A note on naive set theory in an expansion of **LP**

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Institute of Computer Science
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Outline

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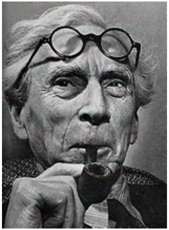
The problem



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It is proved by Russell that

- Axiom (COMP) of naive set theory and
- classical logic

are incompatible in the sense that theory turns out to be trivial.

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- Axiom (COMP) of naive set theory and
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Dialetheic approach!

Priest's motivation

I wish to claim that (COMP) and (EXT) are true, and in fact that they analytically characterise the notion of set. [In Contradiction, p.30]

Call for dialetheias

There are true contradictions (dialetheias) such as $R \in R \wedge {\sim}(R \in R)$.

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Which paraconsistent logic?

- Stanisław Jaśkowski (1948)
- Newton da Costa (1960s)
- Alan Anderson & Nuel Belnap (1975)
- Graham Priest (1979)
- etc.

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Priest's criticism against da Costa systems and Boolean negation

- And in da Costa systems, C_i, for finite i, an operator behaving like classical negation, $\neg^*$ can be defined. The usual arguments establish contradictions of the form $A \wedge \neg^* A$, and so again the theories explode. [PL, pp.350–351]
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An observation

Priest can love da Costa! (Omori, 2015)

- Take **LP**-based naive set theory (Restall, 1992).
- Add the consistency operator to **LP** to get **LFI1**.
- Keep the comprehension as it is in **LP**-based theory.
- Naive set theory based on **LFI1** is non-trivial by following the proof of Restall!

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Question: which logic shall we use?

If Priest can love da Costa . . .

Now we have even more options to choose an underlying logic!

What is logic?

In the context of considering formal theories, one may view propositional logic as the most abstract structure in the following sense.

- As an illustration, consider arithmetic.
- First, strip off all the axioms unique to arithmetic. This leaves us with predicate logic.
- Second, ignore the internal structure of the sentences. This leaves us with the propositional logic.

Then, in the case of classical arithmetic, we have $\top$ in propositional logic. But what is the characteristic feature of dialetheic theories?

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CL?

No, since we need to deal with contradictions.

FDE?

No, since we wish to take realistic attitude toward mathematics.

LP?

No, since we want to keep the possibility of truth-untruth talk.

LP plus '$\circ$'?

No, since we want to reflect the presence of dialetheias, just as we have just true and just false sentences.

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Outline

Definition

The languages $\mathcal{L}$, $\mathcal{L}_\perp$ and $\mathcal{L}_\circ$ consist of a denumerable set, Prop, and the set of logical symbols $\{\sim, \wedge, \vee, \rightarrow\}$, $\{\sim, \wedge, \vee, \rightarrow, \perp\}$ and $\{\sim, \wedge, \vee, \rightarrow, \circ\}$ respectively.

Definition

- **CLuNs** in $\mathcal{L}$ consists of the following axioms plus $\mathbf{CL}^+$:

$$A \vee \sim A \qquad \sim(A \wedge B) \leftrightarrow (\sim A \vee \sim B) \qquad \sim(A \vee B) \leftrightarrow (\sim A \wedge \sim B)$$
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Dialetheic extension of **LFI1**

Definition

A logic **L** is dialetheic iff for some A, $\vdash_{\mathbf{L}} A$ and $\vdash_{\mathbf{L}} \sim A$.

Fact

LFI1 is not dialetheic.

Definition

Let **dLP** be a variant of **LFI1** obtained by replacing

$$\sim(A \to B) \leftrightarrow (A \wedge \sim B)$$

by

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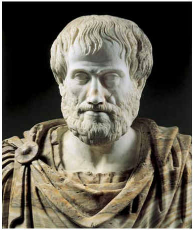

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The new axiom is not new, but used by Heinrich Wansing in developing a system of connexive logic **C**. Connexive logics has theorems such as:

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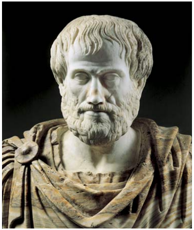

