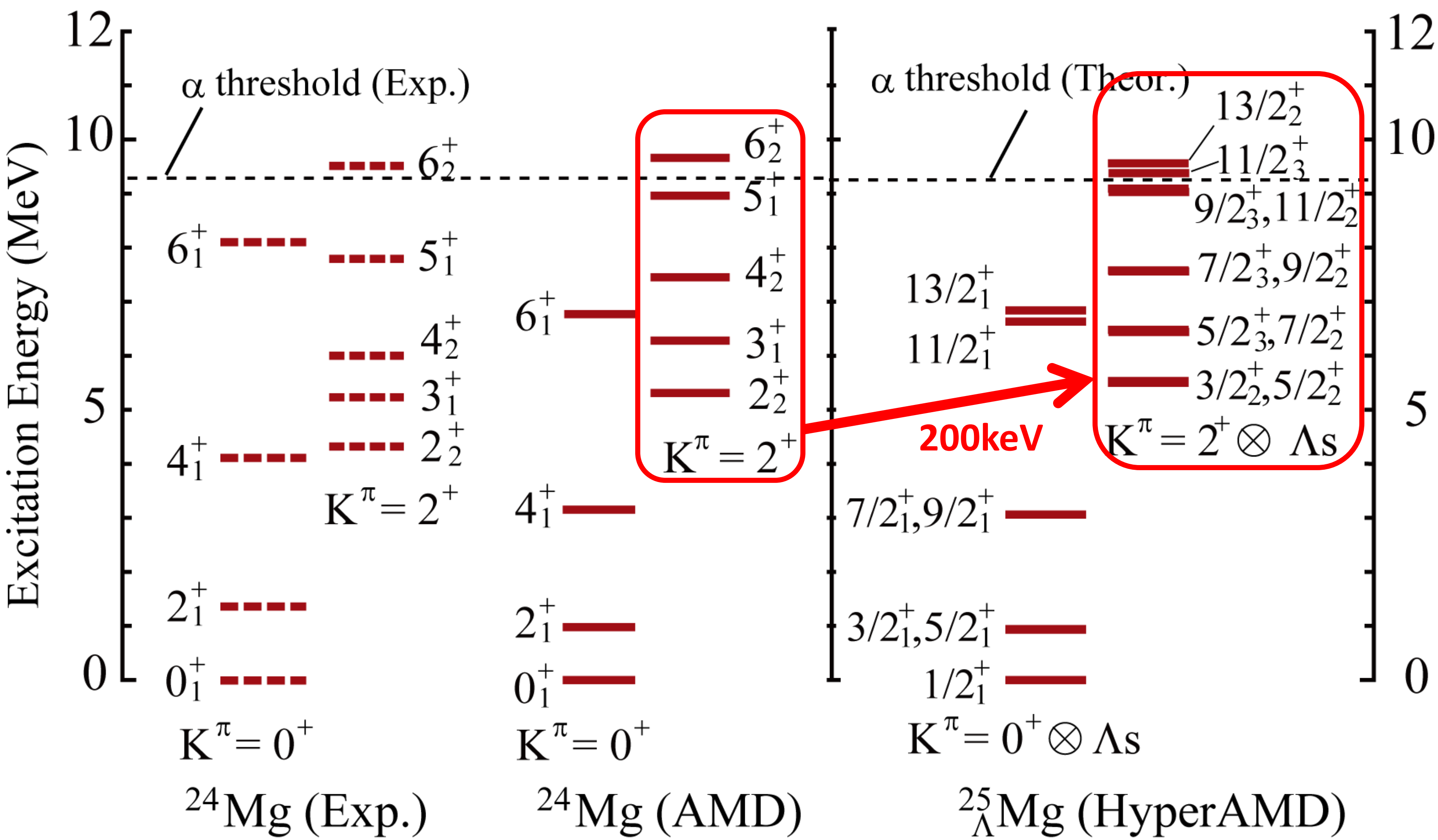




Difference in B(E2) and B_{Λ}

Other example of "impurity effects"

Changes of triaxial deformed hypernucleus $^{25}{}_{\Lambda}Mg$



Difference of deformation change leads to shift up of the side band

M. Isaka, M. Kimura, A. Dote and A. Ohnishi, PRC **85** (2012), 034303.

Summary

◆ Summary

- AMD + GCM was used to study deformations of *sd-pf* shell Λ hypernuclei

Λ as a probe to study triaxial deformation: $^{25}{}_{\Lambda}$Mg

- Λ in p-orbit generates three different p states in $^{25}{}_{\Lambda}$Mg
  - This is due to the triaxial deformation of the core nucleus

Structure change in neutron-rich hypernucleus $^{12}{}_{\Lambda}$Be

- The abnormal parity of $^{11}\mathrm{Be}$ ground state is reverted in $^{12}{}_{\Lambda}\mathrm{Be}$
  - B_{Λ} is different between the $1/2^+$ and $1/2^-$ states of ^{11}Be with different molecular orbital structure

◆ Future plan

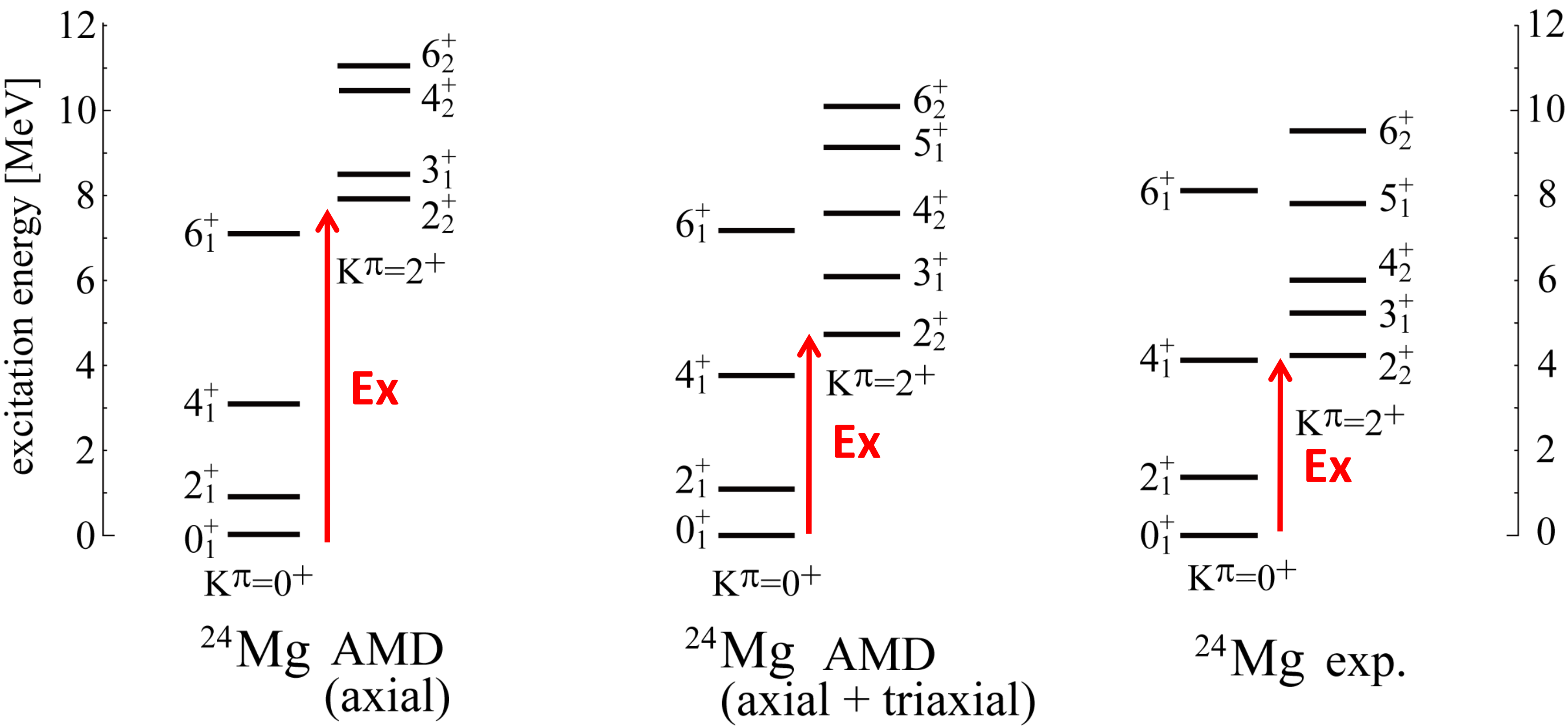
- To predict the production cross sections
- Structure of Be hyper-isotopes: How does Λ modify the 2α structure?

Backup

Structure of ^{24}Mg

- **Low lying $K^\pi=2^+$ band: a sign of triaxial deformation**
- **Excitation energy of $K^\pi=2^+$ band depends on the triaxial deformation**

