Modelling the Kaon Production

Dalibor Skoupil, Petr Bydžovský

Nuclear Physics Institute of the ASCR
Řež, Czech Republic

SPHERE Meeting 2014

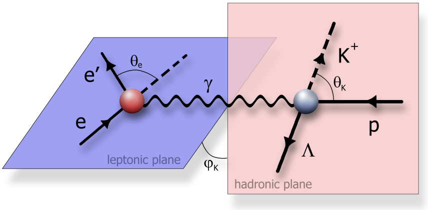
September 9 - 11, 2014, Prague, Czech Republic

Motivation

- Thanks to a rising value of α_s with decreasing energy the perturbation theory in QCD is not suited for small energies $\rightarrow$ introduction of effective theories and models
- Models for description of elementary electroproduction of hyperons provide a useful tool in hypernuclear-physics calculations
- New precise data from LEPS, GRAAL and particularly from CLAS collaborations are available for tuning free parameters of the models
- Constituent Quark Model predicts more excited nucleon states than observed in the pion production experiments $\rightarrow$ "missing resonance" problem (some of these unobserved resonances can possibly play a role in the strangeness production channels)

$e + N \to e' + K + Y$

- 6 channels: $N = p,\ n;\ K = K^+,\ K^0;\ Y = \Lambda,\ \Sigma^0,\ \Sigma^+$
- One-photon-exchange approximation allows the separation of leptonic and hadronic part of the process

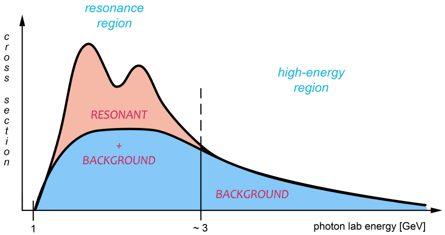


Final state only with $K^+\Lambda$ discussed

- in the other channels, due to the production of Σ hyperon there should be Δ resonances included
- for $n(\gamma, K^0)\Lambda$ channel no data are available (experiments are running on deuteron targets)

$p(p) + \gamma(k) \to K^+(p_K) + \Lambda(p_\Lambda)$

- In the case of photoproduction we study the reaction in the hadronic plane
- The threshold of the $p(\gamma, K^+)\Lambda$ process is $E_\gamma^{lab} = 0.911\,\text{GeV}$
- In the lowest order, the reaction is described through the exchange of intermediate states (resonances)
  - Since there is no dominant resonance, it is needed to consider a number of resonances with mass $\leq$ 2 GeV (*third resonance region*)



- Resonance region dominated by resonant contributions (N^*)
- Many non-resonant contributions (exchange of p, K, Λ; K^* and Y^*) $\Rightarrow$ background

Introduction

Observables

Electroproduction cross section

$$\frac{d^3\sigma}{dE_{e'}\,d\Omega_{e'}\,d\Omega_K^{c.m.}} = \Gamma\left[\sigma_T + \varepsilon\sigma_L + \varepsilon\sigma_{TT}\cos(2\varphi_K) + \sqrt{2\varepsilon_L(\varepsilon+1)}\,\sigma_{LT}\cos\varphi_K\right]$$

Single-polarisation observables

Hyperon polarisation

$$P = \frac{\sigma^{(\lambda_\Lambda=+1)} - \sigma^{(\lambda_\Lambda=-1)}}{\sigma^{(\lambda_\Lambda=+1)} + \sigma^{(\lambda_\Lambda=-1)}}$$

Target polarisation

$$T = \frac{\sigma^{(\lambda_p=+1)} - \sigma^{(\lambda_p=-1)}}{\sigma^{(\lambda_p=+1)} + \sigma^{(\lambda_p=-1)}}$$

Beam asymmetry

$$\Sigma = \frac{\sigma^{(\perp)} - \sigma^{(\parallel)}}{2\sigma_{unpol}}$$

Double-polarisation observables

Beam-recoil polarisation

- γ and Λ polarised

Beam-target polarisation

- γ and p polarised

Target-recoil polarisation

- p and Λ polarised

Introduction

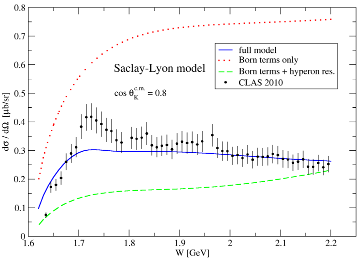
Available Experimental Data for the $p(\gamma, K^+)\Lambda$ Reaction

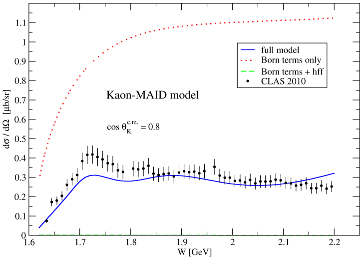
Observable	No. of data	Collaboration	Year
cross section $d\sigma/d\Omega$	56	SLAC	1969
	720	SAPHIR	2004
	1377	CLAS	2006
	12	LEPS	2007
	2066	CLAS	2010
beam asymmetry Σ	9	SLAC	1979
	45	LEPS	2003
	54	LEPS	2006
	4	LEPS	2007
	66	GRAAL	2007
target polarisation T	3	BONN	1978
	66	GRAAL	2008
hyperon polarisation P	7	DESY	1972
	233	CLAS	2004
	66	GRAAL	2007
	1707	CLAS	2010
C_x, C_z	320	CLAS	2007
O_x, O_z	132	GRAAL	2008