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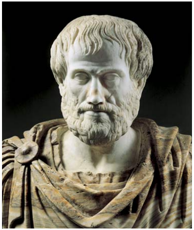

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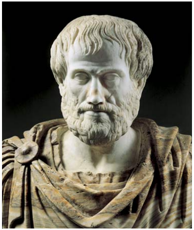

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Basic results (I)

Proposition

- $\vdash_{\mathbf{dLP}} \sim\neg A$ for any A where $\neg A = A \to \bot$ is a classical negation.
- $\nvdash_{\mathbf{dLP}} \sim\neg^* A$ for some A where $\neg^* A = \sim A \wedge \circ A$ is a classical negation.

Propsition

dLP is dialetheic and connexive. In particular, we have the following theorems:

- $\vdash_{\mathbf{dLP}} (A \wedge \neg A) \to B$
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Basic results (II)

Theorem

dLP is complete with respect to the semantics in which the truth table for propositional connectives are as follows:

A	$\sim A$	$\circ A$
t	**f**	**t**
b	**b**	**f**
f	**t**	**t**

$A \wedge B$	**t**	**b**	**f**
t	**t**	**b**	**f**
b	**b**	**b**	**f**
f	**f**	**f**	**f**

$A \vee B$	**t**	**b**	**f**
t	**t**	**t**	**t**
b	**t**	**b**	**b**
f	**t**	**b**	**f**

$A \rightarrow B$	**t**	**b**	**f**
t	**t**	**b**	**f**
b	**t**	**b**	**f**
f	**b**	**b**	**b**

Remark

Semantic clauses for $\rightarrow$ in terms of Dunn semantics are as follows:

$1 \in v(A \rightarrow B)$ iff if $1 \in v(A)$ then $1 \in v(B)$.

$0 \in v(A \rightarrow B)$ iff if $1 \in v(A)$ then $0 \in v(B)$.

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t	**t**	**b**	**f**
b	**b**	**b**	**f**
f	**f**	**f**	**f**

$A \vee B$	**t**	**b**	**f**
t	**t**	**t**	**t**
b	**t**	**b**	**b**
f	**t**	**b**	**f**

$A \rightarrow B$	**t**	**b**	**f**
t	**t**	**b**	**f**
b	**t**	**b**	**f**
f	**b**	**b**	**b**

Remark

Semantic clauses for $\rightarrow$ in terms of Dunn semantics are as follows:

$1 \in v(A \rightarrow B)$ iff if $1 \in v(A)$ then $1 \in v(B)$.

$0 \in v(A \rightarrow B)$ iff if $1 \in v(A)$ then $0 \in v(B)$.

Basic results (II)

Theorem

dLP is complete with respect to the semantics in which the truth table for propositional connectives are as follows:

A	$\sim A$	$\circ A$
t	**f**	**t**
b	**b**	**f**
f	**t**	**t**

$A \wedge B$	**t**	**b**	**f**
t	**t**	**b**	**f**
b	**b**	**b**	**f**
f	**f**	**f**	**f**

$A \vee B$	**t**	**b**	**f**
t	**t**	**t**	**t**
b	**t**	**b**	**b**
f	**t**	**b**	**f**

$A \to B$	**t**	**b**	**f**
t	**t**	**b**	**f**
b	**t**	**b**	**f**
f	**b**	**b**	**b**

Remark

Semantic clauses for $\to$ in terms of Dunn semantics are as follows:

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dLP is complete with respect to the semantics in which the truth table for propositional connectives are as follows:

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t	**f**	**t**
b	**b**	**f**
f	**t**	**t**

$A \wedge B$	**t**	**b**	**f**
t	**t**	**b**	**f**
b	**b**	**b**	**f**
f	**f**	**f**	**f**

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t	**t**	**t**	**t**
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Further results (I): functional completeness

Definition

A matrix $\langle \mathfrak{A}, \mathcal{D} \rangle$ where $\mathfrak{A} = \langle \mathcal{V}, f_1, \ldots, f_n \rangle$, is functionally complete iff every function $f : \mathcal{V}^n \to \mathcal{V}$ is definable by superpositions of $f_1, \ldots, f_n$ alone.

