On an augmented Lagrangian-based preconditioning of Oseen type problems

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We consider a general framework for constructing preconditioners for saddle point matrices, in particular as arising in the discrete linearized Navier-Stokes equations (Oseen's problem). We utilize the so-called augmented Lagrangian approach, where the original linear system of equations is first transformed to an equivalent one, which latter is then solved by a preconditioned iterative solution method.

In the augmented Lagrangian formulation, a scalar regularization parameter is involved, which strongly influences the quality of the preconditioners for the system matrix (referred to as outer), as well as the conditioning and the solution of systems with the resulting pivot block (referred to as inner) which, in the case of large scale numerical simulations has also to be solved using an iterative method. We analyse the impact of the value of the regularization parameter on the convergence of both outer and inner solution methods.

One often used preconditioner exploits the inverse of the pressure mass matrix. We study the effect of various approximations of that inverse on the performance of the preconditioners, in particular that of a sparse approximate inverse, computed in an element-by-element fashion. We analyse and compare the spectra of the preconditioned matrices for the different approximations and show that the resulting preconditioner is independent of problem, discretization and method parameters, namely, viscosity, mesh size, mesh anisotropy. We also discuss possible approaches to solve the modified pivot matrix block.