A self-consistent equation of motion multiphonon method for even and odd mass nuclei

G. De Gregorio
•Università di Napoli Federico II
•INFN

In Collaboration with

N. Lo Iudice, F. Andreozzi ,A. Porrino
Univ. Federico II & INFN Napoli

F. Knapp **Charles University (Prague)**
P. Veselý NPI Řež (Prague)

28th Indian-Summer School of Physics

Semiclassical

$\{\alpha_\lambda, \pi_\lambda\} \longrightarrow \{O_\lambda, O_\lambda^\dagger\}$

EofM

$[H, O_\lambda^\dagger] = \hbar\omega_\lambda O_\lambda^\dagger$

Collective modes

Microscopic

TDA mapping

$O_\lambda^\dagger = \Sigma_{ph}\, c_{ph}(\lambda)\, a^\dagger_p\, a_h$

EoM

$[H, O_\lambda^\dagger]\, |> = \hbar\omega_\lambda\, O_\lambda^\dagger\, | >$

RPA mapping

$O_\lambda^\dagger = \Sigma_{ph} [X_{ph}(\lambda)\, a^\dagger_p\, a_h - Y_{ph}(\lambda)\, a^\dagger_h\, a_p]$

EoM

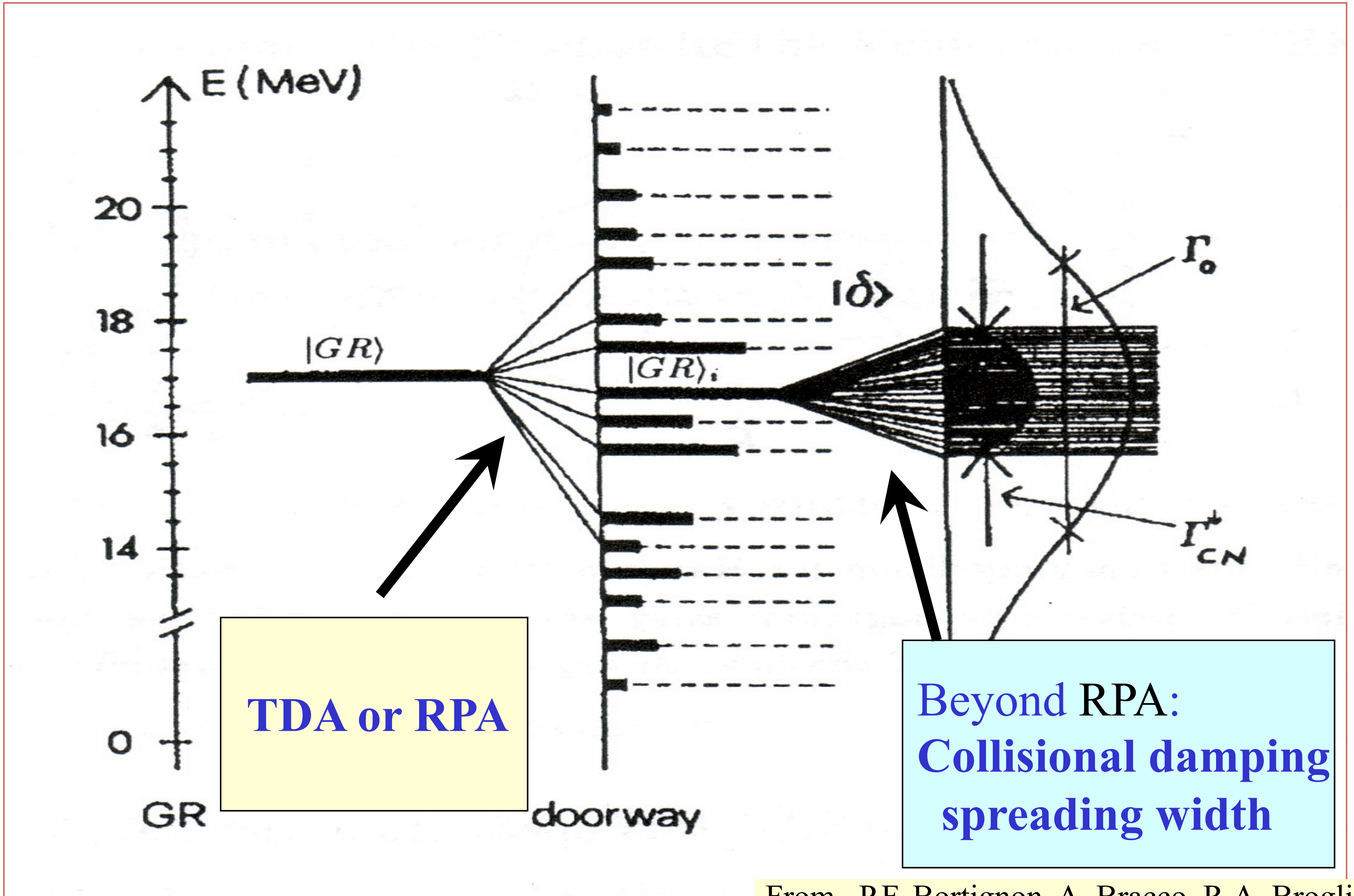
$[H, O_\lambda^\dagger]\, |0> = \hbar\omega_\lambda\, O_\lambda^\dagger\, |0>$

$|0> \equiv$ correlated g.s

Underlying approximation:

$|0> \rightarrow |HF>$ **QBA**

Collective modes: anharmonic features



From P.F. Bortignon, A. Bracco, R.A. Broglia, Giant Resonances (hap, 1998)

Beyond mean field: *Adopted Methods*

Non relativistic

- qp-Phonon

P. F. Bortignon et al. (Milano group)

- 2nd RPA

R. Roth et al.

D. Gambacurta et al.

Phenomenological

- QPM *(Soloviev School (Dubna))*

Relativistic: RTBA

E. Litvinova, P. Ring, D. Vretenar…..

Our proposal: EMPM

*D. Bianco, F. Knapp, N. Lo Iudice, F. Andreozzi, A. Porrino, Phys. Rev. C **85**, 014313 (2012).*

Equation of motion phonon method (EMPM)

Eigenvalue problem

$$H \,|\, \Psi_\nu > = E_\nu \,|\, \Psi_\nu >$$

$| \Psi_\nu > \ni \mathcal{H} = \Sigma_n \oplus \mathcal{H}_n$ $\mathcal{H}_n \in |n; \beta> \equiv$ n-phonon **basis states**

An obvious, but unmanageable, multiphonon basis

$$|\lambda_1,....,\lambda_i,...\lambda_n> \;=\; O^\dagger_{\lambda_1} ... O^\dagger_{\lambda_i} .... O^\dagger_{\lambda_n} |0>$$

$$O^\dagger_\lambda = \Sigma_{ph} c^\lambda_{ph} \; a^\dagger_p a_h$$

A viable route

$$|\alpha_n> \;=\; \Sigma_{\lambda\alpha} C^\beta_{\lambda\alpha} O^\dagger_\lambda \,|\, \alpha_{n-1} >$$

Construction of $|n;\beta\rangle$: EoM

Assuming $|\alpha_{n-1}\rangle$ **known,** we solve the **Eq. of Motion**

$$\langle \alpha_n \| [H, O^\dagger_\lambda] \| \alpha_{n-1}\rangle = (E_\beta^{(n)} - E_\alpha^{(n-1)}) \langle \alpha_n | O^\dagger_\lambda | \alpha_{n-1}\rangle$$

$$\mathcal{A}\,\mathcal{X} = \mathcal{E}\,\mathcal{X}$$

$$\mathcal{X}$$

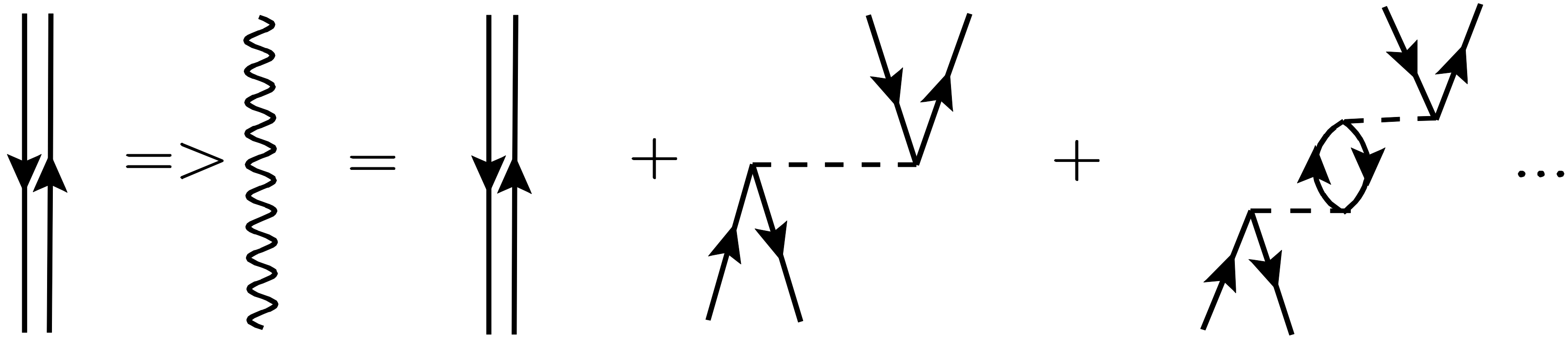
Where

$$\mathcal{A}_{(\mu\alpha)(\nu\gamma)} = (E_\mu + E_\alpha)\delta_{\alpha\gamma}\delta_{\mu\nu} + \mathcal{V}_{(\mu\alpha)(\nu\gamma)}$$