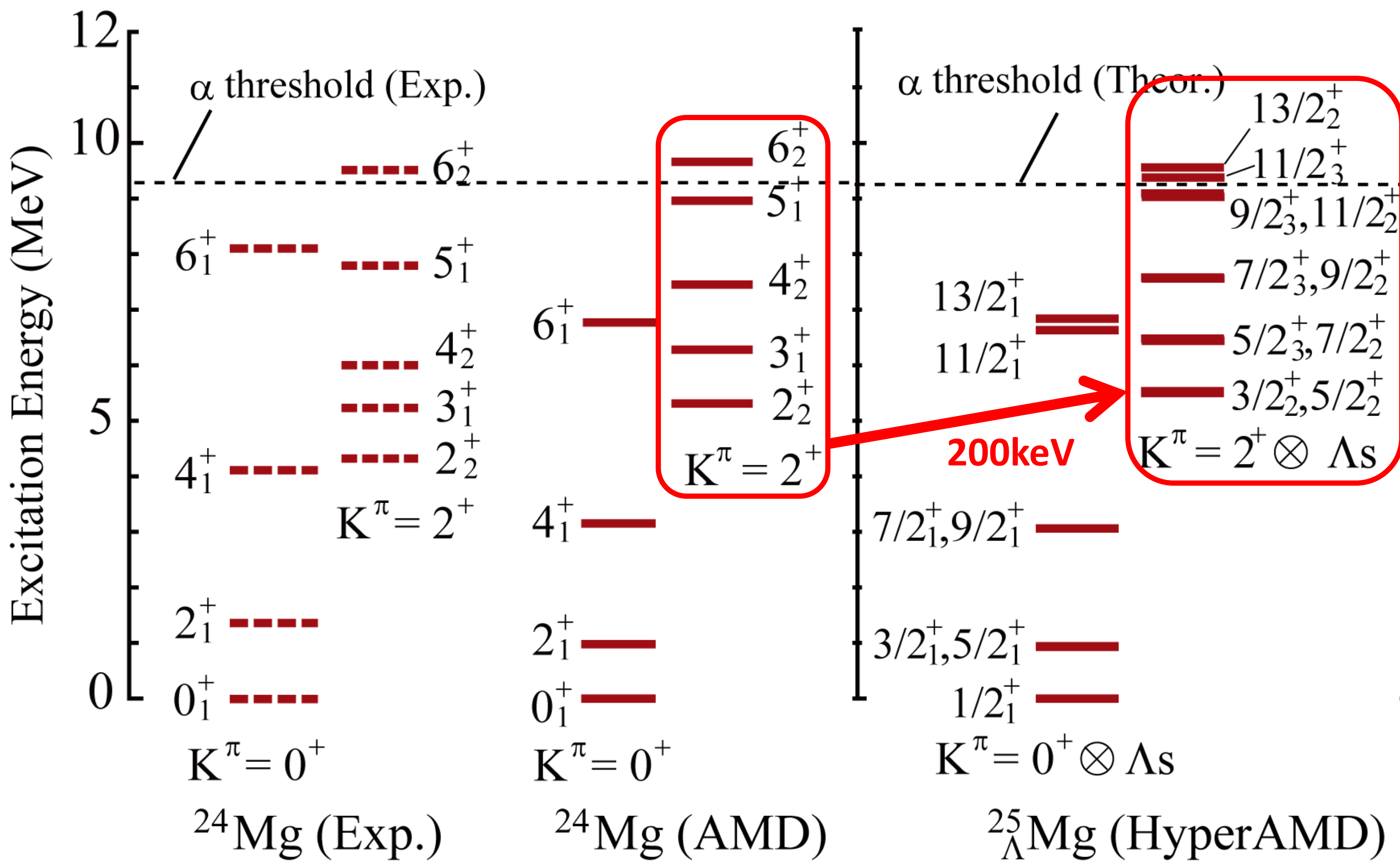
M. Bender, P-H. Heenen, PR**C78**, 024309 (2008). , M. Kimura, R. Yoshida, M.I., PTP **127** , 287(2011).



$K^\pi = 2^+$ band is rigid against the exclusion of the triaxial deformation

Does Λ particle change these two bands?

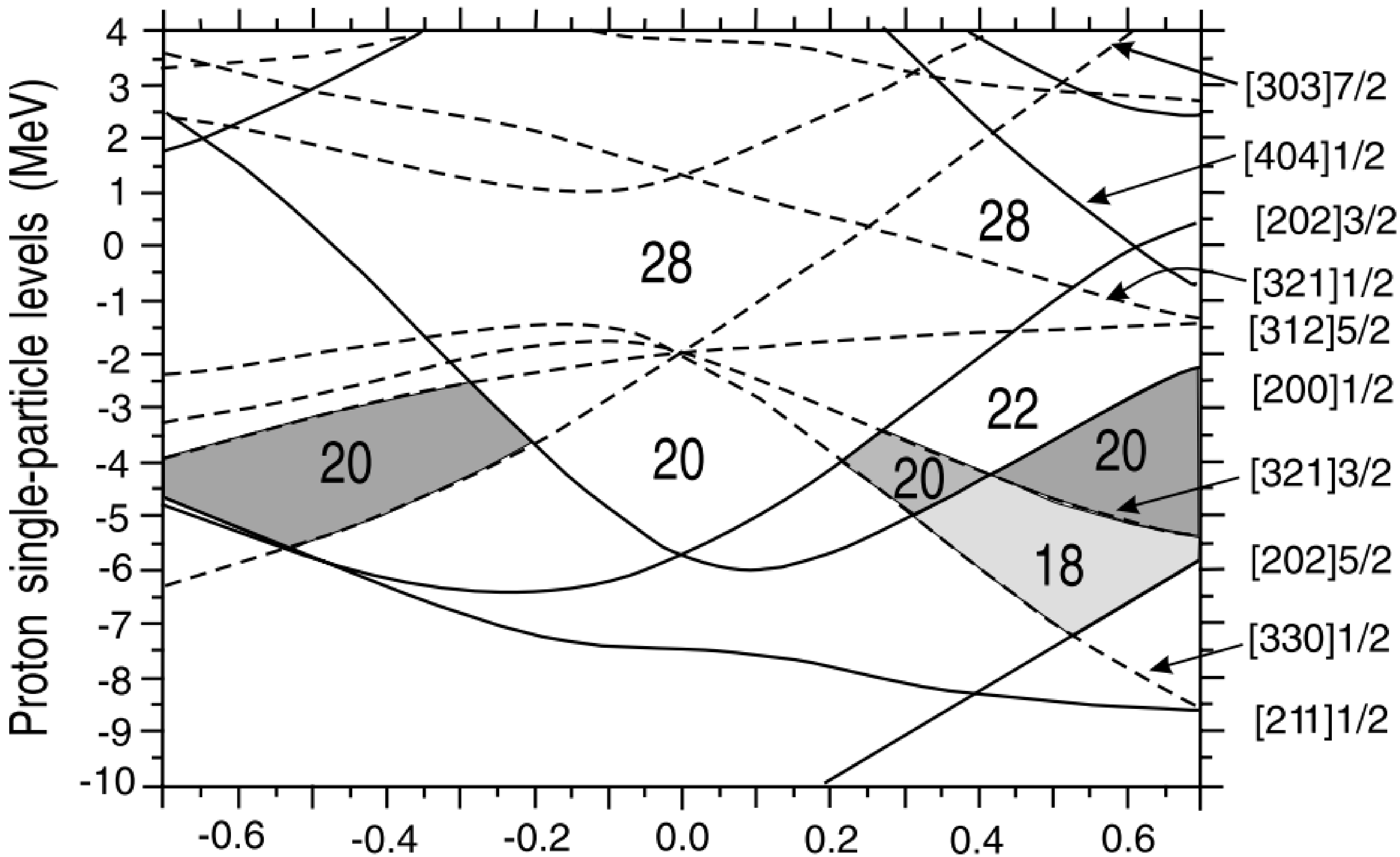
Results: Excitation spectra of $^{25}{}_{\Lambda}Mg$



Excitation energy of $K^\pi=2^+ \otimes \Lambda_s$ band is shifted up by about 200 keV

Back up: Superdeformation

- **SD is due to the large shell gap at certain deformations**



Taken from E. Ideguchi, *et al*., PRL **87**, 222501 (2011)

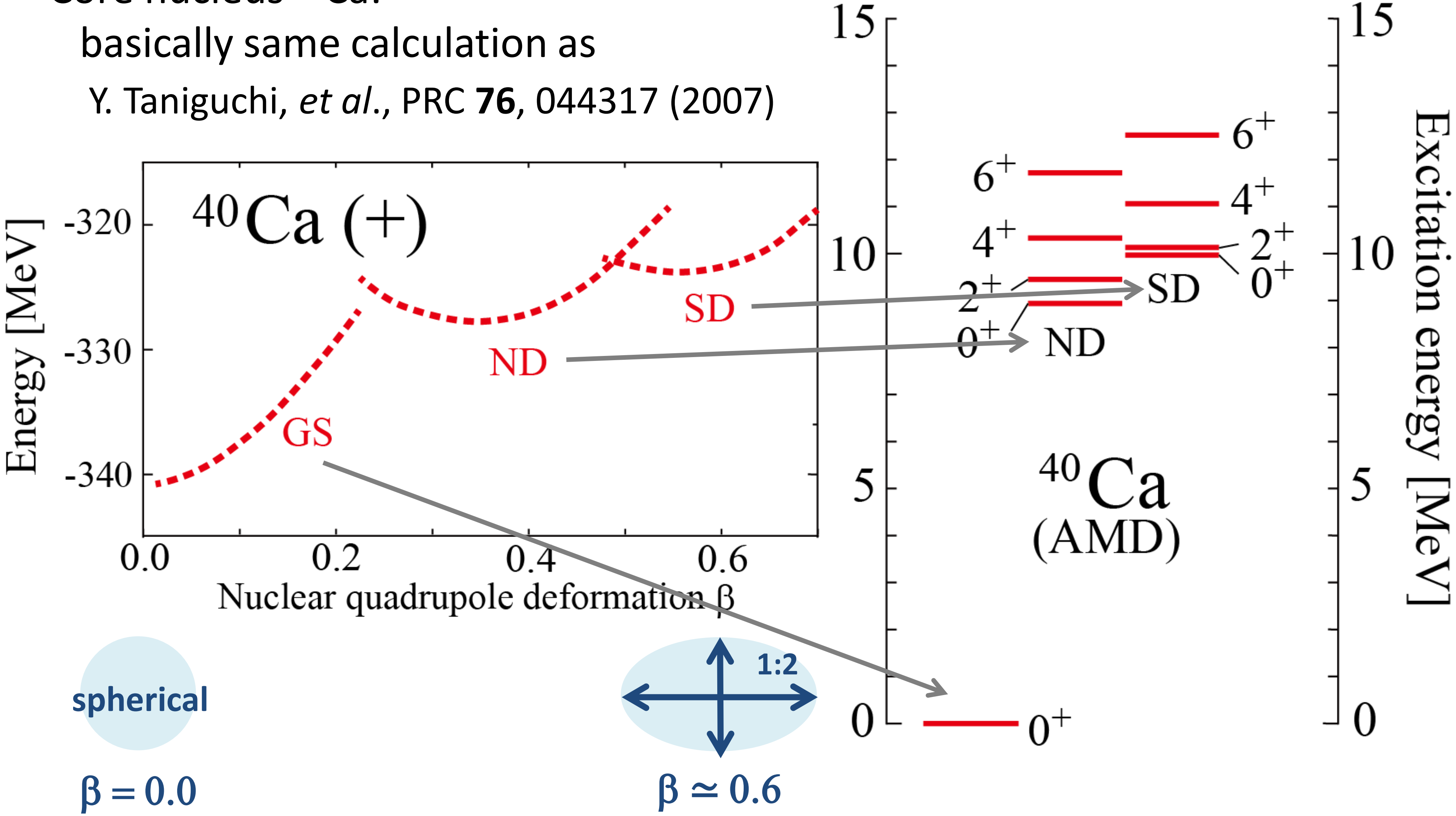
ND and SD states of ^{40}Ca

- **Ground, normal deformed and superdeformed states are obtained**

Core nucleus ^{40}Ca:

basically same calculation as

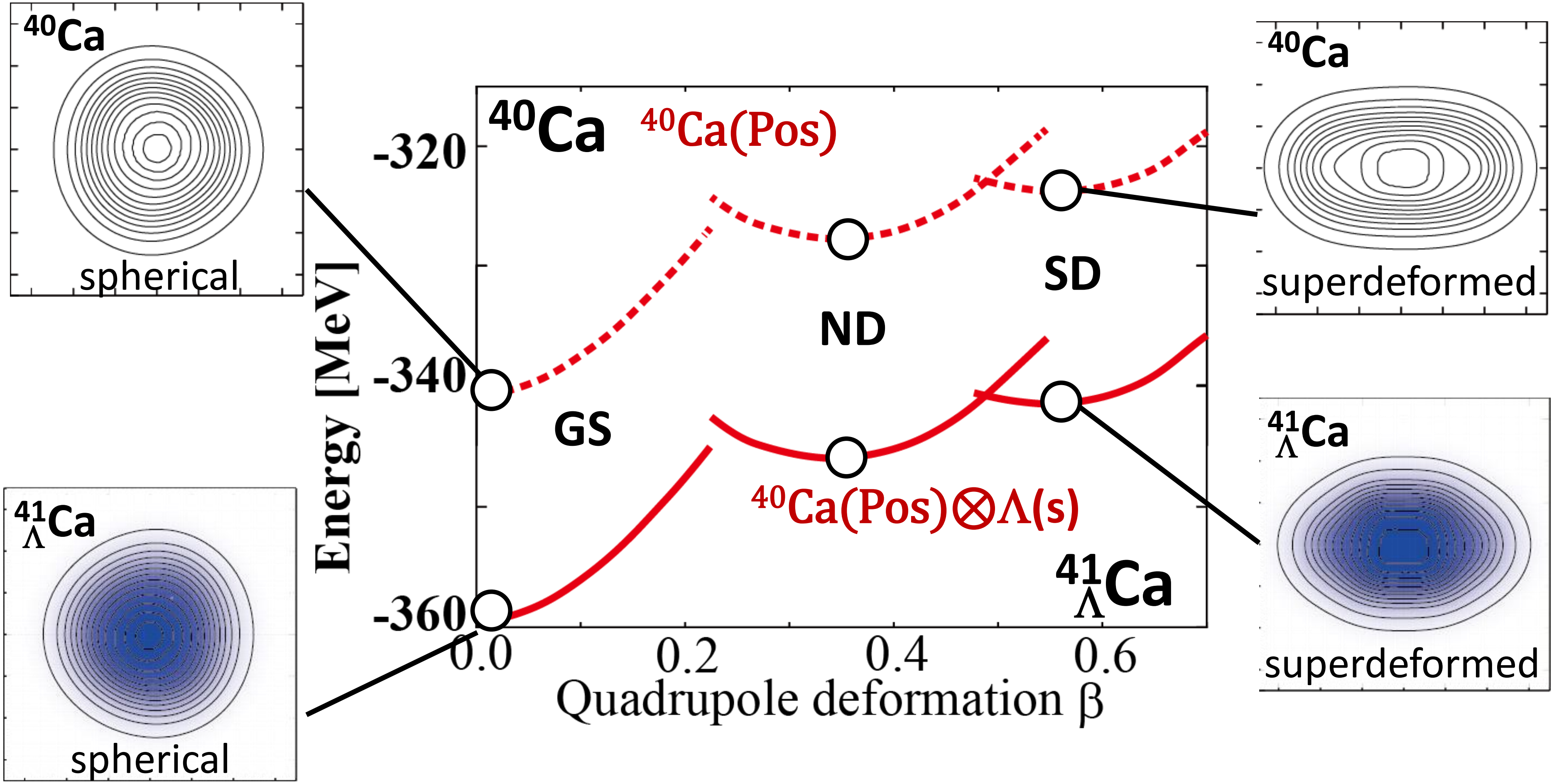
Y. Taniguchi, *et al.*, PRC **76**, 044317 (2007)



Energy curves of $^{41}{}_{\Lambda}$Ca as a function of β

$^{41}_{\Lambda}$Ca

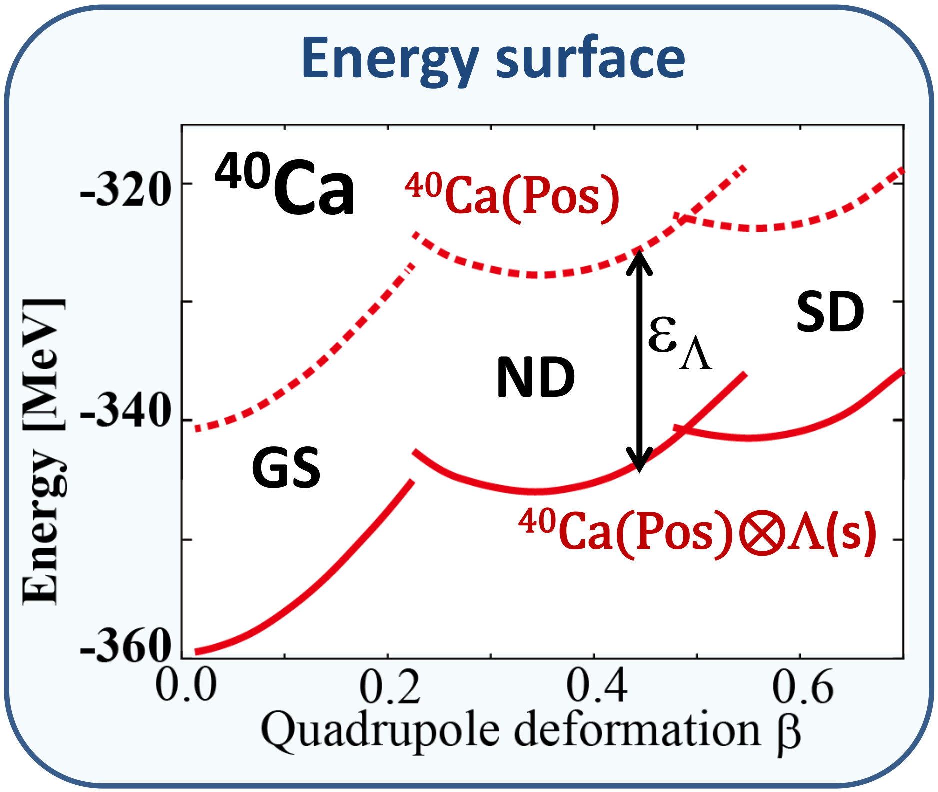
- **"GS⊗Λ", "ND ⊗Λ" and "SD ⊗Λ" curves are obtained**
  → SD states will appear in $^{41}{}_{\Lambda}$Ca
- **Energy (local) minima are almost unchanged**

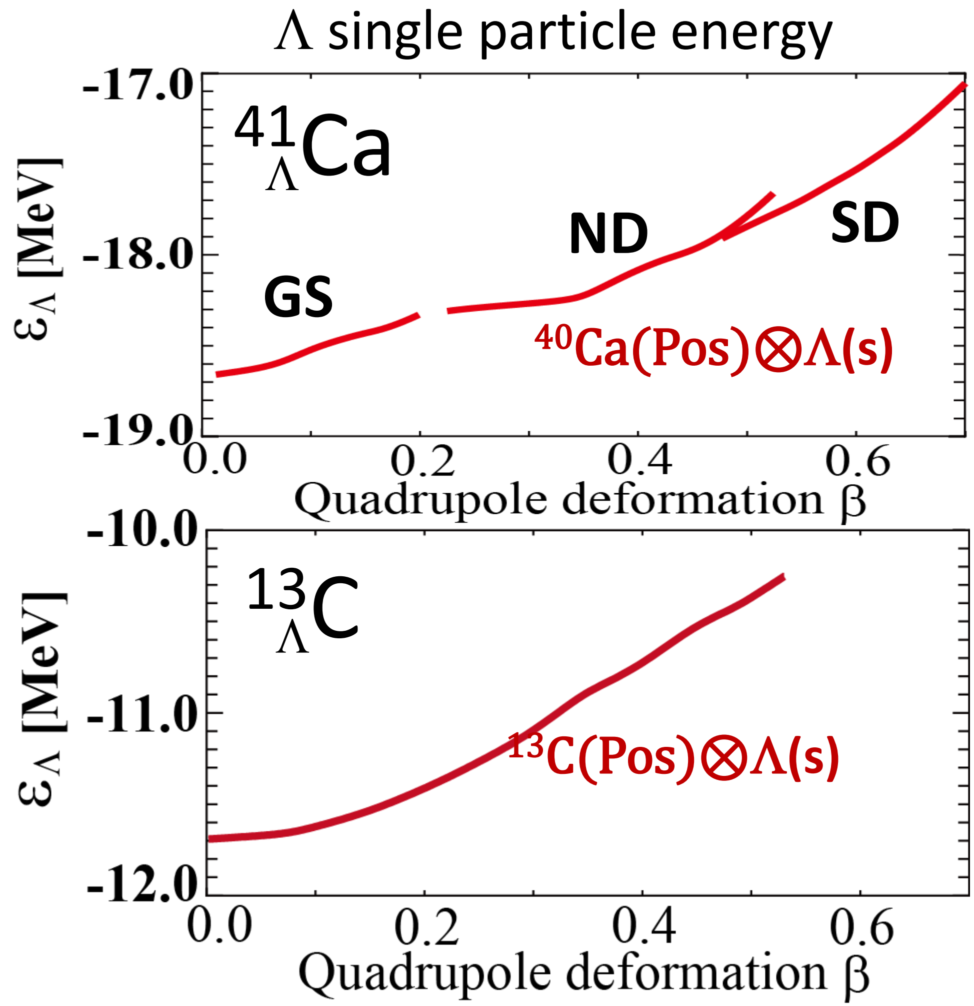


Λ single particle energy

- **Definition:** $\epsilon_\Lambda(\beta) = E(^{46}_{\Lambda}\mathrm{Sc})(\beta) - E(^{45}\mathrm{Sc})(\beta)$
- **General trend: ε_Λ changes within 1 - 2 MeV as β increases**