Ways of Describing the $p(\gamma, K^+)\Lambda$ Reaction

- Constituent Quark Model
  - no need to introduce the resonances, they emerge naturally as excited states of the system
  - "missing resonance" problem
- Chiral Perturbation Theory
  - limited to energies approximately 100 MeV above threshold $\rightarrow$ cannot describe physics in the resonance region
  - contributions from resonances with spin higher than 3/2 cannot be reproduced
- Coupled-channel Analysis
  - takes into account different intermediate processes occuring between the initial and final state (*e.g.* rescattering, final-state interaction)
- Regge-plus-resonance Model
  - description in the energy range from threshold up to $E_\gamma^{lab} \approx 16\,\text{GeV}$
  - the nonresonant part of the amplitude modeled by exchanges of $K^+(494)$ and $K^{*+}(892)$ trajectories
- Isobar Model

Isobar Model





- Use of effective hadron Lagrangian
- Satisfactory agreement with the data in the energy range $E_\gamma^{lab} = 0.91 - 2.5\,\text{GeV}$
- Coupling constants and SU(3)$_f$ symmetry breaking

$$-4.4 \le \frac{g_{K\Lambda N}}{\sqrt{4\pi}} \le -3.0, \quad 0.8 \le \frac{g_{K\Sigma N}}{\sqrt{4\pi}} \le 1.3$$

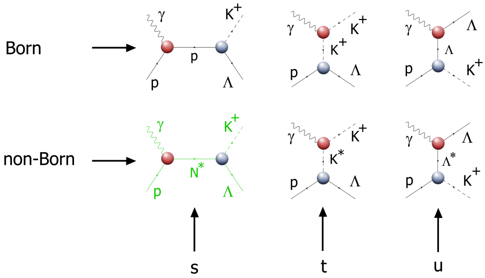
- Problem of large Born contributions (avoided in the RPR approach); solutions:
  - introduction of hyperon resonances in the u-channel (destructive interference with other background terms; Saclay-Lyon model)
  - introduction of hadronic form factors (Kaon-MAID model)
  - ignoring the ranges for $g_{K\Lambda N}$ and $g_{K\Sigma N}$

Isobar Model

Contributing Diagrams

Amplitude constructed as a sum of tree-level Feynman diagrams (higher-order contributions – rescattering, FSI – included by means of effective values of the coupling constants)

- background part: Born terms involving an off-shell proton (s-channel), kaon exchange (t) and hyperon exchange (u); non-Born terms: the exchange of (axial) vector kaon resonances (t) and hyperon resonances (u)
- resonant part: s- channel Feynman diagrams with nucleon resonances in the intermediate state



	mass [MeV]	width [MeV]	spin	isospin	parity	*Kaon-MAID*	*Saclay-Lyon*	*Gent IM*	*RPR-2007*	*RPR-2011A*	*RPR-2011B*
$K^*(892)$	892	50	1	1/2	$-$	✓	✓	✓			
$K_1(1270)$	1270	90	1	1/2	$+$	✓	✓	✓			
$P_{11}(1440)$	1440	300	1/2	1/2	$+$		✓				
$S_{11}(1535)$	1535	150	1/2	1/2	$-$					✓	✓
$S_{11}(1650)$	1655	150	1/2	1/2	$-$	✓		✓	✓	✓	✓
$P_{11}(1710)$	1710	100	1/2	1/2	$+$	✓		✓	✓		✓
$P_{11}(1900)$	1895	200	1/2	1/2	$+$					✓	
$P_{13}(1720)$	1720	250	3/2	1/2	$+$	✓	✓	✓	✓	✓	✓
$D_{13}(1895)$	1895	370	3/2	1/2	$-$	✓		✓	✓	✓	✓
$P_{13}(1900)$	1900	250	3/2	1/2	$+$				✓	✓	
$D_{15}(1675)$	1675	150	5/2	1/2	$-$		✓				
$F_{15}(1680)$	1685	130	5/2	1/2	$+$					✓	
$F_{15}(2000)$	2000	140	5/2	1/2	$+$					✓	
$\Lambda(1405)$	1405	50	1/2	0	$-$		✓				
$\Lambda(1600)$	1600	150	1/2	0	$+$						
$\Lambda(1800)$	1800	300	1/2	0	$-$						
$\Lambda(1810)$	1810	150	1/2	0	$+$		✓				
$\Lambda(1520)$	1520	16	3/2	0	$-$						
$\Lambda(1890)$	1890	100	3/2	0	$+$						
$\Sigma(1660)$	1660	100	1/2	1	$+$		✓				
$\Sigma(1750)$	1750	90	1/2	1	$-$						
$\Sigma(1670)$	1670	60	3/2	1	$-$						

A number of contributing resonances results in several versions:

- *Saclay-Lyon*: hyperon resonances $\Lambda(1407)$, $\Lambda(1670)$, $\Lambda(1810)$, $\Sigma(1660)$; nucleon resonances $P_{11}(1440)$, $P_{13}(1720)$, $F_{15}(1680)$; no hadronic form factors
- *Kaon-MAID*: nucleon resonances only $S_{11}(1650)$, $P_{11}(1710)$, $P_{13}(1720)$ a $D_{13}(1895)$ - "missing" resonance
(both models include exchange of $K^*(890)$ and $K_1(1270)$)
- *Gent Isobar Model*, *Adelseck-Saghai model*, *Mart et al.*, etc.