TDA matrix

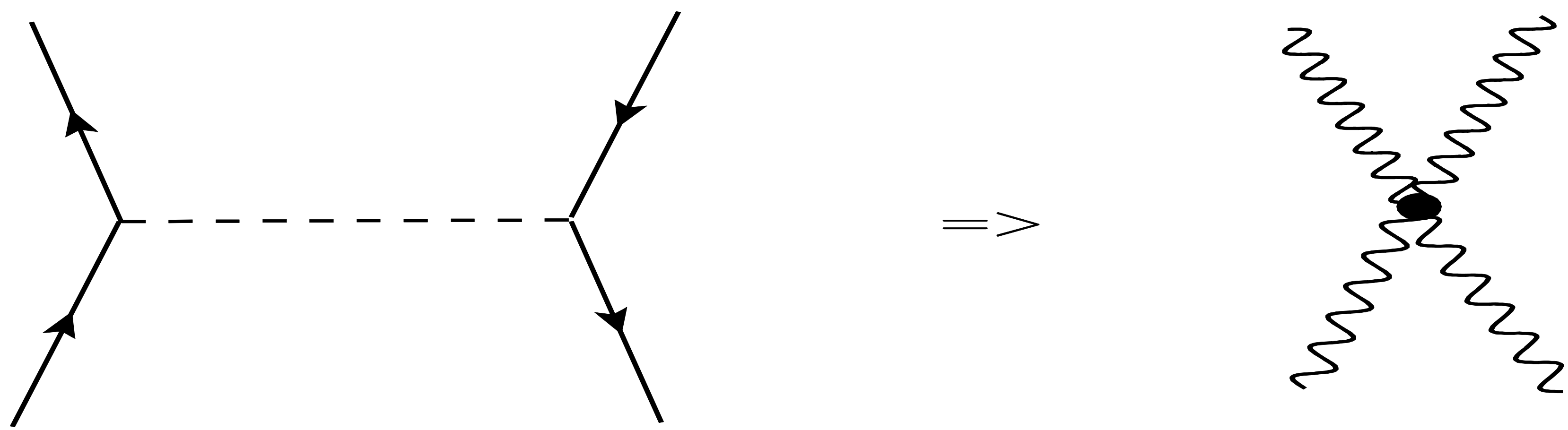
$$A_{(ph)(p'h')} = (\varepsilon_p - \varepsilon_h)\,\delta_{pp'}\,\delta_{hh'} + V_{ph'hp'}$$

From p-h to TDA to EMPM



$\varepsilon_p - \varepsilon_h \Rightarrow E_\mu$

$|ph> \Rightarrow |\mu> = \Sigma_{ph}\, c^{\mu}_{ph}\, a^{\dagger}_{p}\, a_h\, |>$



$V_{ph'hp'} \Rightarrow \mathcal{V}_{(\mu\nu)(\mu'\nu')}$

Construction of $|n; \beta>$: EoM

Problem

$$\mathcal{A}\,\mathcal{X} = \mathcal{E}\,\mathcal{X}$$

is **not** a true Eigenvalue Eq.!

$\{O^{\dagger}_{\lambda}\ |n-1, \alpha>\}$ form a **non-orthogonal redundant** basis

$$\Downarrow$$

$\mathcal{X} = <n, \beta|\ O^{\dagger}_{\lambda}\ |n-1, \alpha>$ is **not** a **true** expansion coefficient

Recipe for solving the problem

1° step

$|\alpha_n\rangle = \Sigma_{\lambda\alpha} C^{\beta}_{\lambda\alpha} \; O^{\dagger}_{\lambda} \; |\alpha_{n-1}\rangle \}$

$|\,\lambda\alpha\rangle = O^{\dagger}_{\lambda} \; |\alpha_n\rangle$

$(\mathcal{X} = \langle \alpha_n \,|\, \lambda \, \alpha_{n-1} \rangle)$

$\mathcal{D} \equiv \{\langle \lambda' \alpha_n' \,|\, \lambda \alpha_n \rangle\}$

$$\mathcal{A}\,\mathcal{X} = \mathcal{E}\,\mathcal{X}$$

$\mathcal{X} = \mathcal{D}\,\mathcal{C}$

$$[\mathcal{H} - \mathcal{E}\mathcal{D}]\,\mathbf{C} = 0$$

where $\mathcal{H} = \mathcal{A}\mathcal{D}$ *But* $\mathcal{D}$ is **singular !**

2° conclusive step: Choleski

$$[\mathcal{H} - E\mathcal{D}]\, C = 0$$

$$\mathcal{H} = \mathcal{A}\mathcal{D}$$

Cholesky ↓

$$\mathcal{D} \rightarrow D$$

$$\mathcal{D} \equiv \{ < \lambda'\alpha' \mid \lambda\alpha > \}$$

$$[H - E]\, C = 0$$

$$H = D^{-1}\mathcal{A}\mathcal{D}$$

↓

$$|n, \beta> = \Sigma_{\lambda\alpha}\, C^{\beta}_{\lambda\alpha}\, O^{\dagger}_{\lambda}\, |\alpha>$$

$$<n',\alpha|n; \beta> = \delta_{nn'}\ \delta_{\alpha\beta}$$

Iterative Generation of n-phonon states

$$\mathbf{A\,c} = \hbar\omega\,\mathbf{c} \quad \Rightarrow \quad |\lambda\rangle = \Sigma_{ph}\, \mathbf{c}^{\lambda}_{ph}\, a^{\dagger}_{p}\, a_{h}\, |\rangle$$

$$\mathbf{H\,C} = \mathbf{E\,C} \quad \Rightarrow \quad |\alpha_n\rangle = \Sigma_{\lambda\alpha(n)}\, C^{\alpha(n)}_{\lambda\alpha(n)}\, O^{\dagger}_{\lambda}\, |\, \alpha_{n-1}\rangle$$

- **No approximations except for truncation!**
- **Pauli** principle fully accounted for:
- **No redundant** states!
- $|\alpha_n\rangle$ form an **orthonormal** basis

n= 2, 3,…..

Eigenvalue problem in Multiphonon basis $\{|\alpha_n>\} \equiv \{|\alpha_0>, |\alpha_1>, .. \}$

$$\Sigma_{n' \beta(n')} [(E_{\alpha(n)} - \varepsilon_\nu) \delta_{nn'} \delta_{\alpha(n) \beta(n')} + V_{\alpha(n) \beta(n')}] C^{\nu}_{\beta(n')} = 0$$

$$|\Psi_\nu > = \Sigma_{\alpha(n)} C^{\nu}_{\alpha(n)} |\alpha_n >$$

where

$$|\alpha_n > = \Sigma_{\lambda\alpha(n-1)} C^{\beta}_{\lambda\alpha(n-1)} O^{\dagger}_{\lambda} | \alpha_{n-1} >$$

EMPM in q-p scheme

One proceeds as in p-h scheme

$$< \alpha_n \|[H, O^\dagger_\lambda]\|\alpha_{n-1} > = (E_\beta^{(n)} - E_\alpha^{(n-1)}) < \alpha_n | O^\dagger_\lambda |\alpha_{n-1} >$$

namely with the replacement

$$O_\lambda^\dagger = \Sigma_{ph}\, c_{ph}(\lambda)\, a^\dagger_p\, a_h \quad \Longrightarrow \quad O_\lambda^\dagger = \Sigma_{r \leq s}\, c_{rs}(\lambda)\, \alpha^\dagger_r\, \alpha^\dagger_s$$

