No-Core Shell Model for Hypernuclei

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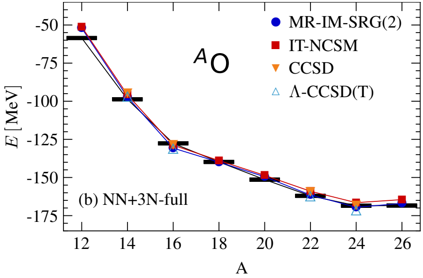
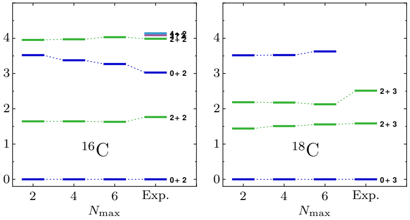
Motivation

Hergert *et al.*, PRL **110**, 242501 (2013)





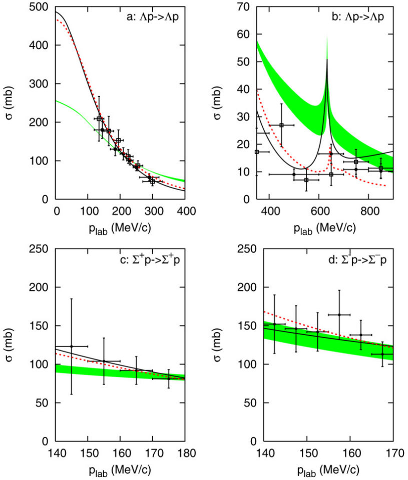
Roth *et al.*, Few-Body Syst. **55**, 659 (2014)

Status of *ab initio* nuclear structure

- Accurate NN+3N Hamiltonians from chiral EFT
- Unitary transformations for converged results
- Many-body methods with controlled uncertainties
- Access to binding energies, spectra, radii, transitions. . .

Extension to hypernuclei?

Hyperon-Nucleon Interaction



Polinder *et al.*, Nucl. Phys. A **779**, 244 (2006)

- Weaker than NN ($\sim 1\,\mathrm{MeV}/N$)
- No YN bound state
- ΛN $\longleftrightarrow$ ΣN conversion
- Spin singlet—triplet transition

Meson-exchange models and chiral EFT interactions available

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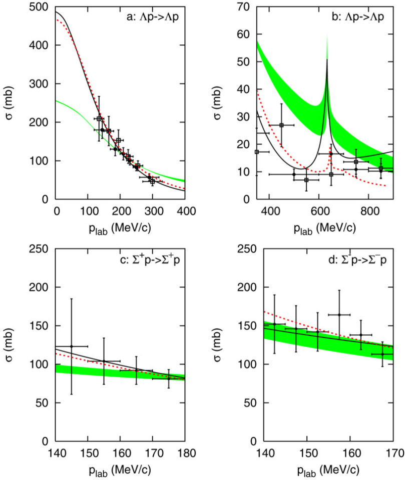

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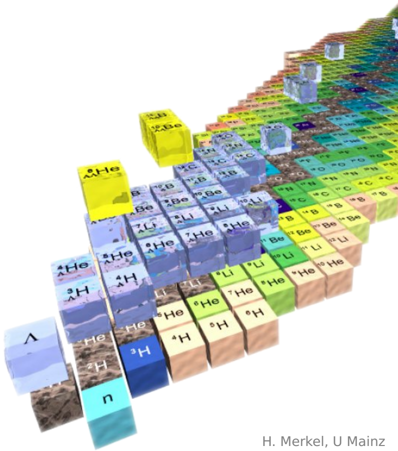
Meson-exchange models and chiral EFT interactions available

YN scattering experiments are challenging
$\Rightarrow$ Few data points
$\Rightarrow$ Large error bars

Necessary Developments

Extend No-Core Shell Model to hypernuclei

- Account for different particle masses
- Include fully active Λ and Σ
  ⇒ Coupled-channel problem
- Adapt unitary transformation framework



H. Merkel, U Mainz

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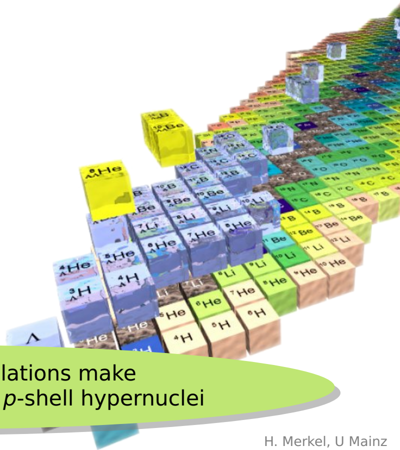

