A POSTERIORI ERROR ESTIMATION FOR STOCHASTIC GALERKIN APPROXIMATION

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Abstract

Stochastic Galerkin finite element approximation is an increasingly popular approach for the solution of elliptic PDE problems with correlated random data. Given a parametrisation of the data in terms of a large, possibly infinite, number of random variables, this approach allows the original PDE problem to be reformulated as a parametric, deterministic PDE on a parameter space of high, possibly infinite, dimension. A typical strategy is to combine conventional (h-) finite element approximation on the spatial domain with spectral (p-) approximation on a finite-dimensional manifold in the (stochastic) parameter space. For approximations relying on low-dimensional manifolds in the parameter space, stochastic Galerkin finite element methods have superior convergence properties to standard sampling techniques. On the other hand, the desire to incorporate more and more parameters (random variables) together with the need to use high-order polynomial approximations in these parameters inevitably generates very high dimensional discretised systems. This in turn means that adaptive algorithms are needed to efficiently construct approximations, and fast and robust linear algebra techniques are essential for solving the discretised problems.

Both strands will be discussed in the talk. We outline the issues involved in a posteriori error analysis of computed solutions and present a practical a posteriori estimator for the approximation error. We introduce a novel energy error estimator that uses a parameter-free part of the underlying differential operator—this effectively exploits the tensor product structure of the approximation space and simplifies the linear algebra. We prove that our error estimator is reliable and efficient. We also discuss different strategies for enriching the discrete space and establish two-sided estimates of the error reduction for the corresponding enhanced approximations. These give computable estimates of the error reduction that depend only on the problem data and the original approximation. We show numerically that these estimates can be used to choose the enrichment strategy that reduces the error most efficiently.