NUMERICAL BEHAVIOR OF INEXACT ITERATIVE SOLVERS

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Abstract

In this contribution we study numerical behavior of several stationary or two-step splitting iterative methods for solving large sparse systems of linear equations. We show that inexact solution of inner systems associated with the splitting matrix may considerably influence the accuracy of computed approximate solutions computed in finite precision arithmetic. We analyze several mathematically equivalent implementations and find the corresponding component-wise or norm-wise forward or backward stable implementations. The theory is then illustrated on the class of efficient two-step iteration methods such as Hermitian and skew-Hermitian splitting methods. We can show that some implementations lead ultimately to errors and residuals on the the roundoff unit level independently of the fact that the inner systems with the splitting matrix were solved inexactly on a much higher level (in practical situations this level corresponds to the uncertainty of input data or imperfection of underlying mathematical model). We give a theoretical explanation for this behavior which is intuitively clear and it is probably tacitly known. Indeed, our results confirm that implementations with simple updates for approximate solutions can solve the algebraic problem to the working accuracy. These implementations are actually those which are widely used and suggested in applications. Our results are examples of rather general fact that it is advantageous to use the update formulas in the form "new_value = old_value + small_correction". Numerical methods are often naturally expressed in this form and in a sense this update strategy can be seen as variant of the iterative refinement for improving the accuracy of a computed approximate solution to various problems in numerical linear algebra.