TRACKING THE TRAJECTORY IN FINITE PRECISION CG COMPUTATIONS

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Abstract

The conjugate gradient method (CG) for solving linear systems of algebraic equations represents a highly nonlinear finite process and thus the general description of the CG rate of convergence using an asymptotic convergence factor has principal limitations. Moreover, the CG method is computationally based on short recurrences and its behaviour in finite precision arithmetic is therefore affected by the possible loss of orthogonality among the computed direction vectors. Consequently, effects of rounding errors must be taken into account in all considerations concerning practical CG computations. As an example, we show that the composite polynomial convergence bounds based on explicit annihilation of the large outlying eigenvalues (which hold assuming exact arithmetic) must inevitably fail in finite precision CG computations.

Whereas the CG convergence rate may substantially differ in finite precision and exact computations, we observe that the trajectory of the approximations is very similar. Results of finite precision computations shifted back by the numerical rank-deficiency of the computed Krylov subspaces seem to closely correspond to the results of exact computations. Moreover, we observe that the computed rank-deficient Krylov subspace span numerically nearly the same subspace as the Krylov subspace of the corresponding rank generated by the CG method in exact arithmetic.