G. De Gregorio,F knapp, N. Lo Iudice, P. Vesely Phys. Rev. C 044314 (2016)

Implementation

1° STEP : Intrinsic Hamiltonian

$$H = T_{int} + V_{NN}$$

where

$$T_{int} = \frac{1}{2m}\sum_i p_i^2 - \frac{P^2}{2M_{cm}} \qquad V_{NN} = V_\chi = NNLO_{opt}$$

2° STEP : HF(B) Self-consistent basis

3° STEP : Construction of TDA phonons

(free of spurious admixtures induced by CM and particle number violation)

$$O_\lambda^\dagger = \Sigma_{ph}\, c_{ph}(\lambda)\, a^\dagger_p\, a_h \qquad O_\lambda^\dagger = \Sigma_{r\leq s}\, c_{rs}(\lambda)\, \alpha^\dagger_r\, \alpha^\dagger_s$$

A. Ekstrom *et al.*, PRL **110**, 192502 (2013)

Implementation

4° STEP : Generation of Multiphonon basis

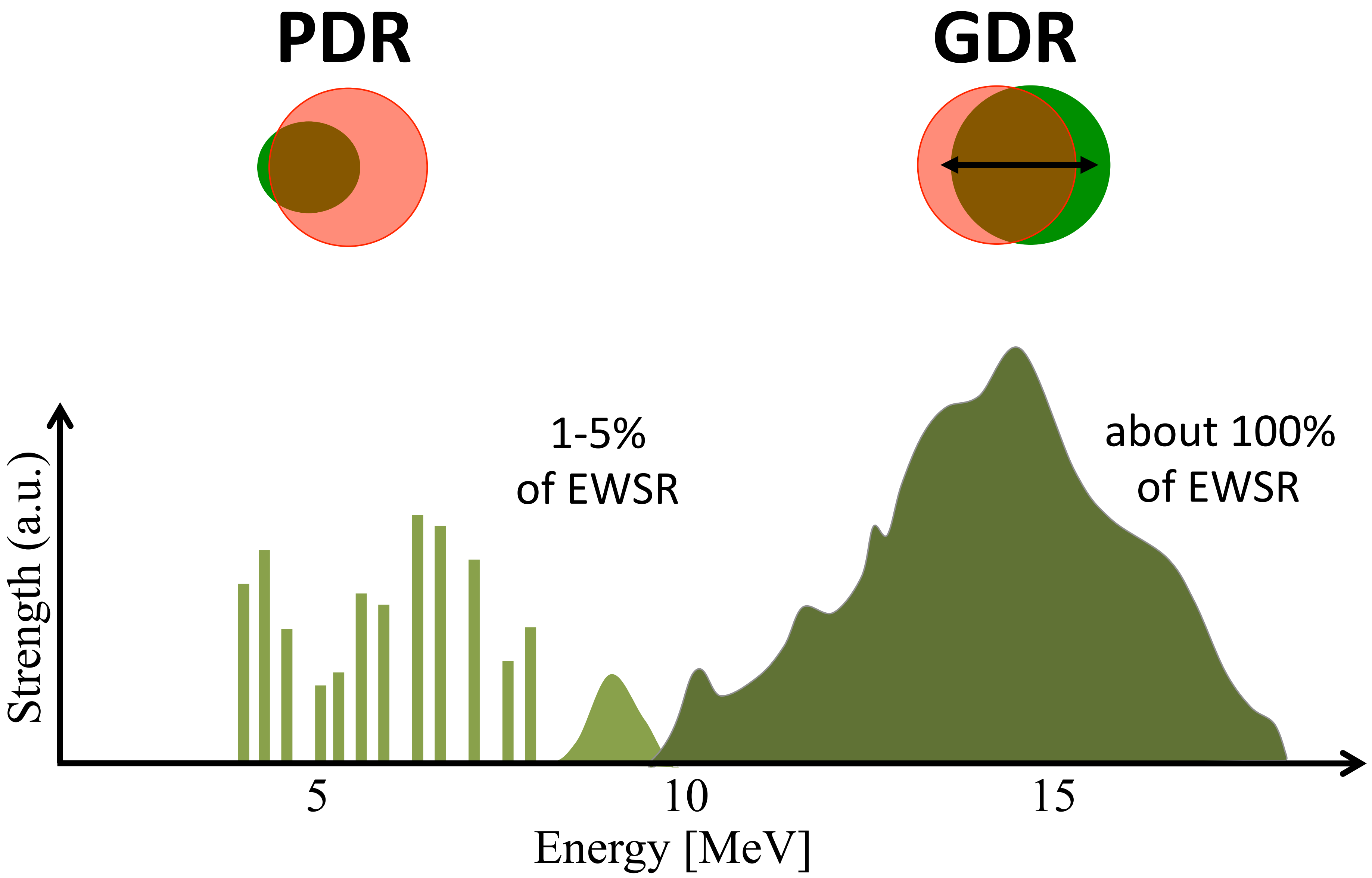
$$\{|\alpha_n>\} \equiv \{|\alpha_0>, \quad |\alpha_1>, \quad |\alpha_2>, \ldots\ldots \}$$

5° STEP : Solution of the eigenvalue problem in Multiphonon basis

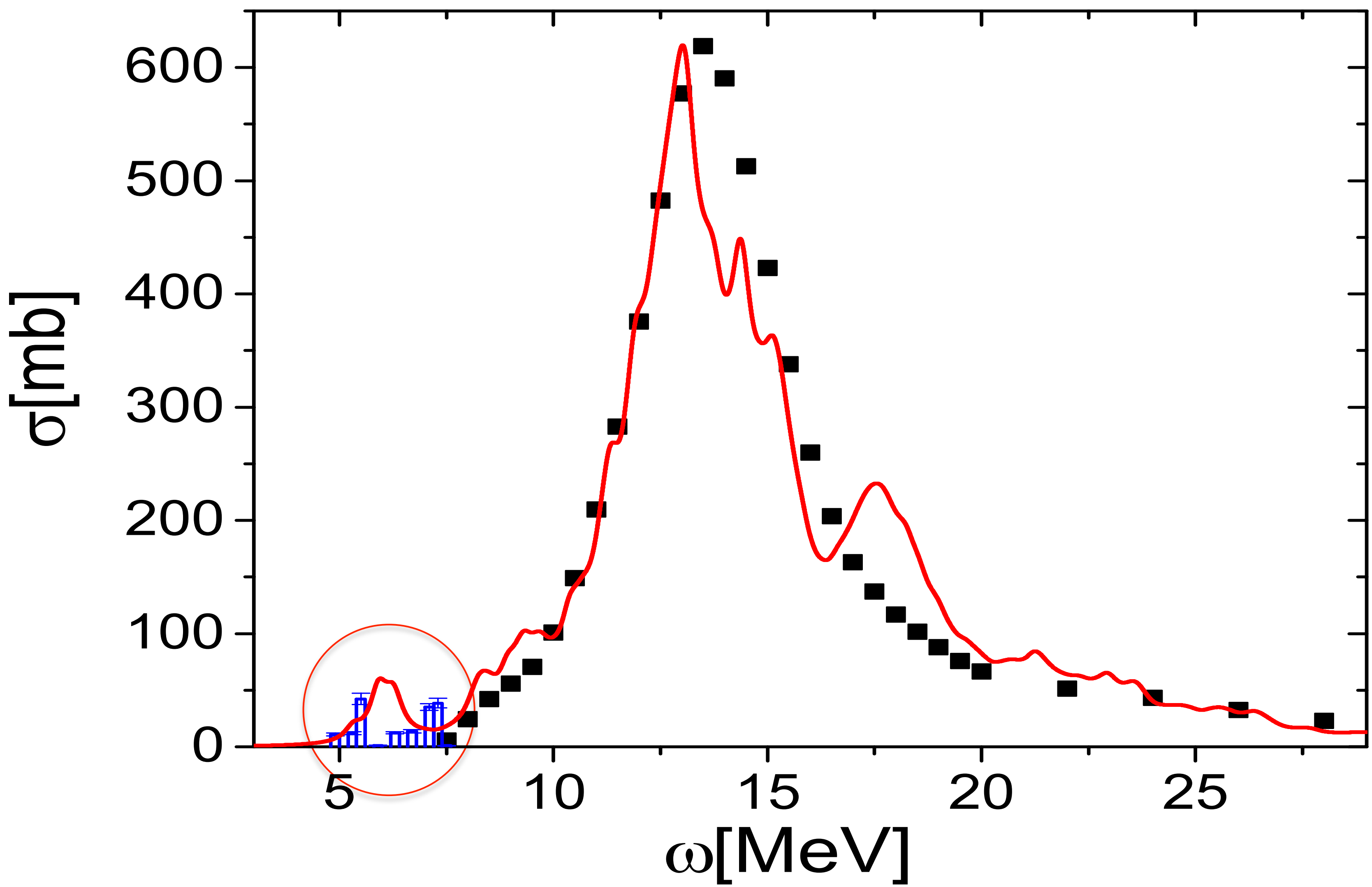
$$| \Psi_\nu\rangle = \sum_{n\alpha} C_\alpha^{(\nu)} \mid n; \alpha\rangle$$