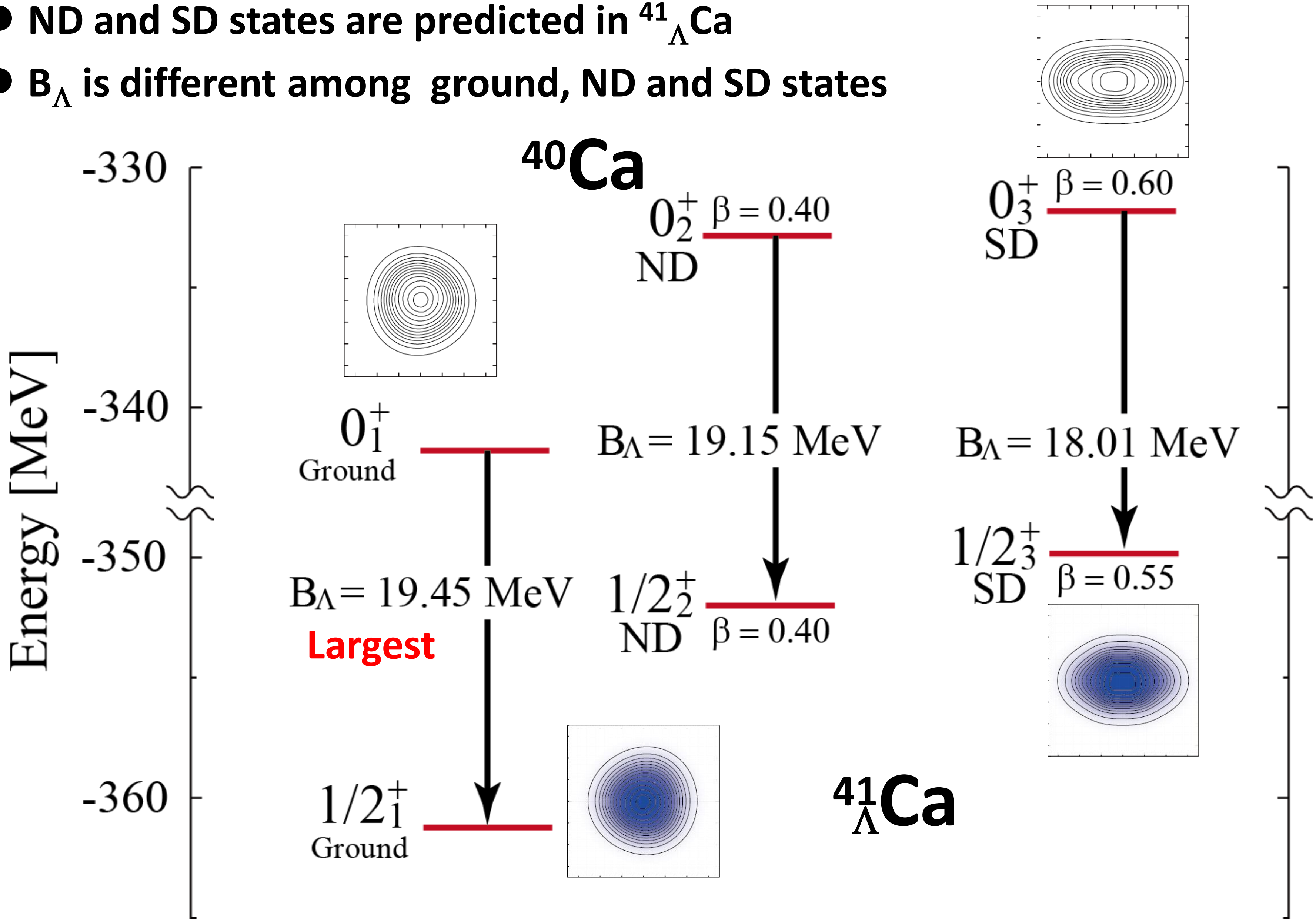
→ **Similar to the *p* shell Λ hypernuclei**





Difference of Λ binding energy B_Λ

- ND and SD states are predicted in ${}^{41}_{\Lambda}$Ca
- B_Λ is different among ground, ND and SD states

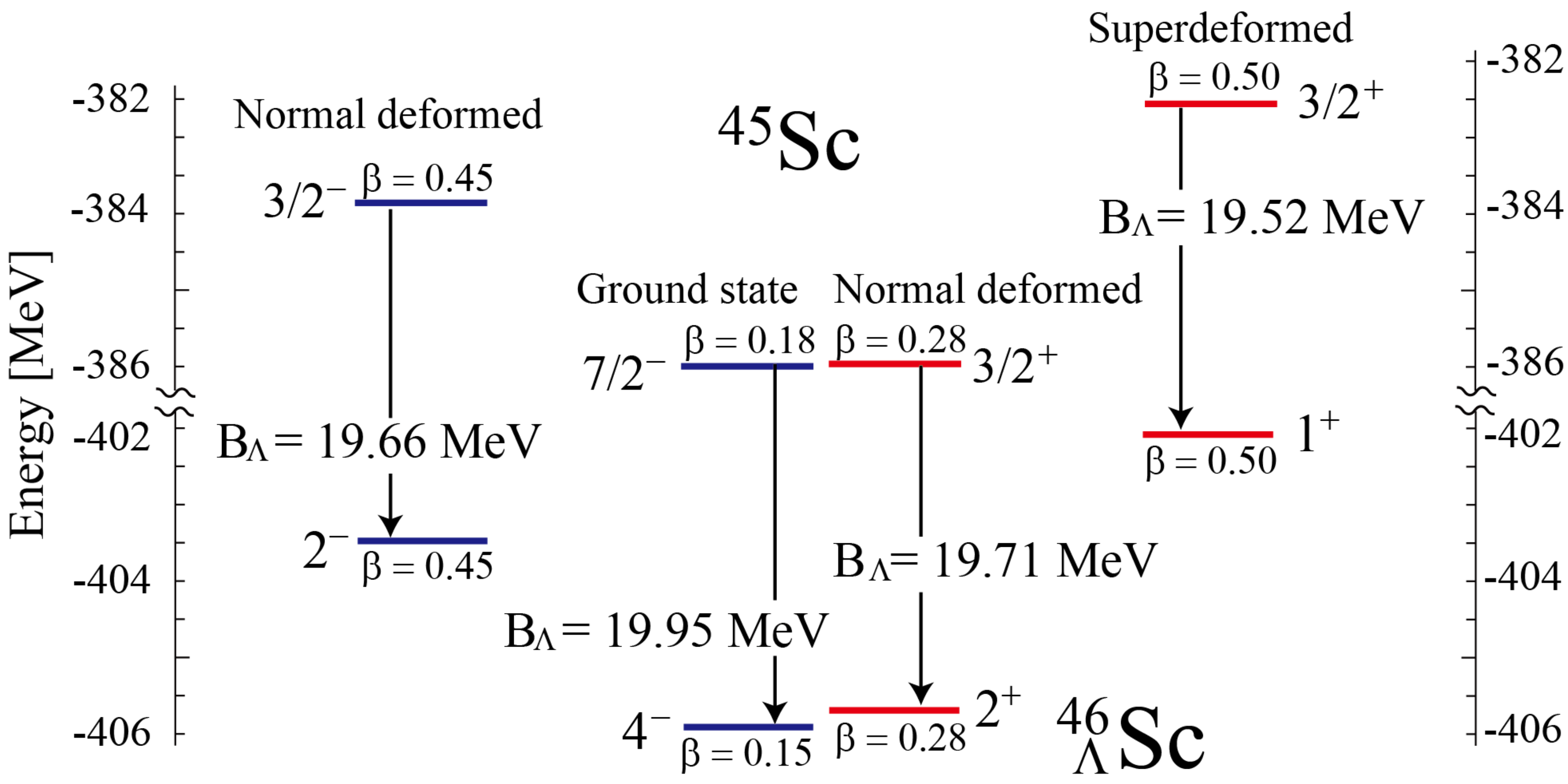


Superdeformed states in Sc hypernuclei

Examples: $^{46}_{\Lambda}$Sc, $^{48}_{\Lambda}$Sc

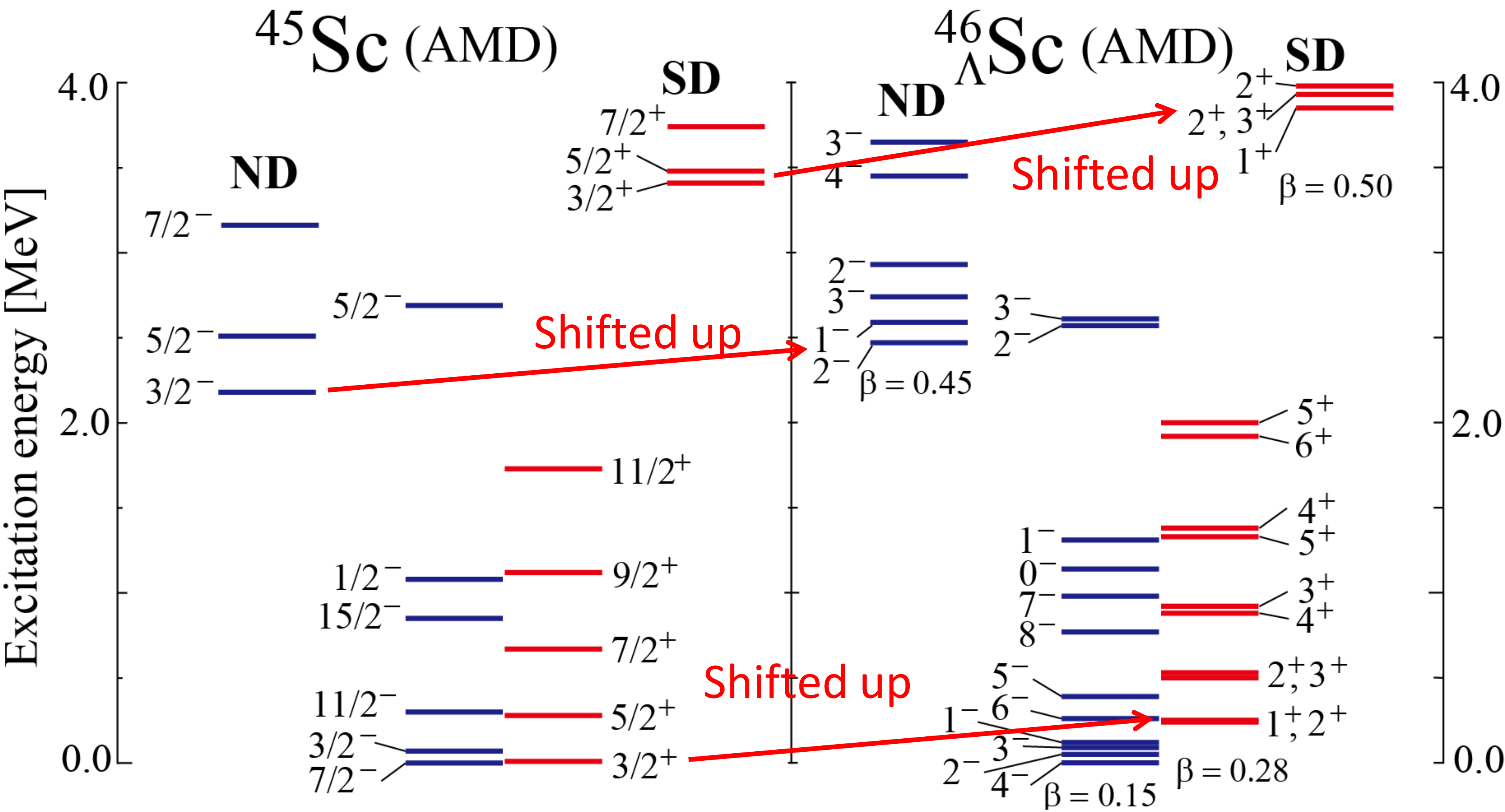
Core nuclei (^{45}Sc, ^{47}Sc)
- Various deformations coexist in the g.s. regions
- We predict ND and SD states with *mp-mh* configuration