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Ab-initio many-body calculations make connection from YN interactions to *p*-shell hypernuclei

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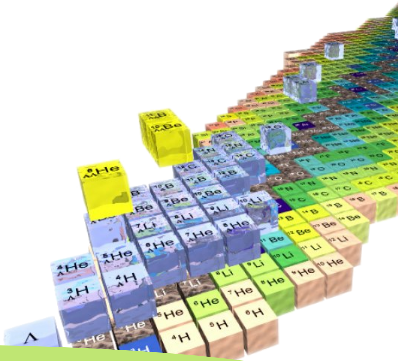

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Ab-initio many-body calculations make connection from YN interactions to *p*-shell hypernuclei

"→" prediction of observables
"←" constraints on YN interactions

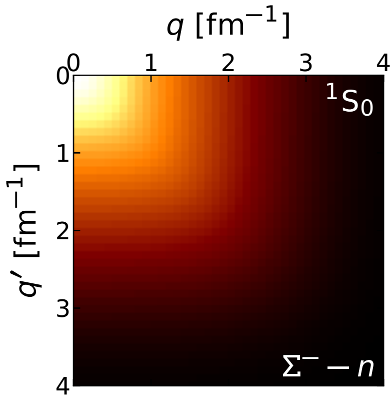
Hypernuclear Hamiltonian

$$\boldsymbol{H} = \Delta\boldsymbol{M} + \boldsymbol{T}_{\text{int}} + \boldsymbol{V}_{\text{NN}} + \boldsymbol{V}_{\text{3N}} + \boldsymbol{V}_{\text{YN}}$$

- NN: chiral N^3LO
  Entem & Machleidt
  Phys. Rev. C **68**, 041001(R) (2003)
  $\Lambda_{\text{NN}} = 500\,\text{MeV}$
- 3N: chiral N^2LO
  Navrátil
  Few-Body Syst. **41**, 117 (2007)
  $\Lambda_{\text{3N}} = 500\,\text{MeV}$
- YN: chiral LO
  Polinder, Haidenbauer & Meißner
  Nucl. Phys. A **779**, 244 (2006)
  $\Lambda_{\text{YN}} = 600\,\text{MeV}, 700\,\text{MeV}$

NN+3N yields quantitative description of *p*-shell nuclei

Similarity Renormalization Group



$\alpha = 0.00\,\mathrm{fm}^4$

$$\partial_\alpha \boldsymbol{H}(\alpha) = [\boldsymbol{\eta}(\alpha), \boldsymbol{H}(\alpha)]$$
$$\boldsymbol{\eta}(\alpha) = m_N^2 [\boldsymbol{T}_{\mathrm{int}}, \boldsymbol{H}(\alpha)]$$

- Up to 6 coupled channels
- Decouples high and low momenta
  ⇒ Improved N_{max} convergence
- BUT: Induced many-body terms
  ⇒ Assess via α-dependence
- NN+3N: Induced terms negligible up to $A \approx 10$

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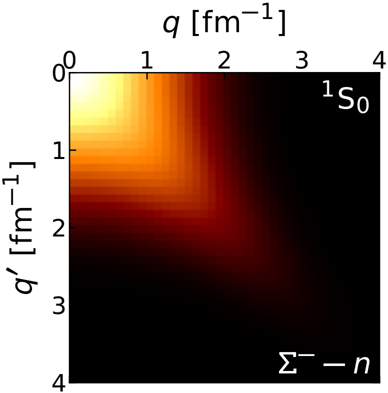

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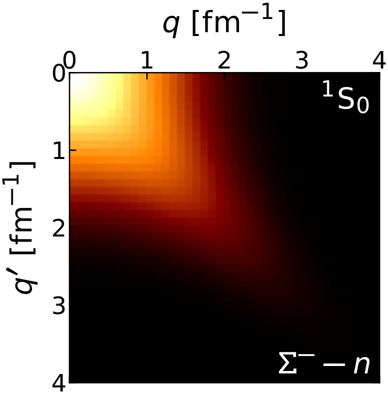

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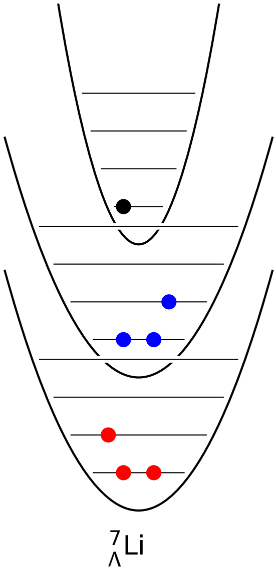
Use $\alpha = 0.08\,\mathrm{fm}^4$