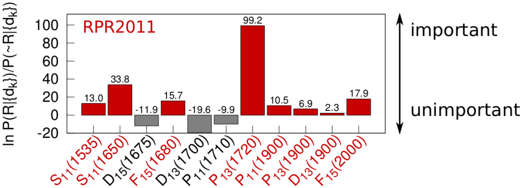
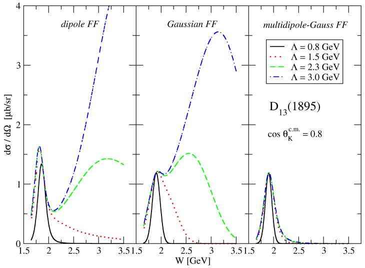


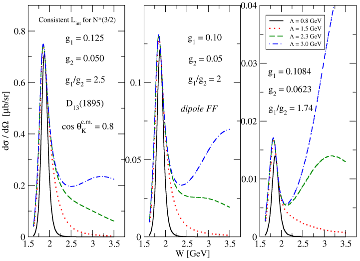
fig. from P. Vancraeyveld: *Bayesian inference of the resonance content of* $p(\gamma, K^+)\Lambda$, MESON 2012

- Claim: dual role of Λ^*
  - reducing the χ^2 value
  - keeping the hadronic cut-off parameter reasonably hard

Isobar Model

Form Factors in the Isobar Model





- Hadrons have internal structure, vertices can not be treated as point-like interactions $\Rightarrow$ use of form factors
- Effectively, the FF ensures that the resonant diagram does not contribute far form the mass pole of the exchanged particle
- With the FF the cut-off dependence is introduced to the cross section $\rightarrow$ the use of **Gaussian** and **dipole** FF creates an artificial cut-off value dependent peak
- The Gent group proposed a **multi-dipole-Gauss** FF as a solution
  - for $J_R = 1/2$ it reduces to the Gauss FF
  - for higher spin, the FF effectively increases the multiplicity of the propagator pole ensuring that the N^* resonates at $s = m_R^2$

- The Rarita-Schwinger propagator for the spin-3/2 fields

$$S_{\mu\nu}(q)=\frac{\not{q}+m}{q^2-m^2+\mathrm{i}m\Gamma}P^{(3/2)}_{\mu\nu}-\frac{2}{3m^2}(\not{q}+m)P^{(1/2)}_{22,\mu\nu}+\frac{1}{\sqrt{3}m}\left(P^{(1/2)}_{12,\mu\nu}+P^{(1/2)}_{21,\mu\nu}\right)$$

- Formerly, inconsistent spin-3/2 $\mathcal{L}_{int}$ were used → propagation of unphysical spin-1/2 modes
- A consistent theory for interacting spin-3/2 fields formulated by V. Pascalutsa (Phys. Rev. **D 58** (1998) 096002)
  → propagation of spin-3/2 modes only (spin-1/2 modes cancel in the amplitude)
- Pascalutsa's spin-3/2 formalism introduced to N*(3/2) as well as to Y*(3/2)
- The spin-3/2-resonance coupling constants have different normalization factors in both prescriptions

$$G_{inc}\sim\frac{g_{K\Lambda R}\,g_{\gamma pR}}{m_R};\quad G_{cons}\sim\frac{g_{K\Lambda R}\,g_{\gamma pR}}{m_R^2 m_K(m_R+m_i)},\quad m_i\equiv m_p \text{ or } m_\Lambda$$

Isobar Model

Newly Introduced Formalism for Spin-3/2 Resonances: $Y^*(3/2)$

Spin-3/2 modes operator

$$P^{(3/2)}_{\mu\nu} = g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{1}{3u}\left(\not{q} q^\nu \gamma^\mu + q^\mu \gamma^\nu \not{q}\right)$$

- u can be zero in the physical region
  $\Rightarrow$ potentially dangerous term
- u vanishes in the amplitude

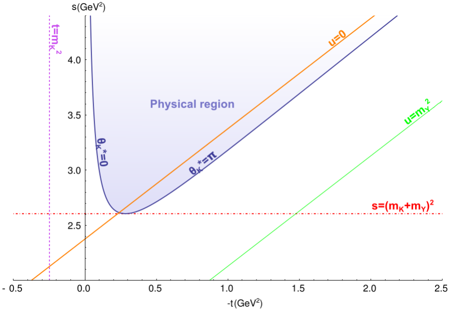


fig. from PhD Thesis of L. De Cruz

$$-f\varepsilon_{\mu\nu\lambda\rho}\mathrm{i}\gamma_5\gamma^\lambda q^\mu p_K^\rho(\not{q}+m_R)\frac{1}{3u}\left(\not{q} q^\beta\gamma^\nu + q^\nu\gamma^\beta\not{q}\right)\left[g_1 q^\alpha(k_\alpha\varepsilon_\beta - k_\beta\varepsilon_\alpha) + g_2\not{q}(k_\beta\not{\varepsilon} - \not{k}\varepsilon_\beta)\right]$$

$$= \frac{-fg_2}{3}\varepsilon_{\mu\nu\lambda\rho}\mathrm{i}\gamma_5\gamma^\lambda q^\mu p_K^\rho(\not{q}+m_R)\left[\gamma^\nu\not{k}(q\cdot\varepsilon) - \gamma^\nu\not{\varepsilon}(q\cdot k)\right]$$