6° STEP : Transition Probabilities and cross section

Applications: Selfconsistent study of Dipole response

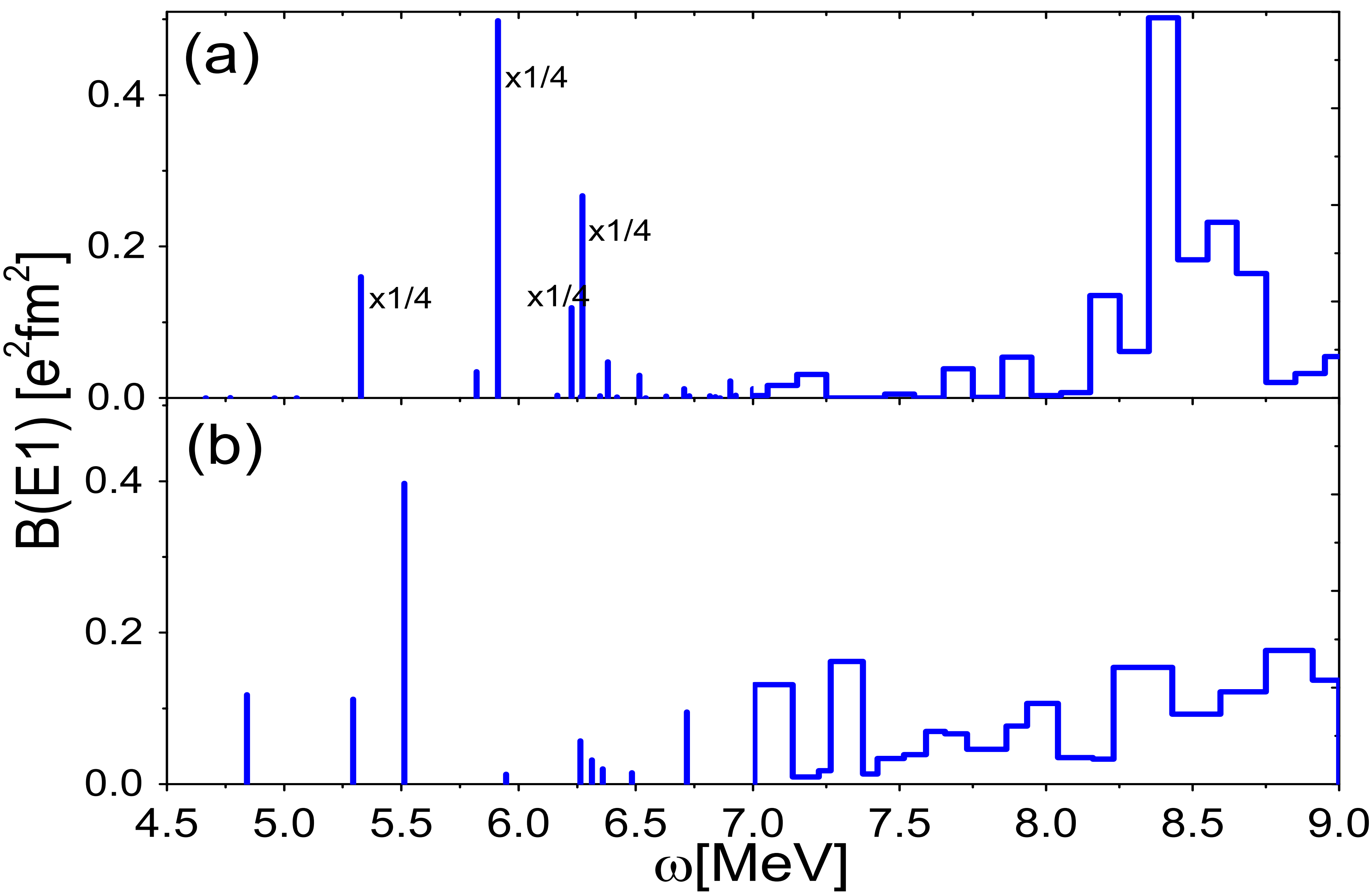


EMPM in p-h scheme: Application to neutron rich ^{208}Pb



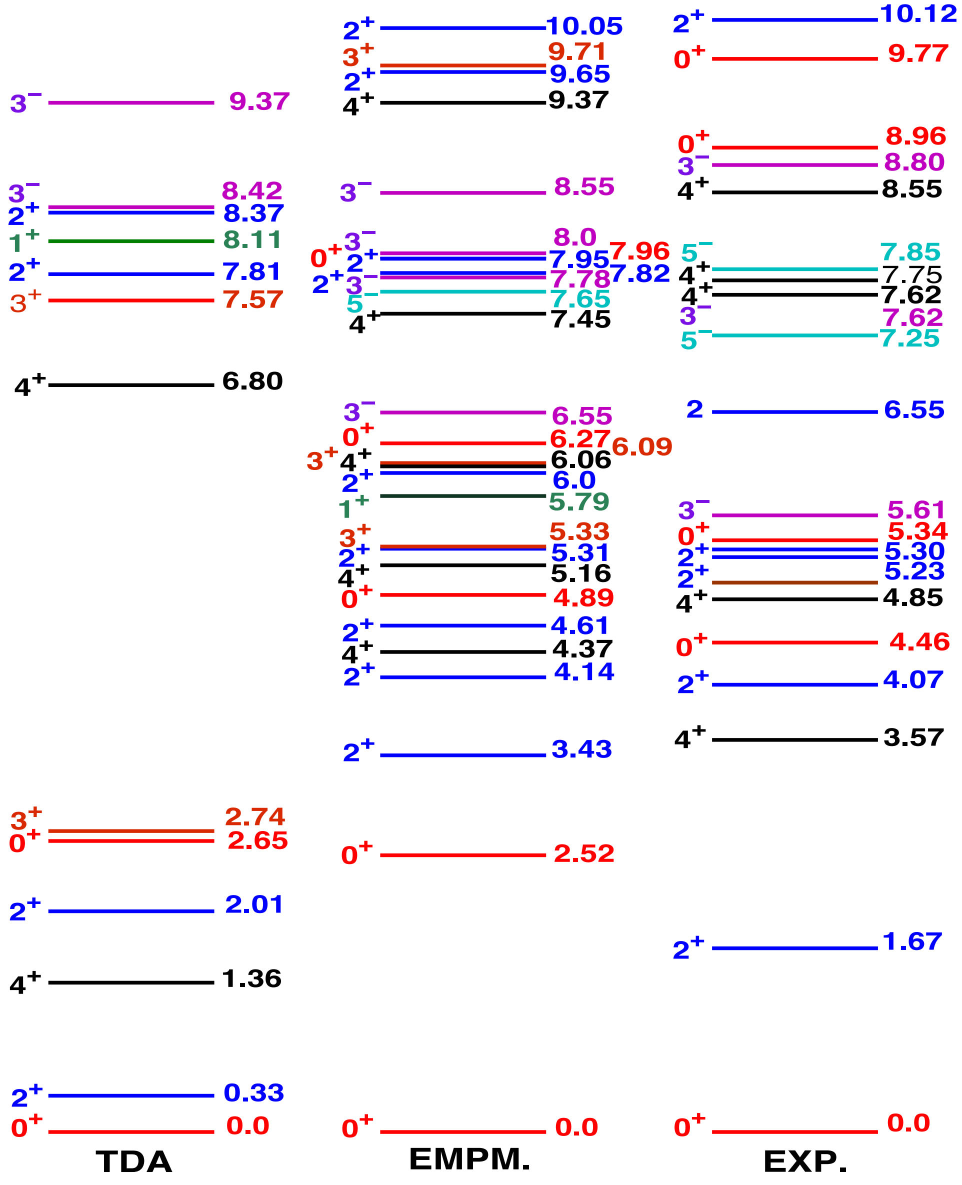
F. Knapp,N. Lo Iudice,P. Vesely,F. Andreozzi, G. De Gregorio,A. Porrino PRC **92**, 054315(2015)

