Paraconsistent Topologies for Games

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Topological semantics appears to be the first semantics suggested for modal logic in 1938 by Tsao-Chen [9]. Picking up from Tsao-Chen's work, McKinsey (later with Tarski) incorporated various other algebraic and topological tools into modal logic, always remaining within the limits of classical logic [4, 6, 5, 7]. However, the strength of topological semantics arguably comes from its versatility. Topological primitives can be used to give meaning for intuitionistic, paraconsistent and modal logics allowing us to analyze topological spaces from a semantical view point [8, 1]. A further step can be taken to use paraconsistent logic with topologies in game theory. This line of thought supports the argument that the theory of games can be viewed as a basis for the ontological possibility of paraconsistency. It also relates the subject to rationality and interactive decision making. Therefore, non-classical logic for game theory suggests a broader (and perhaps relaxed or more precise) reading for the overloaded primitives of game theory.

In this work, I have two foci. First, I will briefly discuss an application of paraconsistent topologies to a well-known epistemic game theoretical *paradox*, called the Brandenburger - Keisler paradox [3, 2]. The paradox can be viewed as a two-person Russell's paradox which relates game theoretical agents' beliefs and assumptions in a self-referential fashion. First, I will consider it from a paraconsistent view point supported with a topological semantics for epistemic modal operators. I will allow inconsistent beliefs and assumptions in games and benefit from a variety of topological tools to express multiple players' interaction. The paradox, within classical logic, suggests for various impossibility results, and I will offer a framework that can render some of such impossibilities possible. I will also relate this discussion to a broader solution concept in game theory: backward induction. I will show how backward induction can be obtained in paraconsistent models for dynamic epistemologies under certain assumptions.

My second focus will be to reformulate the Brandenburger - Keisler paradox, which is a self-referential paradox, in the style of Yablo's paradox, which is a *non*-self-referential paradox [10]. I will observe that the methods which renders the paradoxical Brandenburger - Keisler argument satisfiable in topological paraconsistent models may not be sufficient to give a paraconsistent model for the paradox. *En passant*, I will underline the game theoretical interpretation of Yablo's paradox, and raise some questions about strict finitism within game theory.

Finally, I will conclude arguing why paraconsistent approaches to game theory make sense, and how/why the mathematics of rational interactive decision making and reasoning must be given a paraconsistent (and paracomplete) analysis.

References

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