Resonance selection

- K^*: $K^*(892)$, $K_1(1272)$
- N^*: $S_{11}(1535)$, $S_{11}(1650)$, $P_{11}(1900)$, $P_{13}(1720)$, $P_{13}(1900)$, $D_{13}(1900)$, $F_{15}(1680)$, $F_{15}(2000)$ ...inspired by RPR2011 choice
  - inconsistent $N^*(3/2)$ $\mathcal{L}_{int}$: $\chi^2/n.d.f. \approx 2.7$; consistent: $\chi^2/n.d.f. \approx 1.8$
  - no $N^*(5/2) \Rightarrow \chi^2/n.d.f. \approx 2.2$; with $N^*(5/2) \Rightarrow \chi^2/n.d.f. \approx 1.8$
- Y^*: $\Lambda(1405)$, $\Lambda(1600)$, $\Lambda(1800)$, $\Lambda(1810)$, $\Sigma(1660)$, $\Sigma(1750)$, $\Lambda(1520)$, $\Lambda(1890)$, $\Sigma(1670)$
  - interchanging $Y^*(1/2)$ with $Y^*(3/2)$ leads to reducing the Y^* coupling constant values by a factor of 10

20 to 25 free parameters:

- $g_{K\Lambda N}$, $g_{K\Sigma N}$
- K^* have vector and tensor couplings
- spin-1/2 resonance $\rightarrow$ 1 parameter; spin-3/2 and 5/2 resonance $\rightarrow$ 2 parameters
- 2 cut-off parameters for form factor

Around 3400 data points

- cross section (CLAS 2005 & 2010; LEPS, Adelseck-Saghai: Phys. Rev. **C 42** (1990) 108)
- hyperon polarisation (CLAS 2010)
- beam asymmetry (LEPS)

Isobar Model

Fitting Procedure

	n96	n103	n65	n75	f1	f1 (inc)	f3	f4
g(S_{11}(1535))	0.39	0.55	0.51	0.43	0.51	0.57	0.45	0.44
g(S_{11}(1650))	-0.25	-0.28	-0.26	-0.22	-0.26	-0.28	-0.28	-0.27
g_1(P_{13}(1720))	0.08	0.08	0.09	0.07	0.09	0.05	0.10	0.10
g_2(P_{13}(1720))	0.01	0.01	0.01	0.01	0.01	0.66	0.01	0.01
g_1(D_{13}(1895))	0.06	0.08	0.10	0.14	0.10	0.10	0.10	0.11
g_2(D_{13}(1895))	0.04	0.04	0.06	0.08	0.06	0.05	0.06	0.06
g(P_{11}(1900))	0.27	0.18	0.11	0.15	0.11	0.05	0.28	0.32
g_1(P_{13}(1900))	0.10	0.09	0.09	0.08	0.09	0.12	0.07	0.07
g_2(P_{13}(1900))	0.04	0.03	0.03	0.02	0.03	0.68	0.03	0.03
g_1(F_{15}(2000))	—	—	-1.27	-1.34	-1.28	0.33	-0.73	-1.15
g_2(F_{15}(2000))	—	—	-0.83	-1.05	-0.83	-0.03	-0.51	-0.71
g_1(F_{15}(1680))	—	—	-2.63	-4.12	-2.56	0.12	-2.07	-2.99
g_2(F_{15}(1680))	—	—	-0.49	-1.93	-0.43	0.12	-0.92	-1.42
g(Λ(1405))	12.07	15.54	14.01	3.84	14.92	1.40	—	14.58
g(Λ(1600))	—	—	—	-38.38	—	—	—	—
g(Λ(1800))	-24.69	-28.81	-50.00	-50.00	-48.57	-49.99	39.89	—
g(Λ(1810))	50.00	90.00	50.00	50.00	49.89	46.70	—	—
g_1(Λ(1520))	—	—	—	—	—	—	-4.19	—
g_2(Λ(1520))	—	—	—	—	—	—	-1.69	—
g_1(Λ(1890))	—	—	—	—	—	—	4.59	7.96
g_2(Λ(1890))	—	—	—	—	—	—	-0.83	-1.04
g(Σ(1660))	-42.50	-76.89	-43.01	—	-42.97	-49.99	-7.67	-0.47
g(Σ(1750))	—	—	15.60	45.48	11.66	68.78	—	—
g_1(Σ(1670))	—	—	—	—	—	—	9.89	3.62
g_2(Σ(1670))	—	—	—	—	—	—	4.00	1.30
Λ_{bgr}	1.11	1.15	1.19	1.19	1.19	1.08	1.12	1.07
Λ_{res}	1.67	1.65	1.39	1.54	1.38	2.50	1.59	1.49
χ^2/n.d.f.	2.22	2.15	1.77	1.66	1.77	2.71	1.76	1.77

One of the best results → model f4 with $\chi^2/\text{n.d.f.} = 1.77$

- Resonances:
  - K^*: $K^{*+}(892)$, $K_1(1272)$
  - N^*: $S_{11}(1535)$, $S_{11}(1650)$, $P_{11}(1900)$, $P_{13}(1720)$, $D_{13}(1895)$, $P_{13}(1900)$, $F_{15}(1680)$, $F_{15}(2000)$
  - Y^*: $\Lambda(1405)$, $\Sigma(1660)$, $\Lambda(1890)$, $\Sigma(1670)$