Low-lying Dipole strength in ^{208}Pb



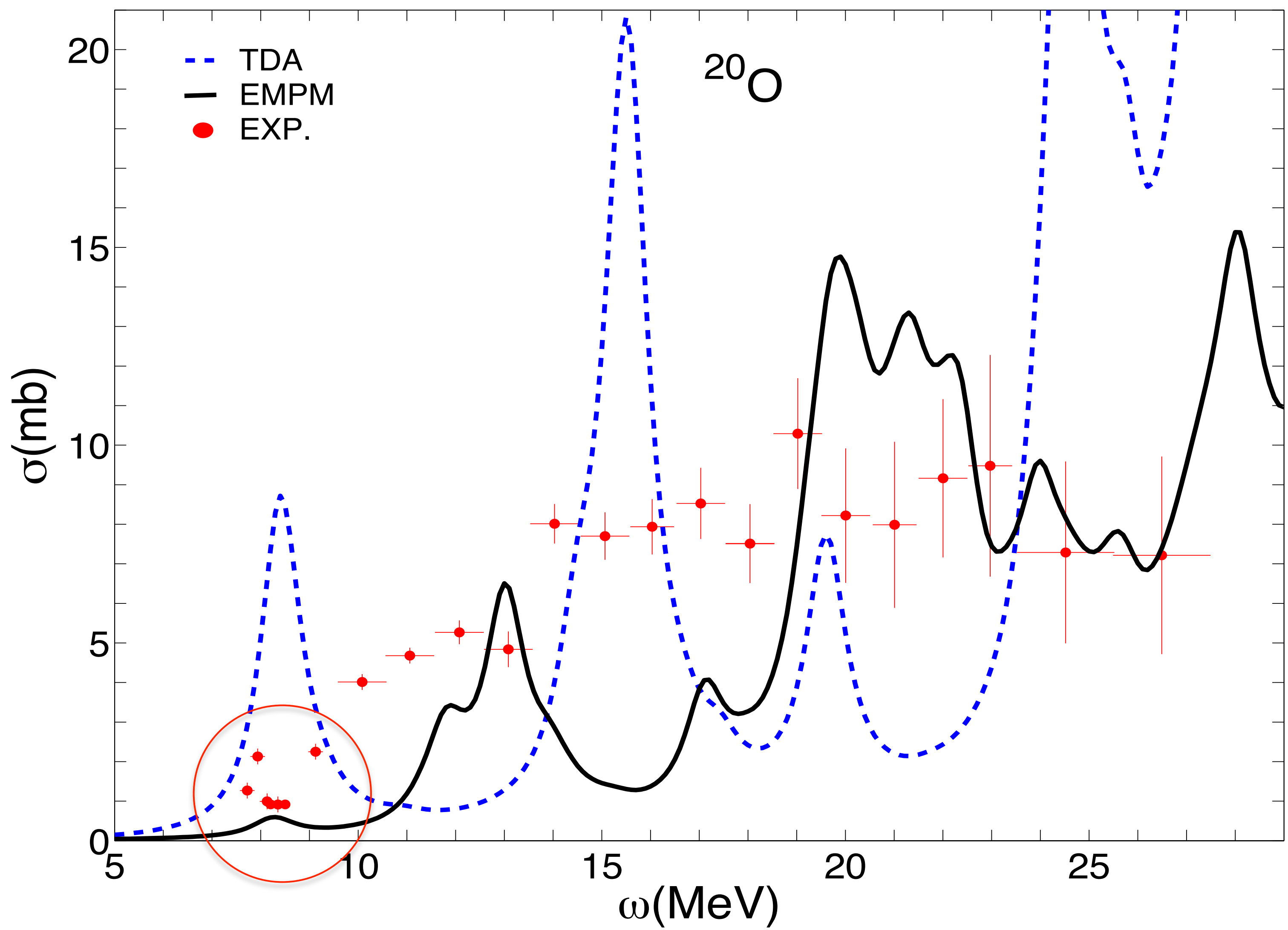
F. Knapp,N. Lo Iudice,P. Vesely,F. Andreozzi, G. De Gregorio,A. Porrino PRC **92**, 054315(2015)

EMPM in q-p scheme: **Application to neutron rich ^{20}O**



G. De Gregorio, F knapp, N. Lo Iudice, P. Vesely Phys. Rev. C 044314 (2016)

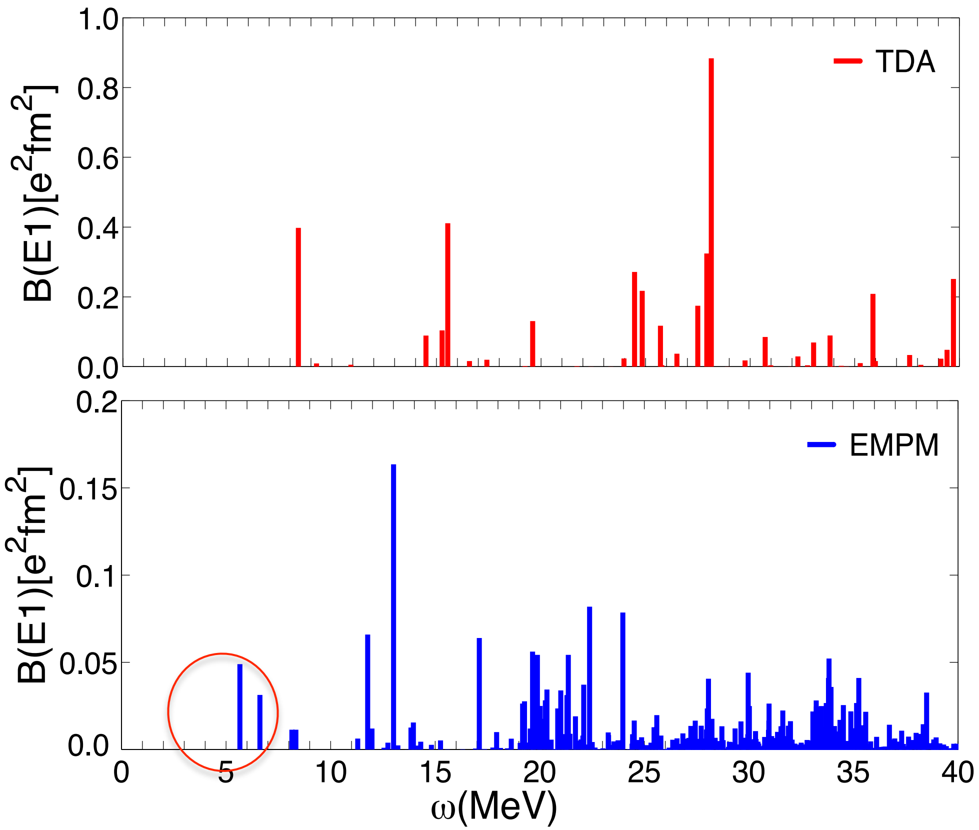
Cross section in ^{20}O



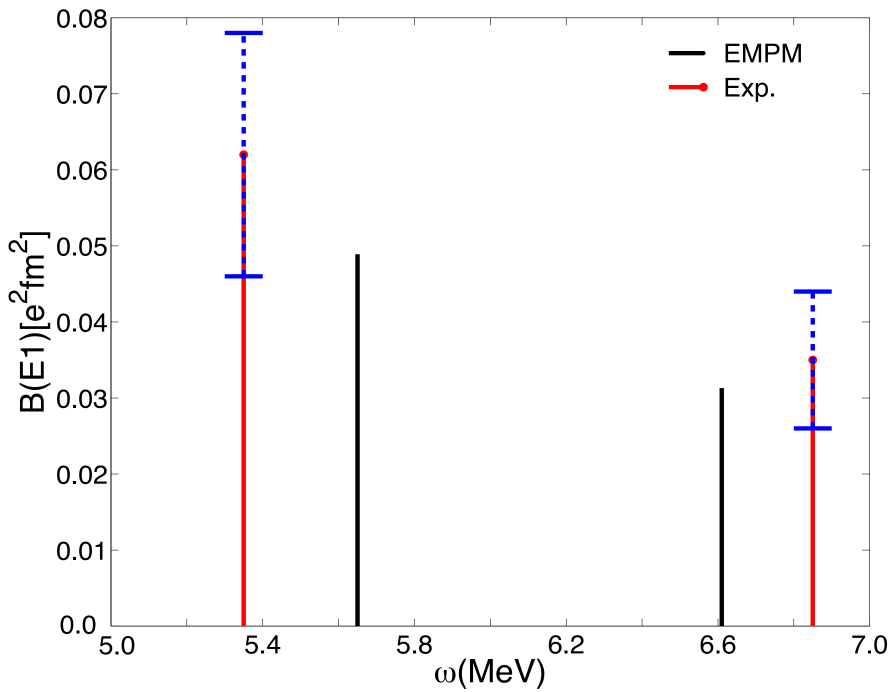
G. De Gregorio,F knapp, N. Lo Iudice, P. Vesely
Phys. Rev. C 044314 (2016)