→ **Difference of B_Λ depending on deformation by adding a Λ**



Excitation spectra of $^{46}{}_{\Lambda}$Sc and $^{48}{}_{\Lambda}$Sc

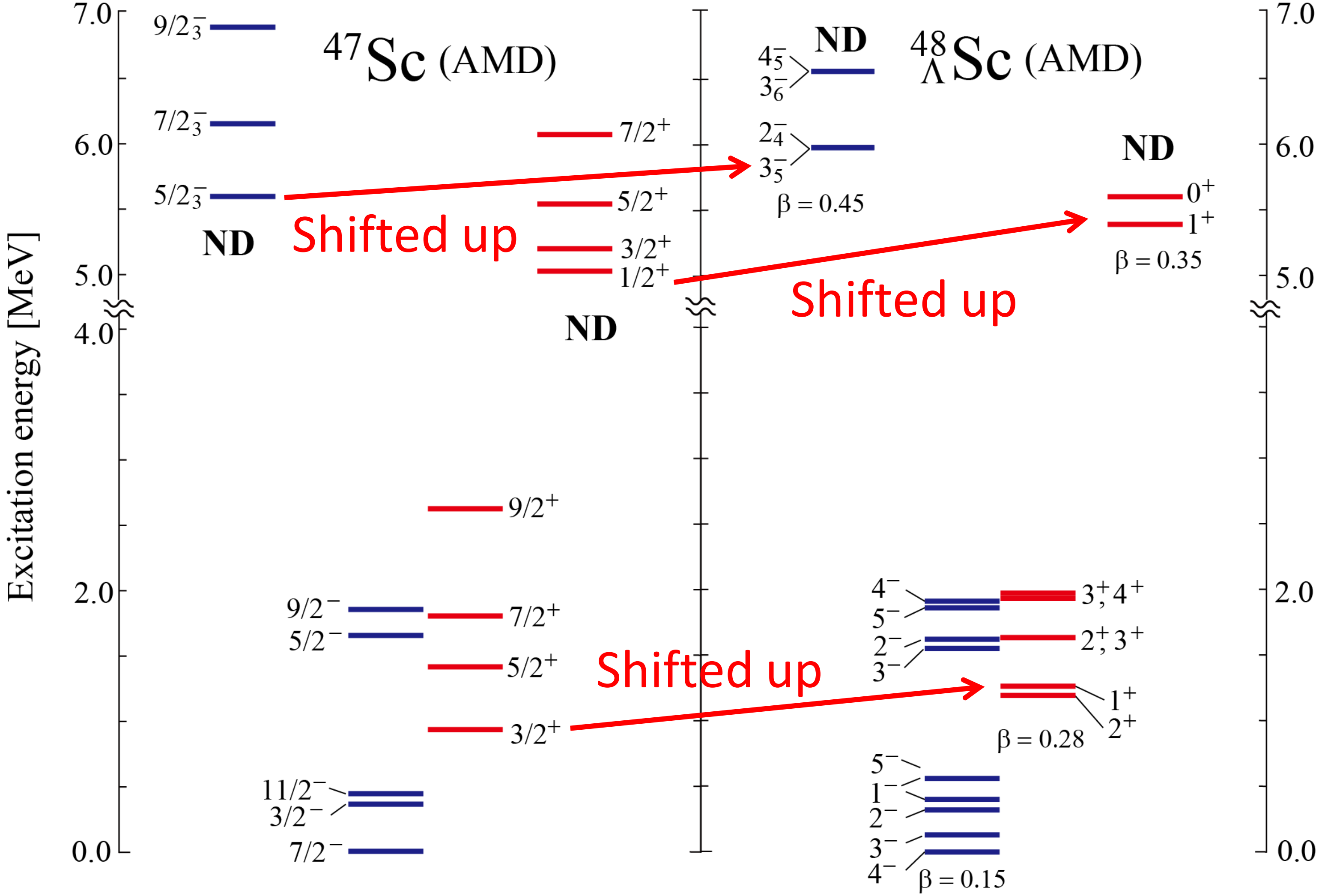
- **Difference of B_{Λ} leads to the energy shift up of the deformed states**
- **Similar phenomena in $^{48}{}_{\Lambda}$Sc**



We hope these states in Sc Λ hypernuclei are observed at JLab

Changes of the excitation spectrum in $^{48}_{\Lambda}$Sc

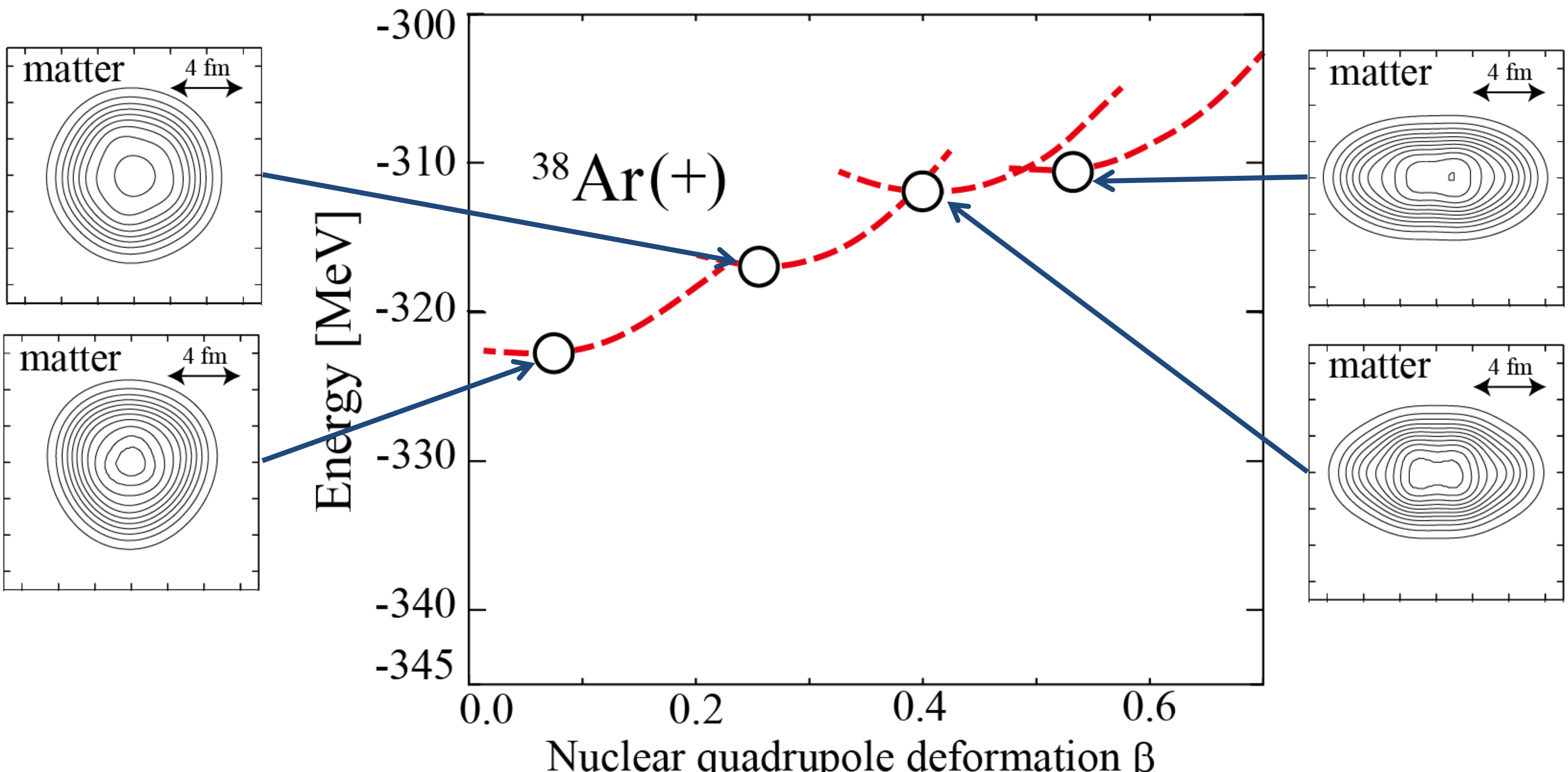
- **We predict *mp-mh* states with various deformations in ^{47}Sc**
- **Difference of B_{Λ} and shift up of the deformed states in $^{48}_{\Lambda}$Sc**