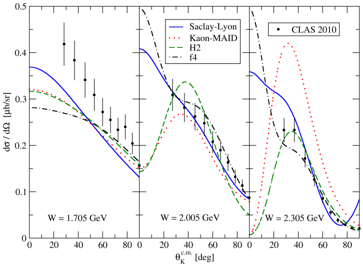
- Hadronic form factor of dipole shape

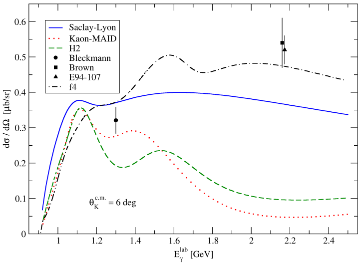
$$F_{dipole}(x) = \frac{\Lambda^4}{\Lambda^4 + (x - m_x^2)^2}$$

  - cut-off parameter for background terms: $\Lambda_{bgr} = 1.0671\,\text{GeV}$
  - cut-off parameter for resonances: $\Lambda_{res} = 1.4942\,\text{GeV}$

Isobar Model

Forward-Angle Predictions





- For $\theta_K < 40°$ predictions of the models differ (more apparent at larger energy)
- Factors concerning model behaviour for small angles:
  - hadron form factor (cut-off dependence: the lower the cut-off value the harder the cross-section suppression)
  - relative sign between vector and tensor coupling of kaon resonances
  - hyperon spin-1/2 resonances
  - nucleon spin-3/2 resonances with consistent formalism
- The lack of experimental data in the very-forward-angle region does not allow to test reliably the models

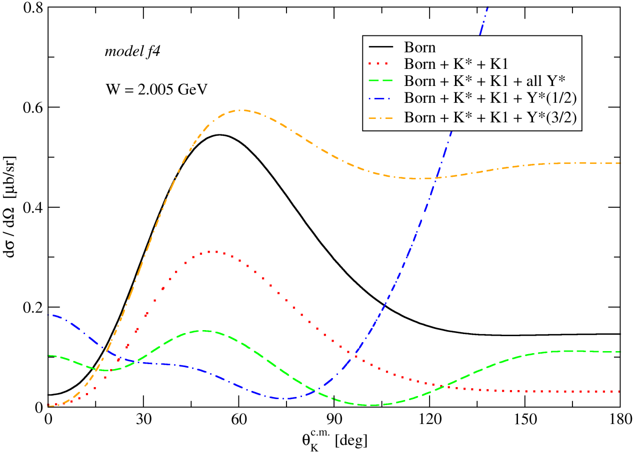


Figure : Forward-angle prediction of the background part of the model f4. The only significant contribution for $\theta_K^{c.m.} \leq 20^\circ$ stems from $Y^*(1/2)$.

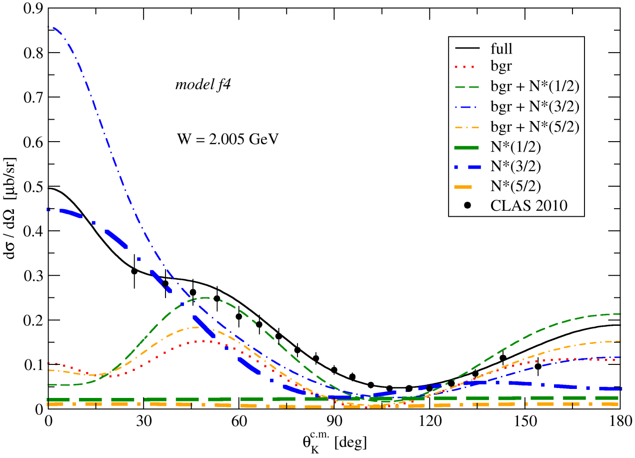


Figure : Forward-angle prediction of N^* with spin 1/2, 3/2 and 5/2.

Isobar Model

Forward-Angle Predictions

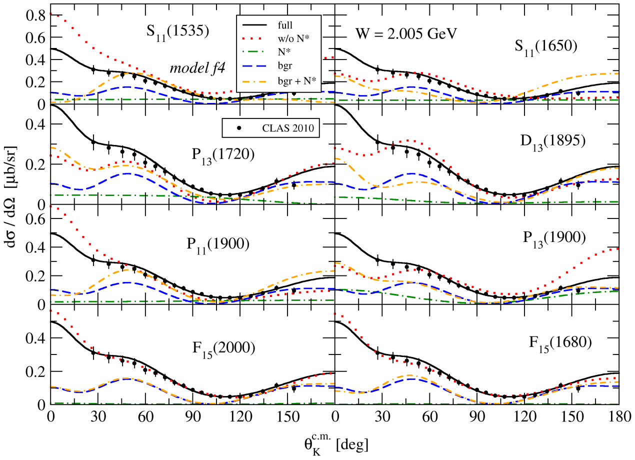


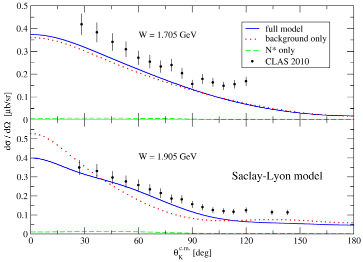
Figure : Contributions of background, different N^* and their combination with background terms to the cross-section prediction in the forward-angle region.

