HOW TO PROVE CONSERVATIVITY BY MEANS OF KRIPKE MODELS

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We say that a classical first-order theory is conservativity over its intuitionistic counter-part with respect to a class of formulae Γ, if both theories prove exactly the same formulae of this class. A typical example of a conservativity result states that Peano Arithmetic (PA) is Π_2-conservative over Heyting Arithmetic (HA). It can be proven in several ways. For example, the so-called (Gödel-Gentzen) negative translation together with the Gödel functional interpretation of HA or proof theoretic analysis of HA can be used. This fact can be also proven by means of the negative translation and the so-called Friedman translation. This approach can be applied also to other theories including set theory.

In this talk I will describe conservativity of classical first-order theories over their intuitionistic counterparts from a semantic perspective. In particular, we consider properties of a class of Kripke models for a given intuitionistic theory that are sufficient to prove conservativity results. We also describe a class of formulae for which such results can be proven.

Our idea is as follows: In order to prove classically that a classical theory T^{c} is Γ-conservative over its intuitionistic counterpart T^{i}, we may show that any formula from Γ which is not derivable intuitionistically in T^{i} is also not derivable classically in T^{c}. So, assume that A is a formula that belongs to the given class Γ and that A is not derivable intuitionistically from Γ. By the strong completeness theorem for Kripke semantics, we can find a Kripke model $\mathcal{M}$ of T^{i} such that $\mathcal{M}$ refutes A. Now we need to find a classical structure which is a counter-model for A and a model of the theory T^{c}. The most natural idea is to look for such a counter-model among the worlds of the Kripke model $\mathcal{M}$. We show that under suitable assumptions concerning models of the theory T^{i} (and some assumptions on T^{i} itself) this can be done.

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