E. Tryggestad, et al., Phys Rev. C 67, 064309 (2003).

Dipole strength in ^{20}O



E. Tryggestad, et al., Phys Rev. C 67, 064309 (2003).



G. De Gregorio,F knapp, N. Lo Iudice, P. Vesely Phys. Rev. C 044314 (2016)

Even Nuclei: Summary

EMPM

- Extends Mean Field without **approximations,** except for truncation, for any potential

- Includes **multiphonon states** **explicitly**

Results

2-phonon configurations are **needed** for

- describing the fine structure of the GDR and PDR
  - obtaining a complete low-energy spectrum

EMPM: Odd nuclei

$| \lambda \, \alpha > = O^{\dagger}_{\lambda} \, | \alpha >$ → $| p \, \alpha > = a^{\dagger}_{p} \, | \alpha >$

$< \alpha_n || [H, O^{\dagger}_{\lambda}] || \alpha_{n-1} >$ → $< \nu_n || [H, a^{\dagger}_{p}] || \alpha_n >$

$$\mathcal{X} = \mathcal{D} \, \mathcal{C}$$

$\mathcal{A} \, \mathcal{X} = \mathcal{E} \, \mathcal{X}$ → $[\mathcal{H} - \mathcal{E}\mathcal{D}] \, \mathbf{C} = 0$

$$\mathcal{D} \equiv \{ < p \, \alpha \, | \, p' \, \alpha' > \}$$

Cholesky

$\mathcal{D} \rightarrow \mathbf{D}$

EMPM: Odd nuclei

$$[H - E]\, C = 0$$

$$H = D^{-1} \mathcal{A} \mathcal{D}$$

$$|\nu_n> = \Sigma_{p\alpha} C^{\nu}_{p\alpha} \, a^{\dagger}_{p} \, |\alpha>$$

$$<\nu'|\nu> = \delta_{\nu\nu'}$$

$$|\Psi_\mu> = \Sigma_i \, C^{(\mu)}_{\nu_i} \, |\nu_i>$$

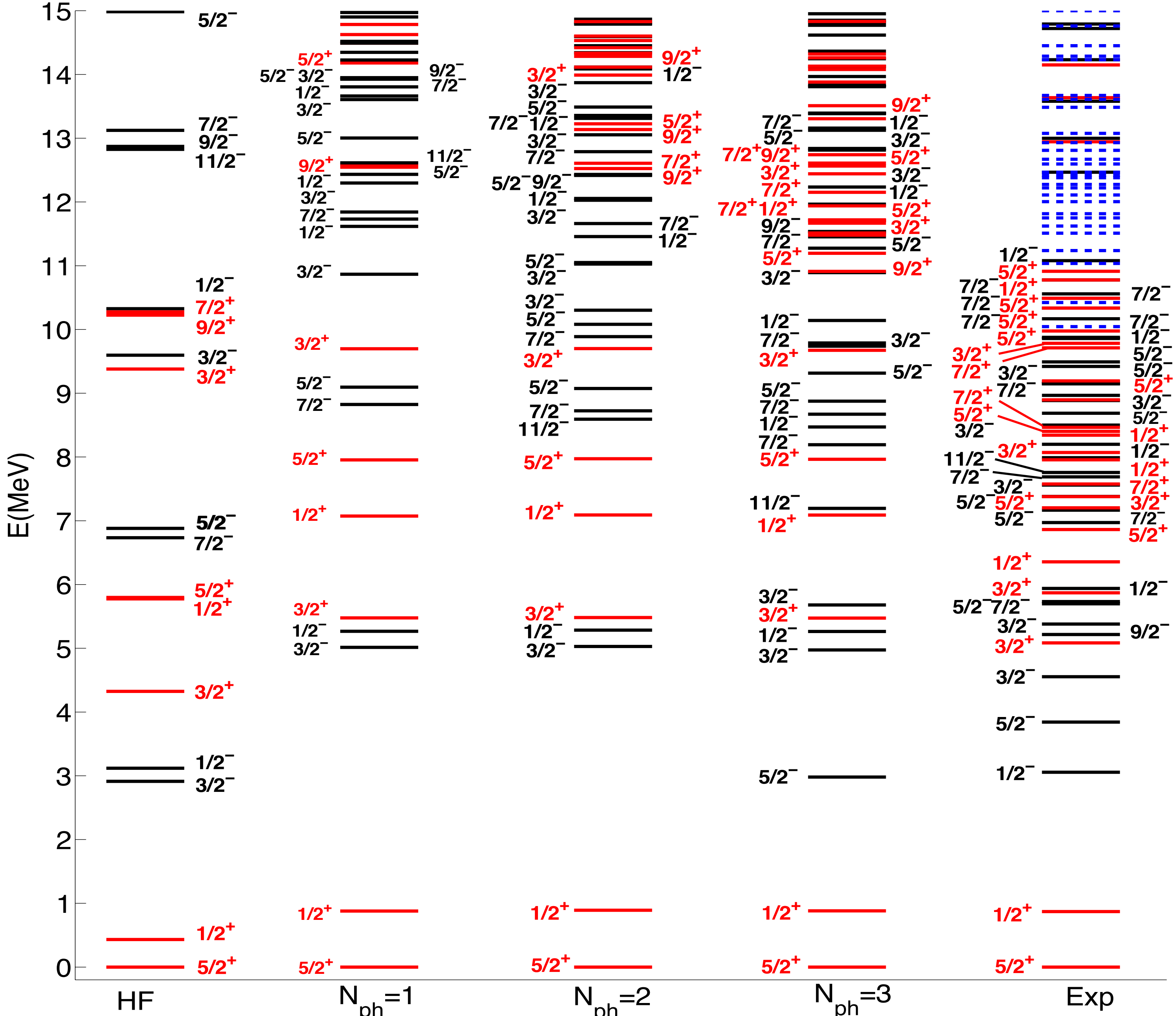
- **No approximations except for truncation !**
- **Pauli** principle fully accounted for:
- **No redundant** states**!**
- $|\nu_n>$ form an **orthonormal** basis

Implementation

Potential: **$NNLO_{opt}$**

- Perform **HF**
- Construct **TDA** phonons (**free** of **CM** spurious admixtures)
- Generate the multiphonon basis $\{|\alpha_n>\} \equiv \{|\alpha_0>, \; |\alpha_1>, \; |\alpha_2>, \; |\alpha_3>\}$
- Generate the **orthornormal** particle-phonon basis $\{|\nu_n>\}$
- Full diagonalization

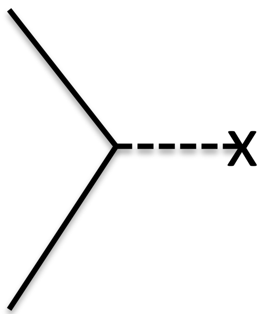
^{17}O spectrum



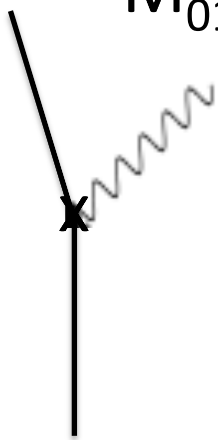
Transitions Amplitudes

$$\left\langle \Psi_{\Omega_f} \,\|\, M(\lambda) \,\|\, \Psi_{\Omega_i} \right\rangle = M_{00}(\lambda) + M_{01}(\lambda) + M_{10}(\lambda) + M_{11}(\lambda) + M_{12}(\lambda) + M_{21}(\lambda) + M_{22}(\lambda)$$