Summary

- Main features of isobar models discussed
  - new consistent formalism for spin-3/2 resonances
    - unphysical lower-spin components vanish
  - the choice of contributing resonances and their role
    - consistent formalism for $N^*(3/2)$ gives lower χ^2 values
    - introducing $Y^*(3/2)$ leads to lower Y^* coupling constants
- Forward-angle predictions of the models (important for calculations of hypernuclei production cross sections) differ significantly
  - new precise data for $\theta_K^{c.m.} = 0 - 20^\circ$ and $W = 2 - 3\,\text{GeV}$ needed to test the models and properly understand the reaction mechanism in this region

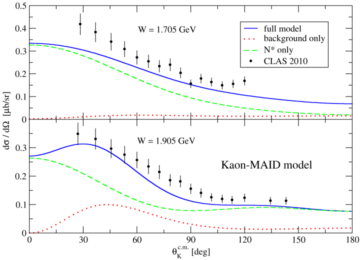
Back Up

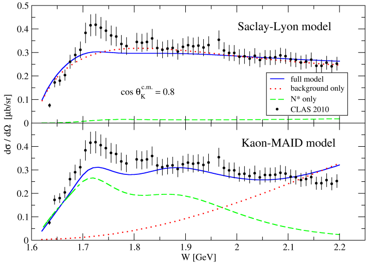
Isobar Model: Contributions of N^* and Background to the Cross Section



- Apart from background and resonant terms, the interference terms also contribute to the cross section
- Schematically:

$$\frac{d\sigma}{d\Omega} \sim \left|\mathbb{M}_{bgr}\right|^2 + \mathrm{Re}\,\mathbb{M}_{bgr}\mathbb{M}^*_{res} + \left|\mathbb{M}_{res}\right|^2$$





- Reaction amplitude consists of several s-, t- and u-channel (non-)Born contributions, *i. e.*

$$\mathbb{M} = \sum_x \mathbb{M}_x, \text{ where } x \equiv s,\ t,\ u,\ N^*,\ K^*,\ Y^*$$

- Each contribution can be written in the compact form

$$\mathbb{M}(p, p_\Lambda, k) = \bar{u}_\Lambda(p_\Lambda)\gamma_5 \left(\sum_{j=1}^{6} \mathcal{A}_j(s,t,u)\mathcal{M}_j \right) u_p(p),$$

where $\mathcal{A}_j$ are scalar amplitudes and $\mathcal{M}_j$ are gauge-invariant operators, *i. e.* $k_\mu \mathcal{M}_j^\mu = 0$, of the form

$$\mathcal{M}_1 = \tfrac{1}{2}\left[\not{k}\,\not{\varepsilon} - \not{\varepsilon}\,\not{k}\right], \qquad \mathcal{M}_2 = (p\cdot\varepsilon) - (k\cdot p)\tfrac{(k\cdot\varepsilon)}{k^2},$$
$$\mathcal{M}_3 = (p_\Lambda\cdot\varepsilon) - (k\cdot p_\Lambda)\tfrac{(k\cdot\varepsilon)}{k^2}, \qquad \mathcal{M}_4 = \not{\varepsilon}(k\cdot p) - \not{k}(p\cdot\varepsilon),$$
$$\mathcal{M}_5 = \not{\varepsilon}(k\cdot p_\Lambda) - \not{k}(p_\Lambda\cdot\varepsilon), \qquad \mathcal{M}_6 = \not{k}(k\cdot\varepsilon) - \not{\varepsilon}k^2.$$

Back Up

Isobar Model: Gauge Invariance

- Gauge invariance related to the principle of charge conservation
- Contributions from the u-channel Born and all non-Born terms are gauge invariant
- Problem arises for the s- and t-channel Born terms which are not individually gauge invariant:

$$\mathbb{M}_{Bs} = \bar{u}_\Lambda(p_\Lambda)\gamma_5 \left[\mathcal{A}_1\mathcal{M}_1 + \mathcal{A}_2\mathcal{M}_2 + \mathcal{A}_4\mathcal{M}_4 + \mathcal{A}_6\mathcal{M}_6 + \mathtt{i}eg_{K\Lambda p}\frac{(k\cdot\varepsilon)}{k^2} \right] u_p(p),$$

$$\mathbb{M}_{Bt} = \bar{u}_\Lambda(p_\Lambda)\gamma_5 \left[\mathcal{A}_2\mathcal{M}_2 + \mathcal{A}_3\mathcal{M}_3 - \mathtt{i}eg_{K\Lambda p}\frac{(k\cdot\varepsilon)}{k^2} \right] u_p(p),$$

- Gauge non-invariant term is canceled in the total amplitude
- Another problem emerges while introducing form factors at hadron vertex $\rightarrow$ addition of contact term needed

- Using de Swart's convention for the unbroken SU(3)$_f$ symmetry, one can determine the two main *KYN* couling constants as

$$g_{K\Lambda N} = \frac{-1}{\sqrt{3}}(3 - 2\alpha_D)g_{\pi NN}, \quad g_{K\Sigma N} = (2\alpha_D - 1)g_{\pi NN}$$

- It is known that the SU(3)$_f$ symmetry is not exact, assuming the symmetry breaking at the level of 20 % one gets the following ranges for the coupling constants (taking experimental value $g^2_{\pi NN}/4\pi = 14.4$ and $\alpha_D = 0.644$)

$$-4.4 \leq \frac{g_{K\Lambda N}}{\sqrt{4\pi}} \leq -3.0, \quad 0.8 \leq \frac{g_{K\Sigma N}}{\sqrt{4\pi}} \leq 1.3$$

- While introducing form factors, the gauge non-invariant terms in s- and t-channel will no longer cancel

$$\mathbb{M}_{s-el} = F_s e\, g_{K\Lambda p} \bar{u}_\Lambda(p_\Lambda)\gamma_5 \frac{\not{p} + \not{k} + m_p}{s - m_p^2}\gamma^\mu u_p(p)\varepsilon_\mu,$$

$$\mathbb{M}_t = F_t e\, g_{K\Lambda p} \bar{u}_\Lambda(p_\Lambda)\gamma_5 \frac{(2p_K - k)^\mu}{t - m_K^2} u_p(p)\varepsilon_\mu$$