Importance-Truncated No-Core Shell Model



- A-body Slater determinants from HO states

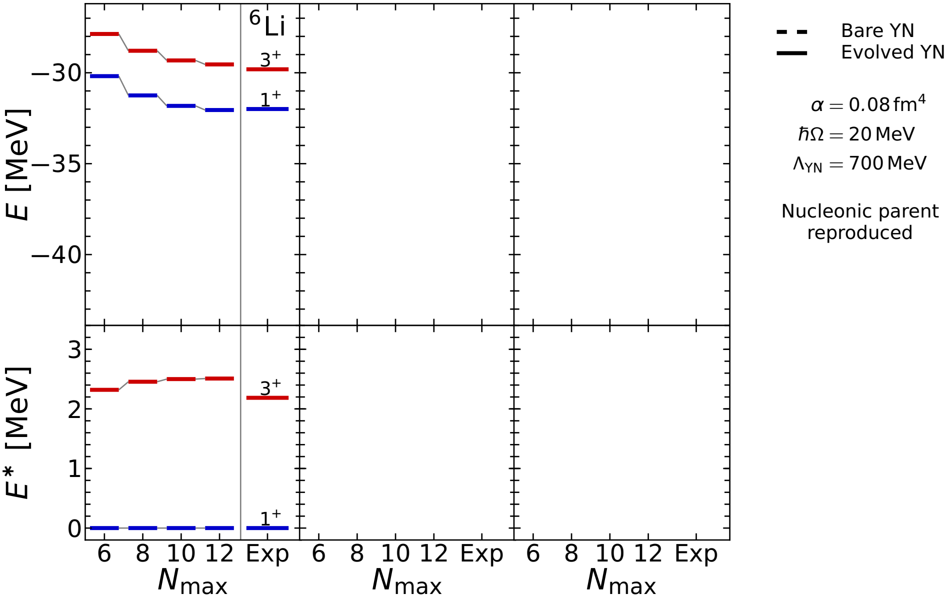
$$|s_1 s_2 \cdots s_A\rangle\,, \quad s_i \equiv |e(l\tfrac{1}{2})j\chi\rangle_i$$

- Λ-Σ conversion, e.g.

$$|pn\Lambda\rangle\,, |pp\Sigma^-\rangle\,, |nn\Sigma^+\rangle \in \mathcal{M}(^{3}_{\Lambda}\mathrm{H})$$

- Impose N_{max} truncation
- Importance truncation: discard irrelevant states + *a posteriori* extrapolation
- Diagonalize Hamilton matrix $\Rightarrow$ Energies & wave functions

$^{7}_{\Lambda}$Li — Energies and Spectra



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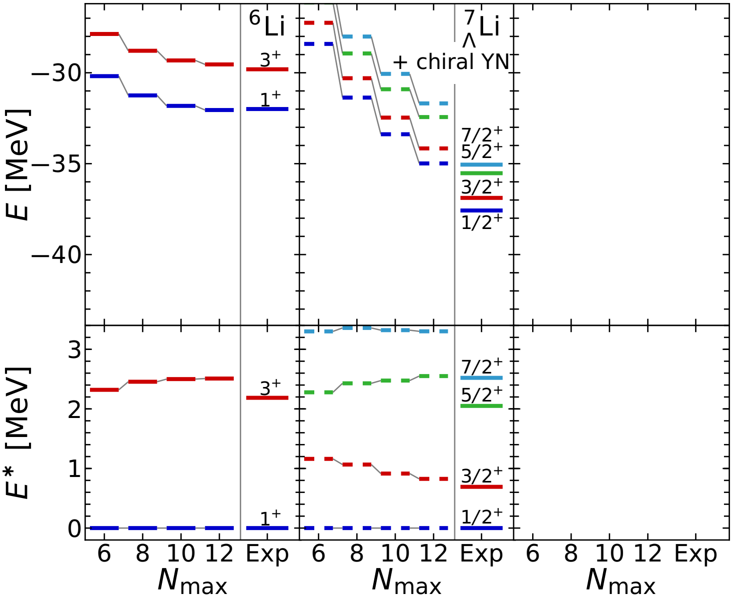

Bare YN
Evolved YN

$\alpha = 0.08\,\mathrm{fm}^4$
$\hbar\Omega = 20\,\mathrm{MeV}$
$\Lambda_{YN} = 700\,\mathrm{MeV}$