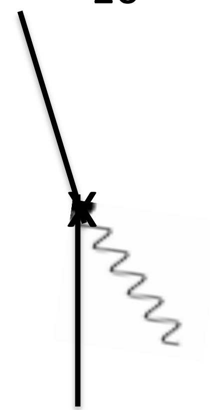
M_{00}



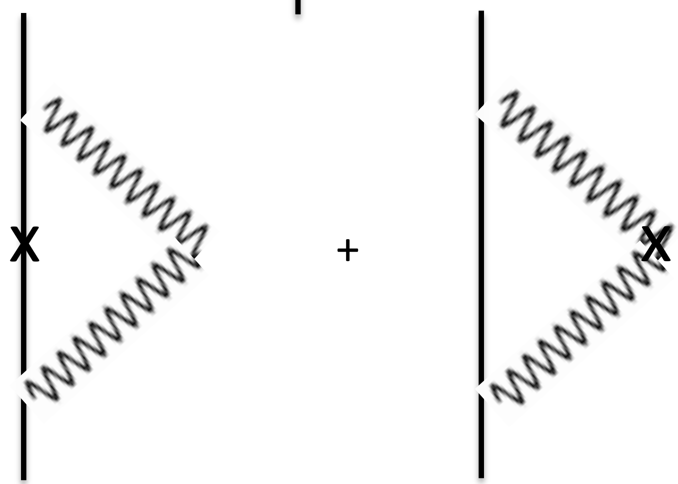
M_{01}



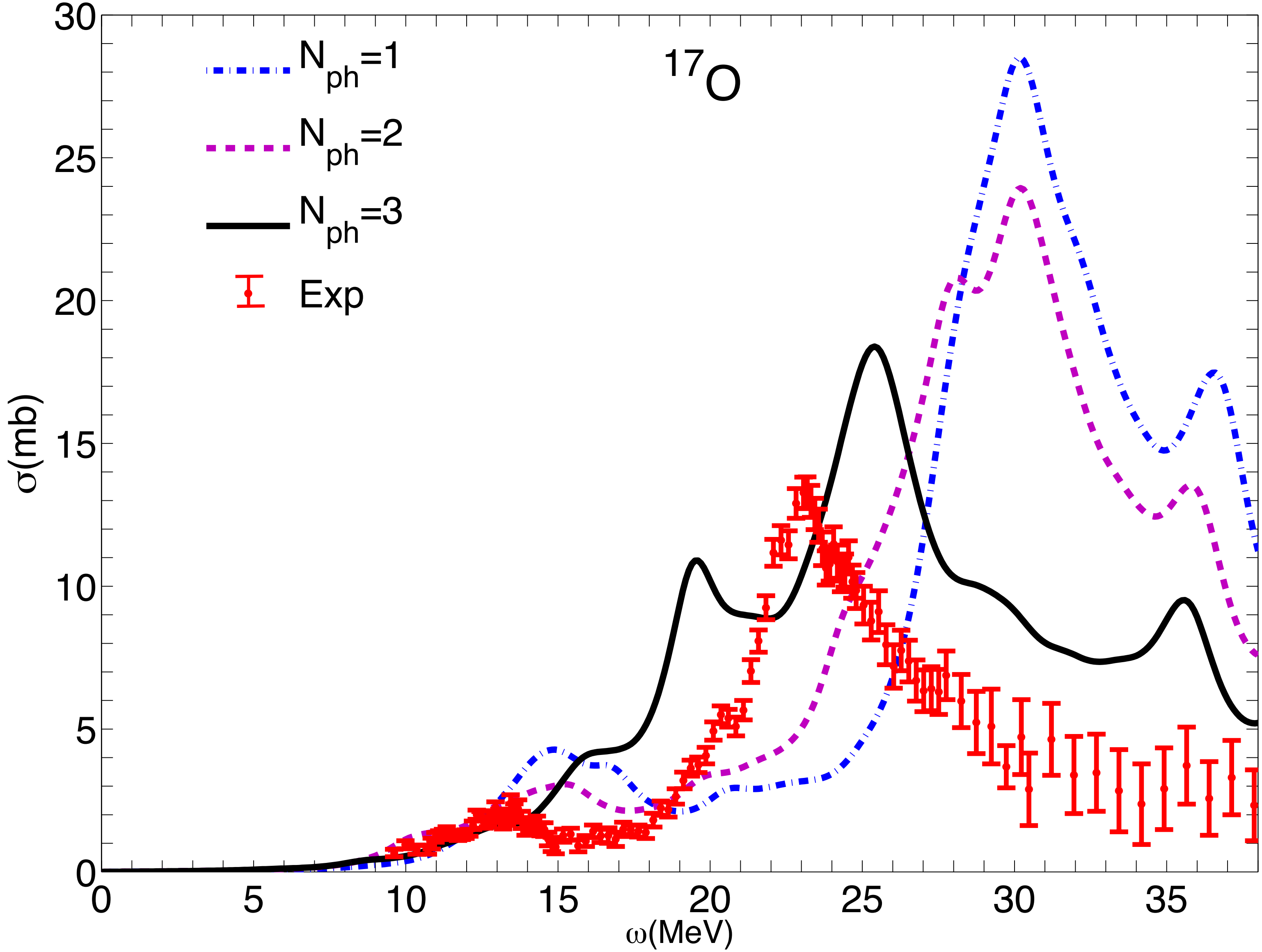
M_{10}



M_{vv} =

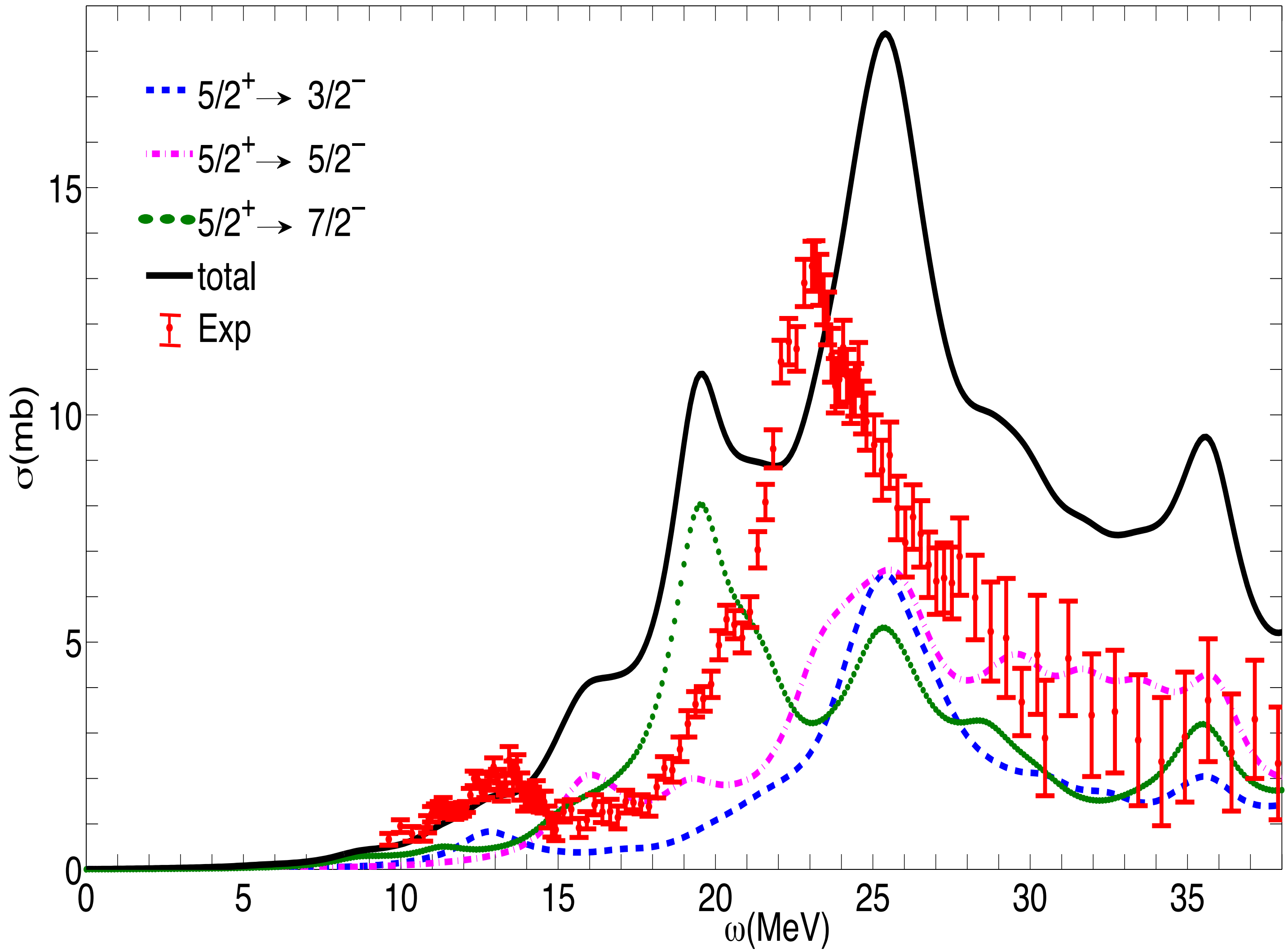


Cross section



J. W. Jury, et al., Phys Rev. C **21**, 503 (1980).

Cross section



J. W. Jury, et al., Phys Rev. C **21**, 503 (1980).

Mean Values and transitions

^{17}O	HF	EMPM	Exp[1]
Q(barn)	0	-0.00841	-0.025
μ (nm)	-1.9130	-1.833	-1.893
B(E2; $5/2^+_1 \rightarrow 1/2^+_1$) (e^2fm^4)	0	0.17	2.18(16)