- The remedy is to introduce a contact term of the form

$$\mathbb{M}_{contact} = e g_{K\Lambda p} \bar{u}_\Lambda \gamma_5 \left[\frac{2p^\mu + \not{k}\gamma^\mu}{s - m_p^2}(\hat{F} - F_s) + \frac{2p_K^\mu}{t - m_K^2}(\hat{F} - F_t) \right] u_p \varepsilon_\mu$$

- This ensures the gauge invariance:

$$k_\mu(\mathbb{M}^\mu_{s-el} + \mathbb{M}^\mu_t + \mathbb{M}^\mu_{contact}) = 0$$

- Considered Λ^*:

	mass [GeV]	width [GeV]	spin	isospin	parity
$S_{01}(1405)$ 'L1'	1.405	0.050	1/2	0	-1
$P_{01}(1600)$ 'L2'	1.600	0.150	1/2	0	1
$S_{01}(1670)$ 'L3'	1.670	0.035	1/2	0	-1
$S_{01}(1800)$ 'L4'	1.800	0.300	1/2	0	-1
$P_{01}(1810)$ 'L5'	1.810	0.150	1/2	0	1

- Claim: dual role of Λ^*
  - reducing the χ^2 value
  - keeping the hadronic cut-off parameter reasonably hard

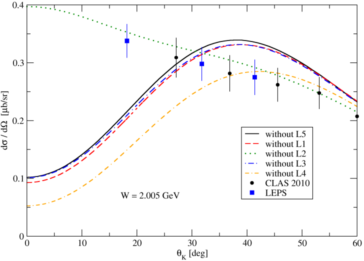
Back Up

Isobar Model: The Role of Hyperon Resonances

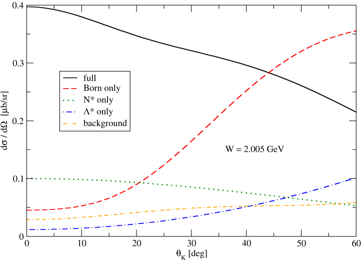
Λ^*	Λ_{bgr} [GeV]	χ^2/n.d.f.	Λ^*	Λ_{bgr} [GeV]	χ^2/n.d.f.
no Λ^*	0.55	1.38	L2+L3+L4	1.09	1.10
L1	0.64	1.29	L3+L4+L5	0.78	1.12
L2	0.90	1.26	L1+L3+L4	0.54	1.03
L3	0.66	1.42	L1+L4+L5	0.73	1.13
L4	0.65	1.49	L2+L4+L5	0.78	1.18
L5	0.87	1.25	L1+L3+L5	0.73	1.15
L1+L2	0.77	1.18	L1+L2+L5	0.75	1.16
L1+L3	0.62	1.41	L1+L2+L3	0.99	1.03
L1+L4	0.65	1.19	L2+L3+L5	0.76	1.17
L1+L5	0.75	1.15	L1+L2+L4	0.76	1.14
L2+L3	0.76	1.17	L1+L2+L3+L4	0.74	1.11
L2+L4	0.77	1.19	L2+L3+L4+L5	0.79	1.13
L2+L5	0.81	1.32	L1+L3+L4+L5	1.25	1.03
L3+L4	0.67	1.40	L1+L2+L4+L5	0.77	1.18
L3+L5	0.77	1.15	L1+L2+L3+L5	1.10	1.42
L4+L5	1.23	1.10	L1+L2+L3+L4+L5	0.98	1.12

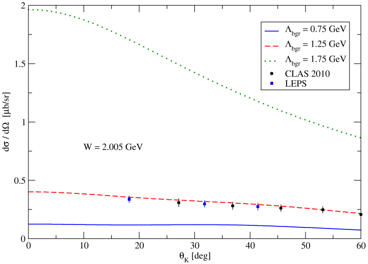
Back Up

Isobar Model: The Role of Hyperon Resonances



- Cross-section predictions of fit without L2 differ significantly from predictions of other fits
  - the difference caused mainly by the value of $\Lambda_{bgr} = 1.25$ (background terms weakly regularised) $\rightarrow$ strong cut-off dependence





- Hadronic form factors
  - dipole: $F_{dipole}(x) = \frac{\Lambda^4}{\Lambda^4+(x-m_x^2)^2}$
  - Gaussian: $F_{Gauss}(x) = \exp\left(-\frac{(x-m_x^2)^2}{\Lambda^4}\right)$
  - Multi-dipole-Gauss:

$$F_{mG}(x) = \left[\frac{m_x^2\tilde{\Gamma}^2}{(x-m_x^2)^2+m_x^2\tilde{\Gamma}^2}\right]^{J-1/2} F_{Gauss}(x),\ \tilde{\Gamma}(J) = \frac{\Gamma}{\sqrt{2^{(1/2J)-1}}}$$