Results: Energy curve

^{38}Ar

- **Energy (local) minima with different deformations appear**



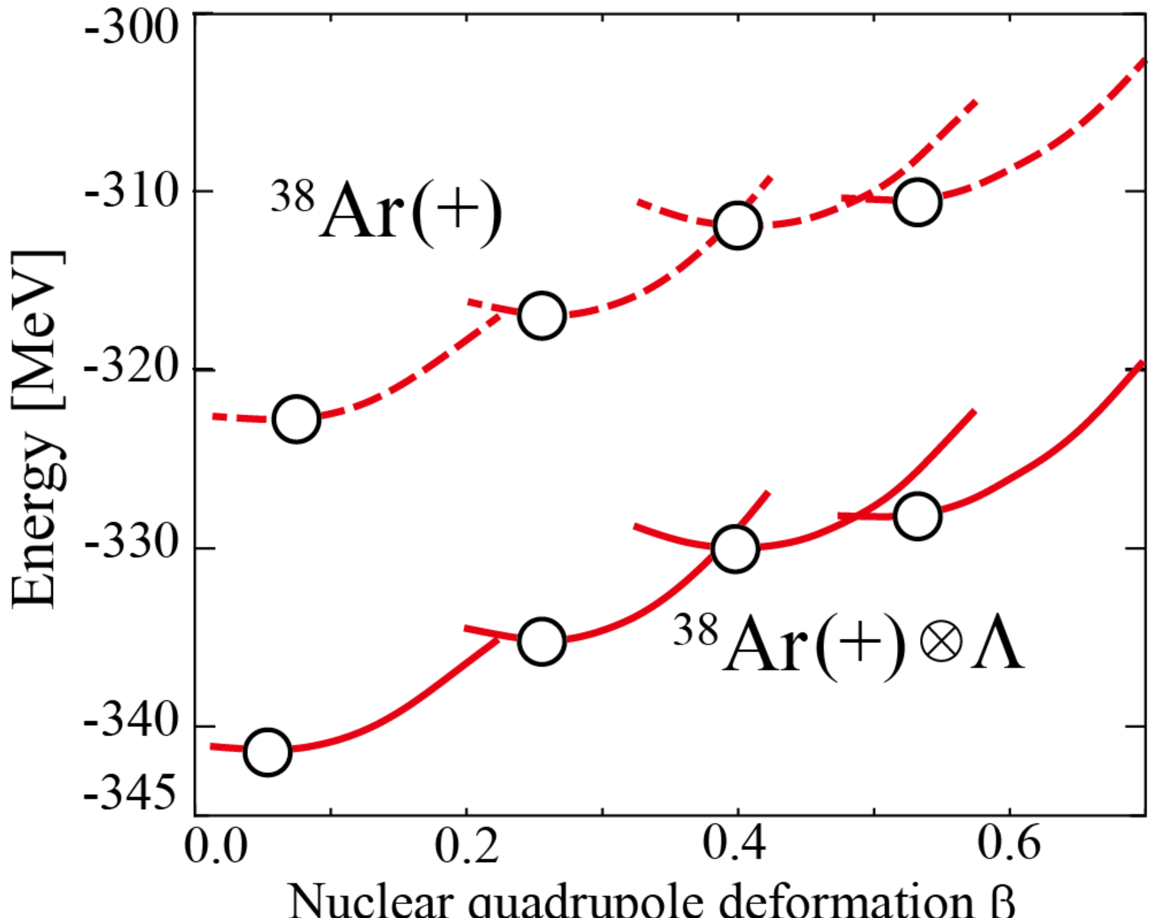
Results: Energy curve

^{38}Ar

- **Energy (local) minima with different deformations appear**

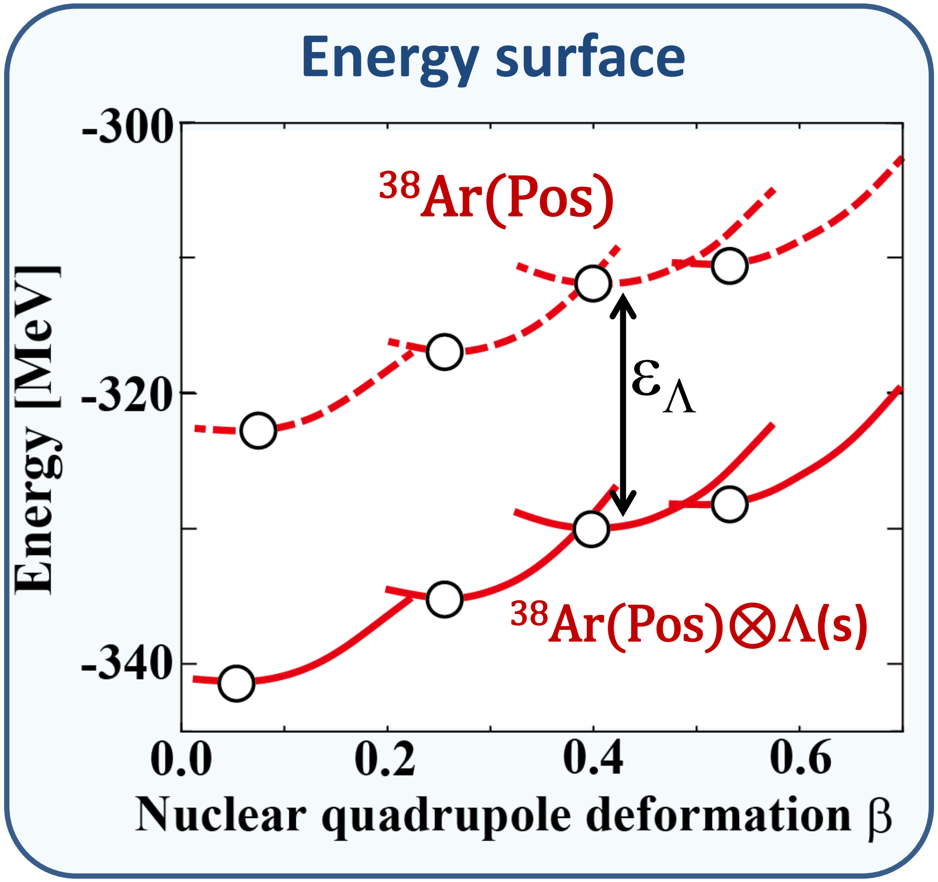
$^{39}_{\Lambda}$Ar

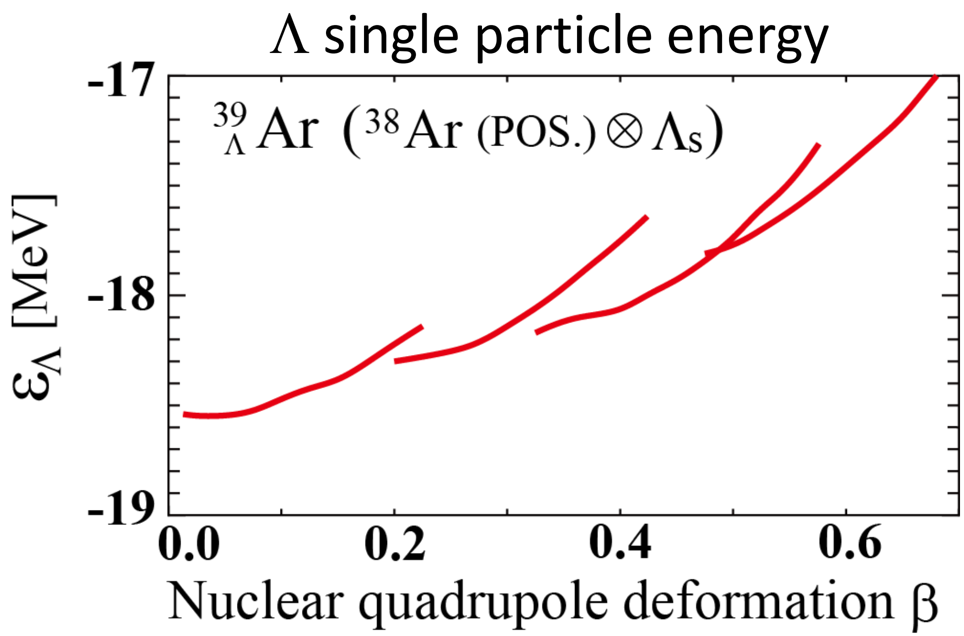
- **"GS⊗Λ", "ND ⊗Λ " and "SD ⊗Λ " curves are obtained**
- **Energy (local) minima are almost unchanged**



Results: Λ single particle energy

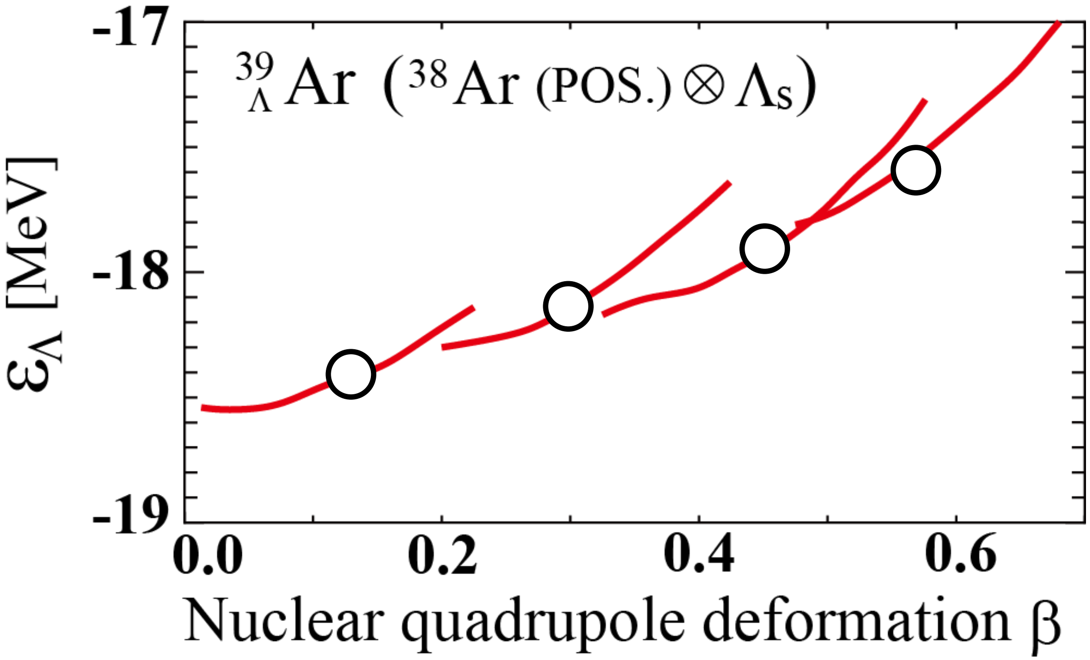
- **Definition:** $\epsilon_\Lambda(\beta) = E({}^{39}_{\Lambda}\mathrm{Ar})(\beta) - E({}^{38}\mathrm{Ar})(\beta)$
- **General trend: ε_Λ changes within 1 - 2 MeV as β increases**