Theorem (Słupecki)

$\mathfrak{A}$ ($\sharp\mathcal{V} \geq 3$) is functionally complete iff in $\mathfrak{A}$

(i) all unary functions on $\mathcal{V}$ are definable, and

(ii) at least one surjective and essentially binary function on $\mathcal{V}$ is definable.

Theorem

The matrix complete with respect to **dLP** is functionally complete

Remark

The variant of **CLuNs**$^{\perp}$ (cf. Cantwell) is strictly weaker than **dLP**.

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Definition

The logic **L** is Post complete iff for every formula A such that $\nvdash A$, extension of **L** by A becomes trivial, i.e. $\vdash_{\mathbf{L}\cup\{A\}} B$ for any B.

Theorem (Tokarz)

If **L** is complete with respect to a matrix which is functionally complete, then **L** is Post complete.

Corollary

dLP is Post complete.

Remark

Unlike other systems of paraconsistent logic in the literature, **dLP** shares a lot of properties with **CL**.

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Outline

1. Background
2. Logic: a dialetheic expansion of **LP**
3. Naive set theory: a rough sketch
4. Conclusion

Setting up the theory

Formulating naive set theory

Let **N** be the set of all instances of the comprehension schema along with the axiom of extensionality stated as follows:

(COMP) $$\exists x \forall y (y \in x \equiv A(y))$$

for each A in which x is not free, and

(EXT) $$\forall x \forall y ((\forall z (z \in x \equiv z \in y)) \supset x = y)$$

where $x = y := \forall z (x \in z \equiv y \in z)$ and $A \equiv B := (A \supset B) \wedge (B \supset A)$.

Remark

- If we formulate (COMP) in terms of $\leftrightarrow$, then the triviality is back.
- The biconditional $\equiv$ is quite weak.

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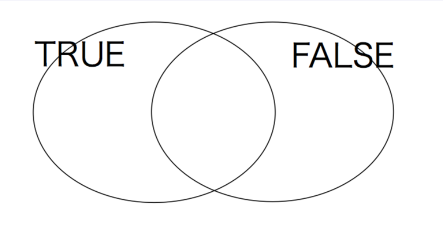
Material biconditional: some remarks

Reading of material biconditional in **dLP**

$A \equiv B$ iff A and B are in the same area:



This also explains the weakness of $\equiv$ as well.

A comparison

- $1 \in v(A \equiv B)$ iff $(1 \in v(A)$ & $1 \in v(B))$ or $(0 \in v(A)$ & $0 \in v(B))$.
- $1 \in v(A \leftrightarrow B)$ iff $(1 \in v(A)$ & $1 \in v(B))$ or $(1 \notin v(A)$ & $1 \notin v(B))$.

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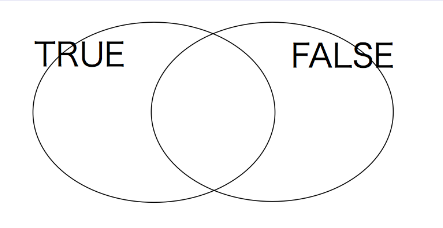

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Some possible enrichments

Definition

Let $\mathbf{N}_i$ be the set of all instances of the comprehension schema (COMP) along with one of the axioms of extensionality (EXTi) ($1 \leq i \leq 5$) stated as follows:

(EXT1)	$\forall x \forall y((\forall z(z \in x \equiv z \in y)) \rightarrow x = y)$
(EXT2)	$\forall x \forall y((\forall z(z \in x \leftrightarrow z \in y)) \supset x = y)$
(EXT3)	$\forall x \forall y((\forall z(z \in x \leftrightarrow z \in y)) \rightarrow x = y)$
(EXT4)	$\forall x \forall y((\forall z(z \in x \equiv z \in y)) \rightarrow x =_+ y)$
(EXT5)	$\forall x \forall y((\forall z(z \in x \leftrightarrow z \in y)) \rightarrow x =_+ y)$

where $x = y := \forall z(x \in z \equiv y \in z)$ and $x =_+ y := \forall z(x \in z \leftrightarrow y \in z)$.