HFF introduced by the method of Davidson and Workman

$$F = F_s(s) + F_t(t) - F_s(s)F_t(t)$$

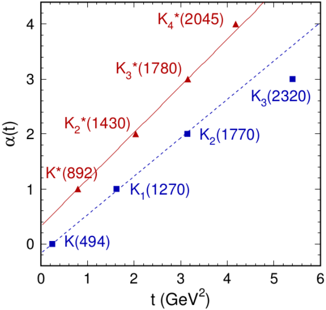
- Framework for the high-energy ($E_\gamma^{lab} > 4\,\mathrm{GeV}$) forward-angle kinematical region
- Instead of the exchange of single particles, the reaction dynamics is governed by the exchange of entire **Regge trajectories**
  - This is achieved by replacing the usual Feynman propagator of a single particle with a Regge one

$$\frac{1}{t - m_X^2} \rightarrow \mathcal{P}^X_{Regge}(\alpha_X(t))$$

  - The Regge amplitude can then be written as

$$\mathbb{M}_{Regge} = \beta_X \mathcal{P}^X_{Regge}(\alpha_X(t))$$

  - The focus on the forward-angle kinematical region implies the exchange of kaonic trajectories in the t-channel



- Regge trajectory ≡ set of mesons sharing the same internal quantum numbers
- The $K(494)$ and $K^*(892)$ trajectories are dominant contributions to the high-energy amplitudes

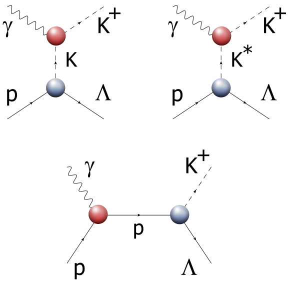
$$\alpha_{K^+}(t) = 0.70\,\mathrm{GeV}^{-2}(t - m_{K^+}^2)$$

$$\alpha_{K^*}(t) = 1 + 0.85\,\mathrm{GeV}^{-2}(t - m_{K^*(892)}^2)$$

- Corresponding Regge propagators are

$$\mathcal{P}_{Regge}^{K(494)}(s,t) = \frac{(s/s_0)^{\alpha_K(t)}}{\sin(\pi\alpha_K(t))}\frac{\pi\alpha'_K}{\Gamma(1+\alpha_K(t))}\left\{\begin{array}{c} 1 \\ e^{-i\pi\alpha_K(t)} \end{array}\right\}$$

$$\mathcal{P}_{Regge}^{K^*(892)}(s,t) = \frac{(s/s_0)^{\alpha_{K^*}(t)-1}}{\sin(\pi(\alpha_{K^*}(t)-1))}\frac{\pi\alpha'_{K^*}}{\Gamma(\alpha_{K^*}(t))}\left\{\begin{array}{c} 1 \\ e^{-i\pi(\alpha_{K^*}(t)-1)} \end{array}\right\}$$



- Exchange of the $K^+(494)$ in the t-channel breaks gauge invariance
  - in the typical effective-Lagrangian theory, the Born terms do not individually obey gauge invariance, but their sum does
  - in addition to the $K^+(494)$ and $K^{*+}(892)$ trajectory exchanges, the Regge amplitude should also include a contribution from the Reggeized electric part of the s-channel Born term

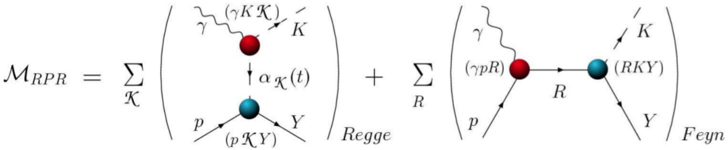
Regge amplitude

$$\mathbb{M}_{Regge} = \mathbb{M}^{K^+}_{Regge} + \mathbb{M}^{K^{*+}}_{Regge} + \mathbb{M}^{p,el}_{Feyn} \cdot \mathcal{P}^{K^+}_{Regge} \cdot (s - m_p^2)$$

Back Up

Regge-plus-resonance Model

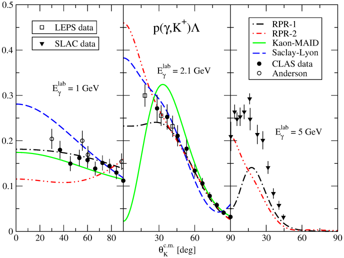
- Description in the energy range from threshold up to $E_\gamma^{lab} \approx 16\,\text{GeV}$
- The nonresonant part of the amplitude modeled by exchanges of $K^+(494)$ and $K^{*+}(892)$ trajectories → only three free parameters of background: $g_{K\Lambda p}$, $G^v_{K^*}$, $G^t_{K^*}$
  - the resonant part described by adding resonant s-channel diagrams with standard Feynman propagators
  - in the high-energy regime, all resonant contributions vanish (thanks to inclusion of a form factor at $K\Lambda R$ vertices) and only the Regge part of the amplitude remains

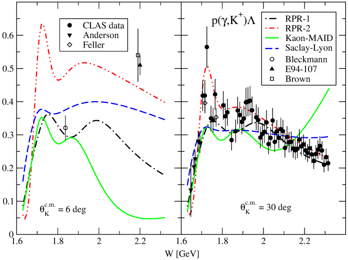


- The unreasonably large Born contribution is absent → no hadronic form factors for the background needed

Back Up

Regge-plus-resonance Model: Comparison with Isobar Models





- Bayesian analysis inspired us to construct new RP models assuming nucleon resonances $S_{11}(1535)$, $S_{11}(1650)$, $P_{11}(1710)$, $P_{13}(1720)$, $D_{13}(1895)$ selected in the analysis
- Attention to the behaviour of the models in the forward-angle region have been paid
- Both models fitted to LEPS and CLAS cross sect data:
  - RPR-1 fitted to the whole angular range
  - RPR-2 fitted only to the forward-angle data ($\theta_K < 90^\circ$)
- Prediction of the models differ significantly for ka angles smaller than 40° (more apparent at larger energy)