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*Fuzzy Sets and Fuzzy Classes in Universes of Sets*

Traditionally, the concept of fuzzy set is closely related to a fixed set of interesting objects called the “universe of discourse” and fuzzy set theory is developed over the set of all fuzzy subsets of this universe. However, this restriction to one universe has several disadvantages. Among others, the assumption of a fixed universe of discourse for all fuzzy sets brings a certain limitation on fuzzy sets constructions. For example, the concept of power fuzzy set cannot be introduced. It means that a “rich” fuzzy set theory cannot be developed on a set of all fuzzy subsets of a fixed universe of discourse. It should be noted that similar disadvantages have been recognized by other authors, namely, S. Gottwald in [2] where he proposed a cumulative system of fuzzy sets, L. Běhounek and P. Cintula in [1] where they proposed a fuzzy class theory admitting fuzzy sets of higher orders.

In the presentation, we will introduce a universe of sets and demonstrate certain elements of fuzzy set theory built over this universe of sets. The universe of sets (over a complete residuated) is a class of sets that satisfies the axioms of the Grothendieck universe (see [3]) and one additional axiom that ensures the existence of fuzzy sets in this universe. For example, the class of all sets or finite sets form a universe of sets. Fuzzy sets in a universe of sets are identified with functions of sets of a universe of set into the support of a given complete residuated lattice. Since fuzzy sets are special sets (relations) in the universe of sets, all basic constructions for sets as the union, intersection, product, power, exponentiation can be introduced for fuzzy sets. Also basic relations for sets as “to be identical”, “to be a subset” or “to be equipollent” can be introduced for fuzzy sets including their graded versions. For this purpose, we introduce the auxiliary concept of fuzzy class in the universe of sets. Fuzzy classes are identified with functions of subsets of the universe of sets into the support of a given residuated lattice. It can be shown that each fuzzy set is a fuzzy class, but not vice-versa. For example, the graded equipollence is a proper fuzzy class in the universe of sets.

*Acknowledgment*

This work was supported by the European Regional Development Fund in the IT4Innovations Centre of Excellence project (CZ.1.05/1.1.00/02.0070)

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