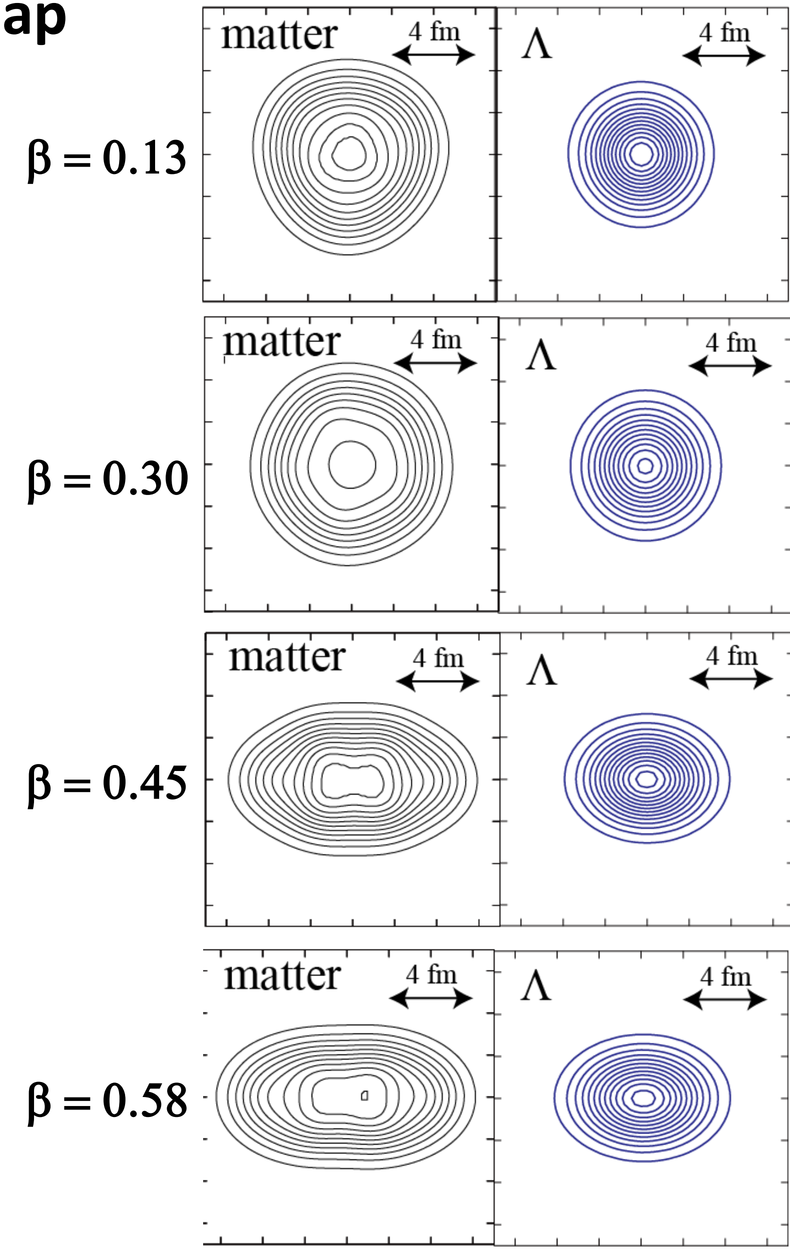




Results: Λ single particle energy

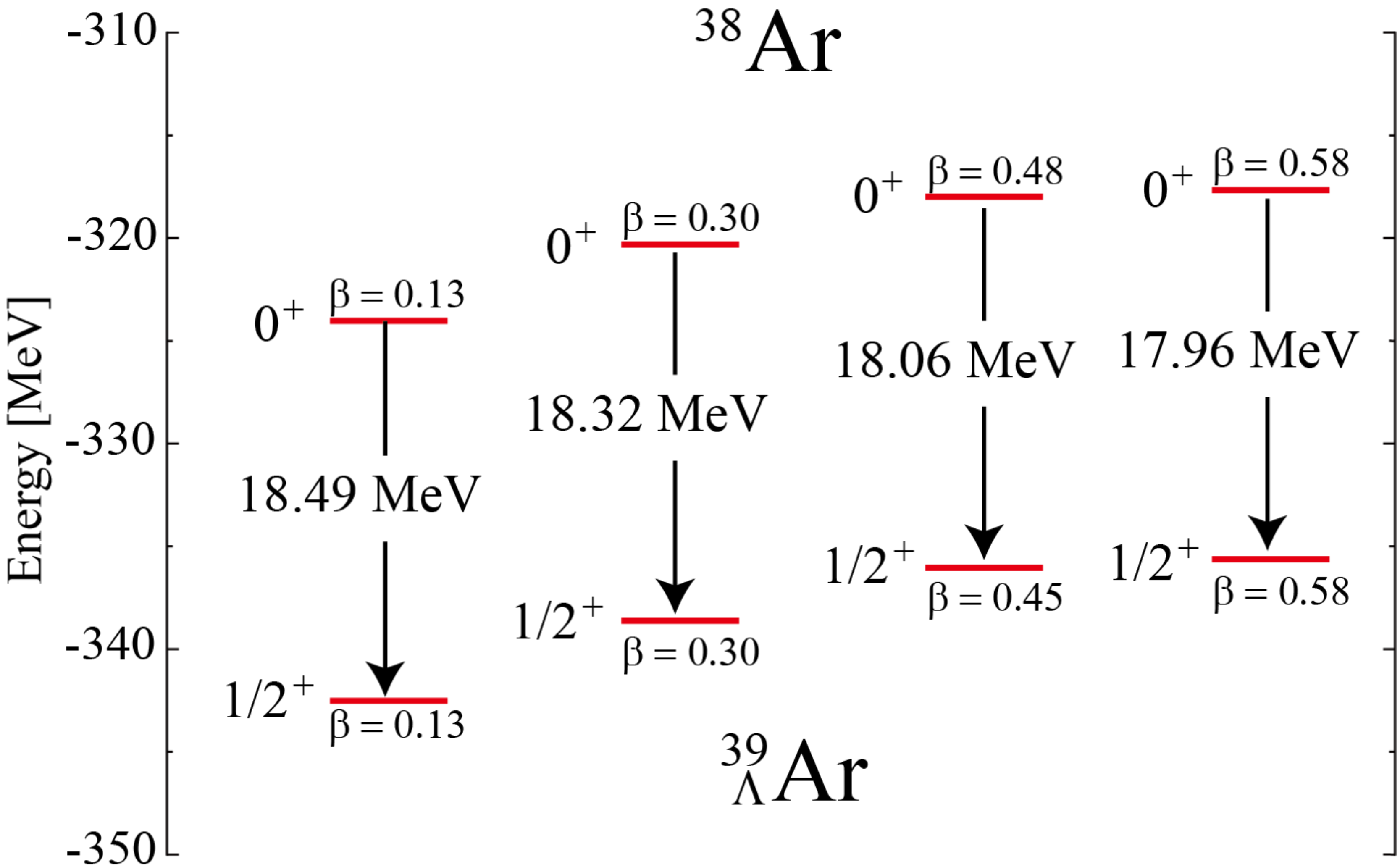
- **ε_Λ varies due to changes of spatial overlap between Λ and N**
  - **Deformation of Λ distribution is small, while nuclear part is deformed**





Results: Difference of B_Λ

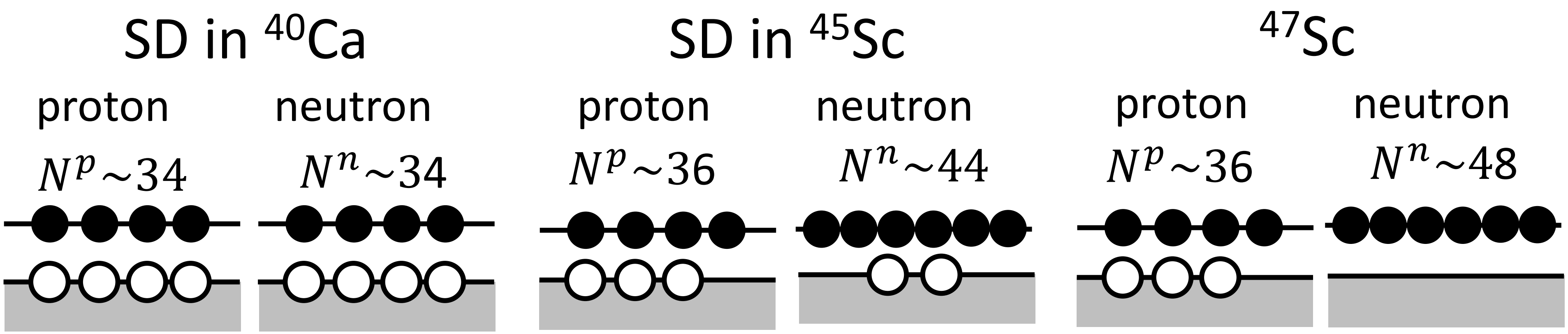
- **Excited staes with different deformations are predicted in $^{39}_{\Lambda}$Ar**
- **B_Λ is different depending on deformations and consistent with $\varepsilon_\Lambda(\beta)$**



Short summary

- **AMD + GCM framework has been applied to ^{45}Sc and ^{47}Sc to investigate deformed excited states.**

