MIXED FINITE ELEMENT METHODS: REDUCTION TO ONE UNKNOWN PER ELEMENT

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Abstract

We consider the lowest-order Raviart–Thomas–Nédélec mixed finite element method for the numerical approximation of the Poisson equation. This method stems from a saddle-point weak variational formulation and naturally leads to unknowns simultaneously approximating the scalar potential (one primal unknown per element) and the vector flux (one dual unknown per face). We show how this method can equivalently be reduced to the primal scalar unknowns only. Several rather surprising properties are unveiled, like the existence of local flux expressions in mixed finite element methods and their tight links to seemingly different numerical schemes (finite volumes, mimetic finite differences). The striking consequences for the linear solvers are discussed and illustrative numerical experiments are presented. Finally, extensions to unsteady nonlinear problems are presented.