[1] N.P. Stone, Atomic Data and Nuclear Data Tables **90** (2005) 75–176

Odd nuclei : Concluding remarks

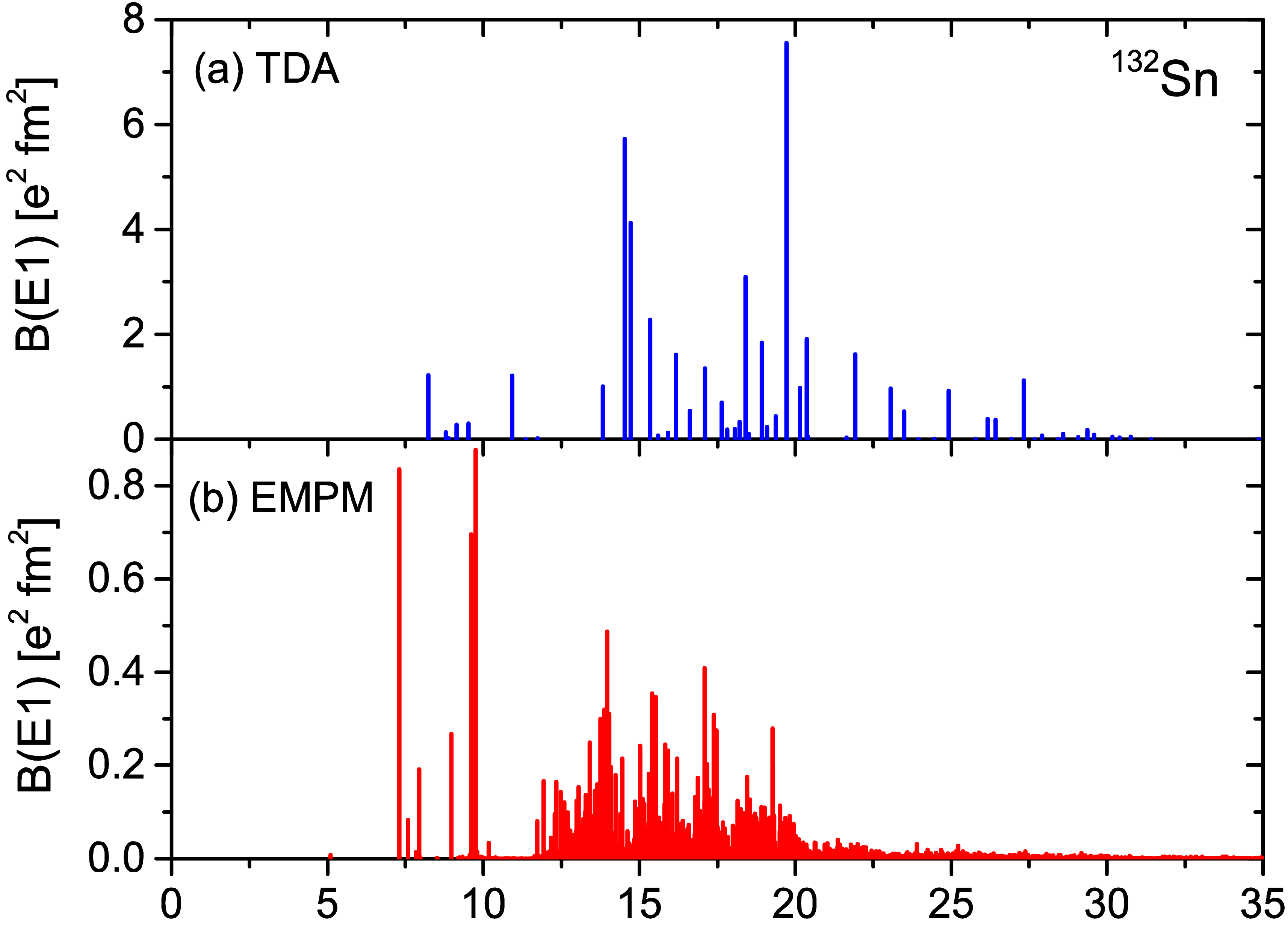
- One-and two-phonon states enhances greatly the density of levels consistently with experiments.

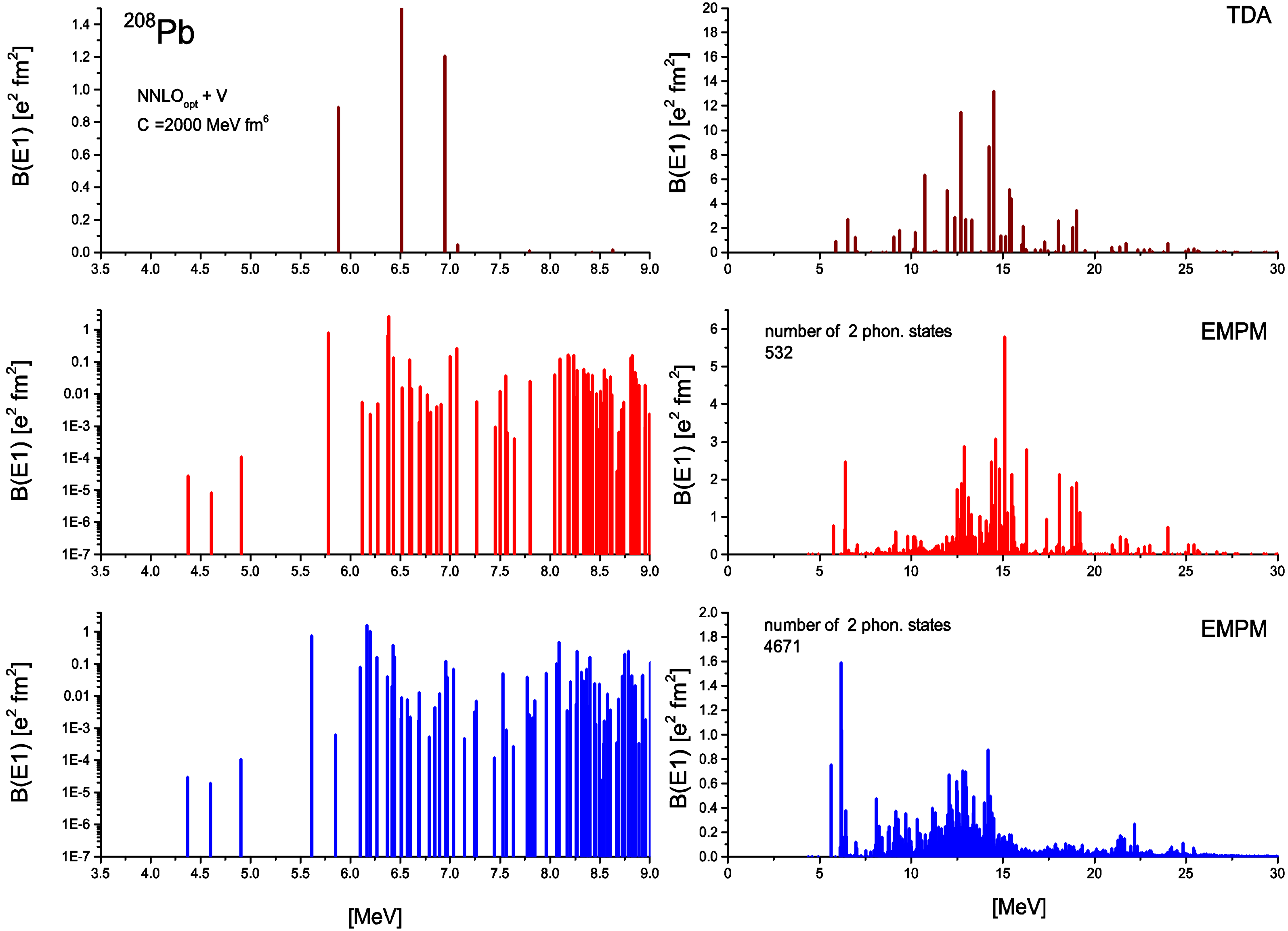
- Three-phonon states play an important role, their (strong) coupling to particle-phonon states $|\nu_1>$

  a) shift downwards the energies of $|\nu_1>$
  b) improve the cross section
  c) increase the density of levels in low-energy part of the spectrum

Thank you

Dipole strength and cross section in ^{132}Sn





^{208}Pb
NNLO$_{opt}$ + V
C =2000 MeV fm^6
TDA
number of 2 phon. states
532
EMPM
number of 2 phon. states
4671
EMPM
B(E1) [e^2 fm^2]
[MeV]