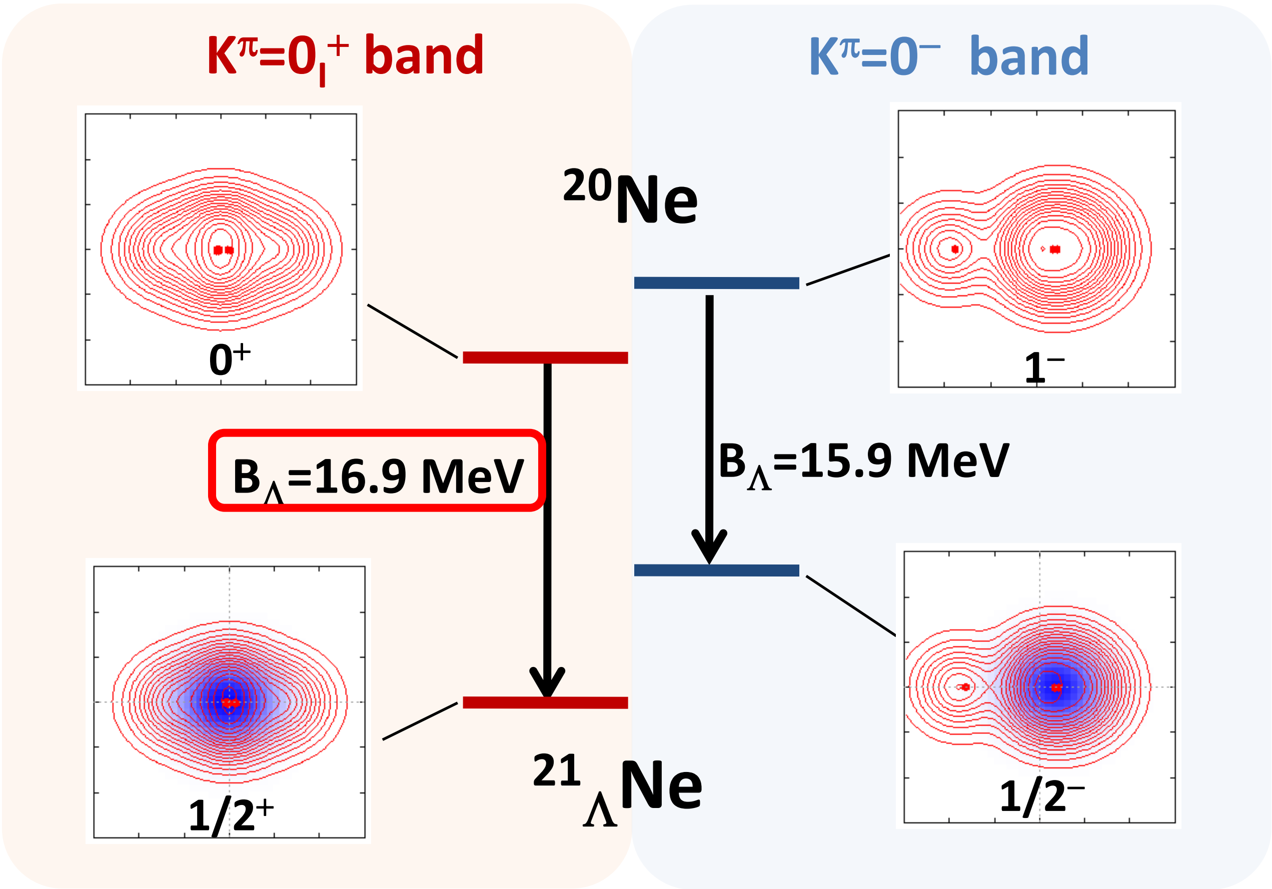
- **Various deformed states with many-particle many-hole configurations**
  - In ^{45}Sc, prediction of the SD states with proton 4*p* configuration
  - In ^{47}Sc, deformed states with 4*p* proton configuration, but neutron configuration is the same as the GS.



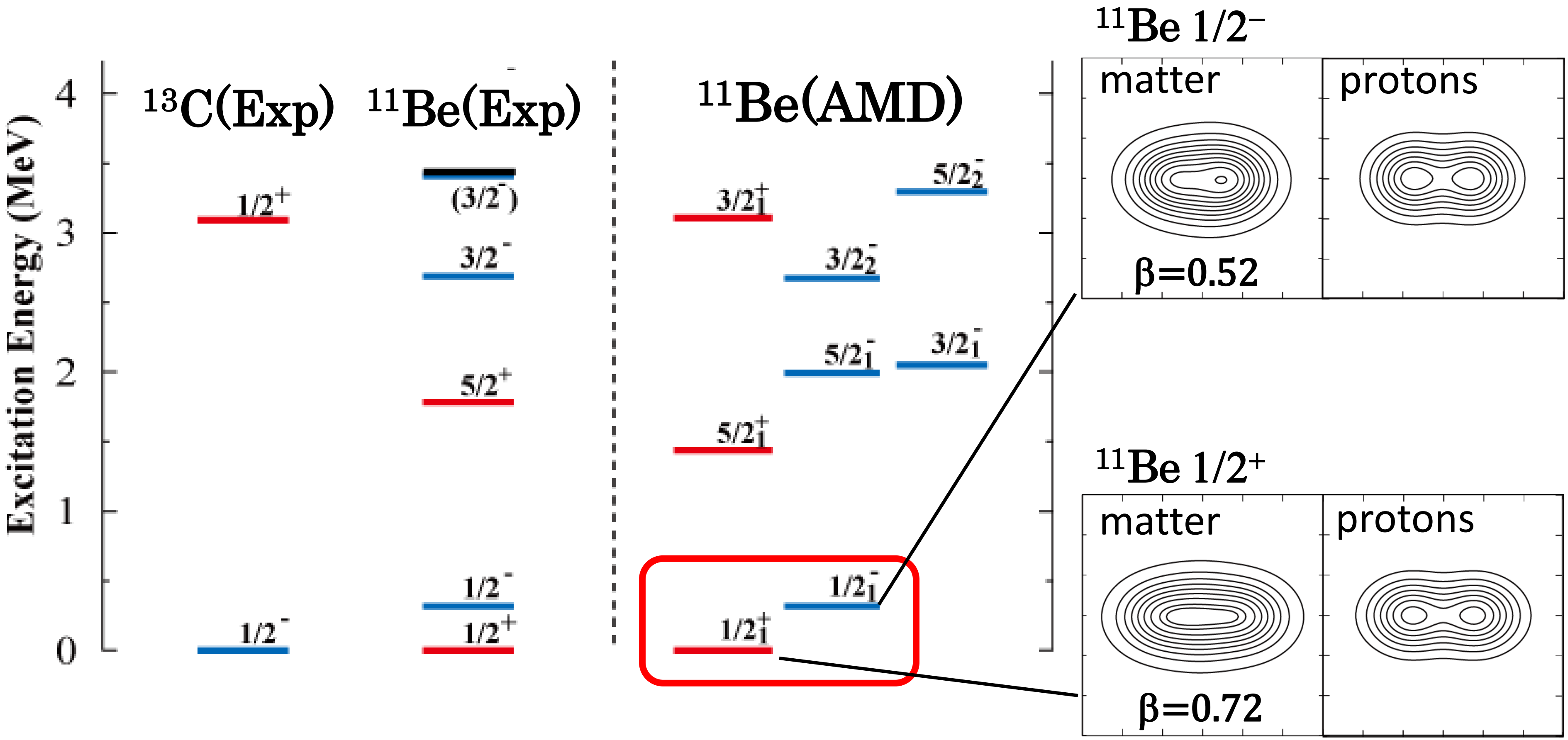
→
- **Increase of neutron number may affect neutron configuration?**
- **Further investigations are required**

Structure dependence

- **The Λ hyperon coupled to the intermediate state is more deeply bound than that coupled to the well developed $\alpha + {}^{16}$O state**
- **Λ is localized around ^{16}O cluster in the α + ^{16}O + Λ cluster state.**
  ⟶ shallow binding in the α + ^{16}O state

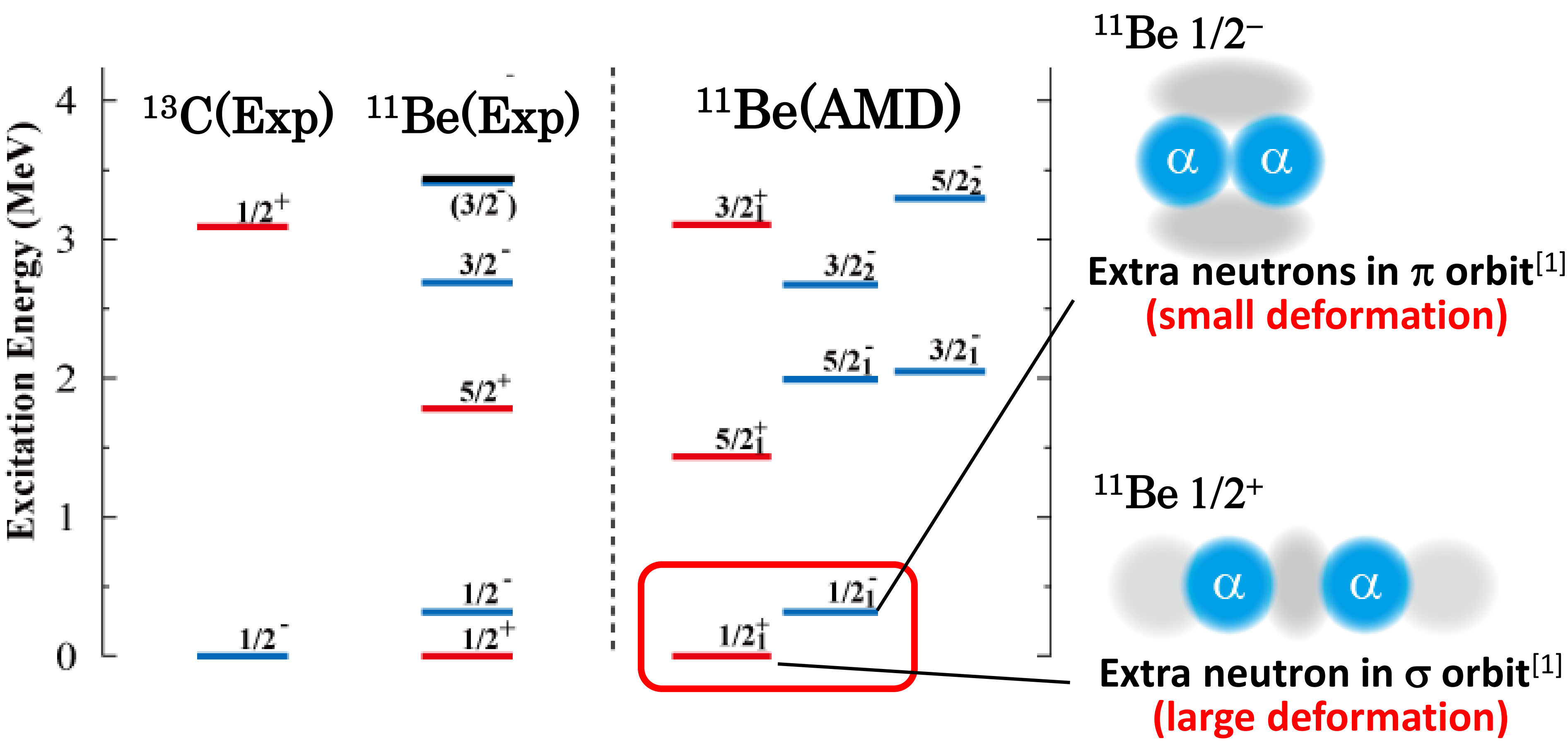


AMD results for ^{11}Be



- **Deformation of the $1/2^-$ state is smaller than that of the $1/2^+$ state**

AMD results for ^{11}Be



- **Deformation of the 1/2$^-$ state is smaller than that of the 1/2$^+$ state**

↳ Difference in the orbits of extra neutrons

[1] Y. Kanada-En'yo and H. Horiuchi, PRC **66** (2002), 024305.