ON SECOND ORDER IN TIME NUMERICAL SCHEMES FOR THE CAHN-HILLIARD EQUATION

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Abstract

The study of interfacial dynamics in the mixture of different fluids, solids, or gas is one of the fundamental issues in hydrodynamics and materials science due it plays an increasingly important role in many current scientific, engineering, and industrial applications. The diffuse interface model describes the interface by a mixing energy represented as a layer of small thickness. This idea can be traced to van der Waals, and is the foundation for the phase-field theory for phase transition and critical phenomena. Thus, the structure of the interface is determined by molecular forces; the tendencies for mixing and de-mixing are balanced through the non-local mixing energy. Our aim is to design numerical schemes to approximate phase field models in different applications such as phase separation, liquid crystals, vesicle membranes, image processing, biofilm deformation . . . The key point is to try to preserve the properties of the original models while the numeri cal schemes are efficient in time. For these purpose, the adaptivity of the mesh (in time and space) is going to be a critical point.

The Cahn-Hilliard model was originally introduced by Cahn and Hilliard and describes the complicated phase separation and coarsening phenomena in the mixture of different fluids, solid or gas where only two different concentration phases can exist stably.

This work is devoted to studying the numerical aspects of the Cahn-Hilliard model, although the main ideas are going to be also valid for other phase field models. In order to obtain numerical schemes with an affordable computational time, we have designed a new second order linear scheme, and we have compared it with other second order in time numerical schemes, that have been previously introduced in the literature. We study the constraints on the physical and discrete parameters that can appear to assure the energy-stability, unique solvability and, in the case of nonlinear schemes, the convergence of Newton?s method to the nonlinear schemes.

Moreover, in order to save computational cost we have developed a new adaptive time stepping algorithm based on the numerical dissipation introduced in the discrete energy law in each time step.

Finally, we compare the behaviour of the schemes and the effectiveness of the adapt-time algorithm through several computational experiments.