Some results of possible enrichments (I): basics

Theorem

N and its variants $\mathbf{N}_i$s based on **dLP** are non-trivial.

Proposition: 'empty' set

$\mathbf{N} \vdash_{\mathbf{dLP}} \exists x \forall y \sim(y \in x)$. Moreover, in $\mathbf{N}_1$ and $\mathbf{N}_4$, the 'empty' set is unique with respect to the equalities $=$ and $=_+$ respectively.

Proposition: 'empty' set is not empty!

$\mathbf{N} \nvdash_{\mathbf{dLP}} \exists x \forall y \neg(y \in x)$.

Proposition: universal set

$\mathbf{N} \vdash_{\mathbf{dLP}} \exists x \forall y (y \in x)$. Moreover, in $\mathbf{N}_1$ and $\mathbf{N}_3$, the universal set is unique with respect to the equality $=$, and in $\mathbf{N}_4$ and $\mathbf{N}_5$, the universal set is unique with respect to both equalities $=$ and $=_+$.

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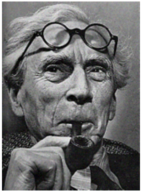
Some results of possible enrichments (II): Russell and Curry




Fact

We get the following through (COMP).

- If $A(x) := \neg(x \in x)$, then $\mathbf{N} \vdash_{\mathbf{dLP}} \exists x(x \in x \wedge \sim(x \in x))$.
- If $A(x) := \sim(x \in x)$, then again $\mathbf{N} \vdash_{\mathbf{dLP}} \exists x(x \in x \wedge \sim(x \in x))$.
- If $A(x) := x \in x \to B$, then $\mathbf{N} \vdash_{\mathbf{dLP}} \exists x(\sim(x \in x) \vee B)$.

Remark

Curry's predicate now does not have anything to do with contradictions!

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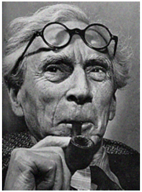

Fact

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$\mathbf{N} \vdash_{\mathbf{dLP}} \forall x({\sim}(x = x) \rightarrow \forall y({\sim}(x = y)))$.

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$\mathbf{N} \vdash_{\mathbf{dLP}} \forall x(x{=_+}x \wedge {\sim}(x{=_+}x))$.

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Maybe, this might be a reason to prefer $=$ over $=_+$. Moreover, if we define equality in terms of material biconditional defined by classical negation, then this will not be the case.

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A glance at further enrichment (I)

Problem

We still don't have any clue for the truth-untruth perspective for $\in$.

Idea

Add some ZFC axioms to talk about truth-untruth aspect of $\in$?

However, we cannot add them directly:

Fact

N' together with (SEP) based on **dLP** is trivial.

(SEP) $\forall z \exists x \forall y (y \in x \leftrightarrow y \in z \wedge A(y))$

Proof.

By the existence of universal set in **N'**. □

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We may consider the following formulations:

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Can we prove the relative non-triviality of extended system with respect to **ZF** (or **ZFC**)?

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If we can prove the above result, then dialetheic mathematics can be seen as an extension of classical mathematics!

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The intuitive reading is lost in the biconditional of **FDE**.

Keep the intuition!

Another biconditional: $A \equiv^* B := (A \wedge B) \vee (\sim A \wedge \sim B)$

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$A \equiv^* A$ does not hold. A and A are not in the same area?

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If we keep dialetheic *and* anti-realistic attitude towards mathematics, then getting an intuitive formulation of naive set theory will be not obvious.

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1. Background
2. Logic: a dialetheic expansion of **LP**
3. Naive set theory: a rough sketch
4. Conclusion

Conclusion

Summary

Under a specific understanding of logic:

- Developed a dialetheic logic **dLP**. Recipe: take Priest, then first da Costize and second Wansingize it! ($\{\sim, \circ, \rightarrow\}$: functionally complete)
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Big picture

We might be able to extend classical mathematics to accommodate some of inconsistencies without falling into triviality.

Future directions

Explore the theory further!

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DĚKUJI!