TIME DOMAIN DECOMPOSITION METHODS

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Abstract

Many problems in science and engineering are time dependent, and time stepping methods are used to obtain approximate solutions. If the problems are large scale, or solutions are needed in real time, it is necessary to use the computing power of parallel computers. The classical strategy to parallelize time integration is to parallelize the solution at each time step, and to advance sequentially from time step to time step. This approach however neglects an entire dimension, the time dimension, which could also be used for the parallelization. In contrast to the spatial dimensions, the time dimension has however a direction: the solution later in time depends only on the solution earlier in time, and not vice versa. It therefore seems difficult to do useful computations at a future time step, before the present time step results are known.

There are several algorithms which nevertheless try to do useful computations later in time, before fully accurate results at the present time step are available, and one of the more recent ones is based on multiple shooting: the parareal algorithm. This algorithm is using an approximation of the Jacobian on a coarse grid in the Newton iteration classically used for solving the shooting equations. After reviewing a compact convergence result for this algorithm, I will illustrate its numerical performance for several examples of systems of ordinary and partial differential equations. These examples reveal that while the algorithm performs well for diffusive problems, convergence is unsatisfactory for hyperbolic equations. I will then explain as possible remedies for this problem the Krylov parareal algorithm, and also a different time parallel method called ParaExp. I will finally show two further developments, a space-time algorithm based entirely on multigrid techniques, and a space-time algorithm based on space-time domain decomposition.