Nucleonic parent reproduced

Slow convergence of bare YN

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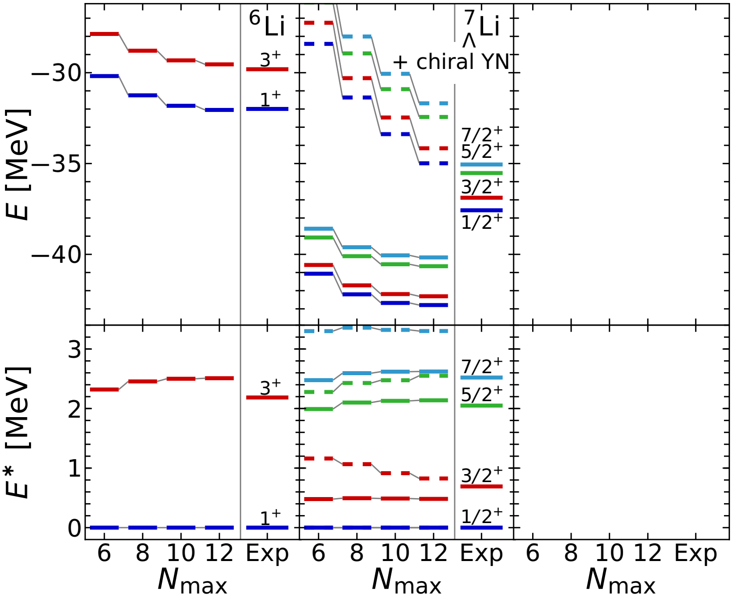

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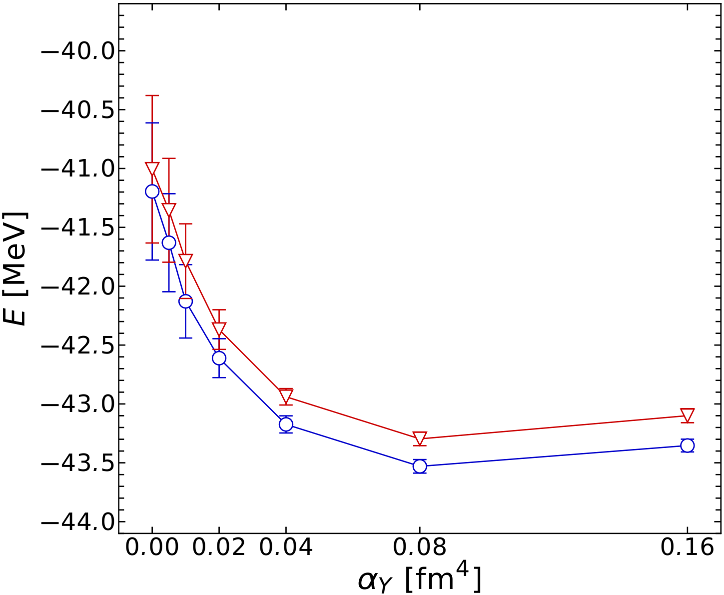
Evolved YN overbinds

SRG Evolution of the YN Interaction



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Strong α_Y-dependence of extrapolated energies

Induced YNN strongly repulsive

Spectra stable

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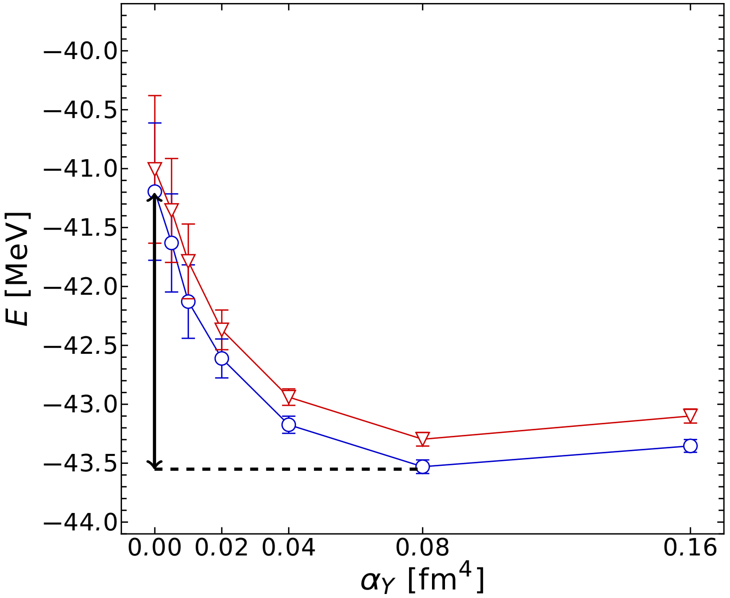

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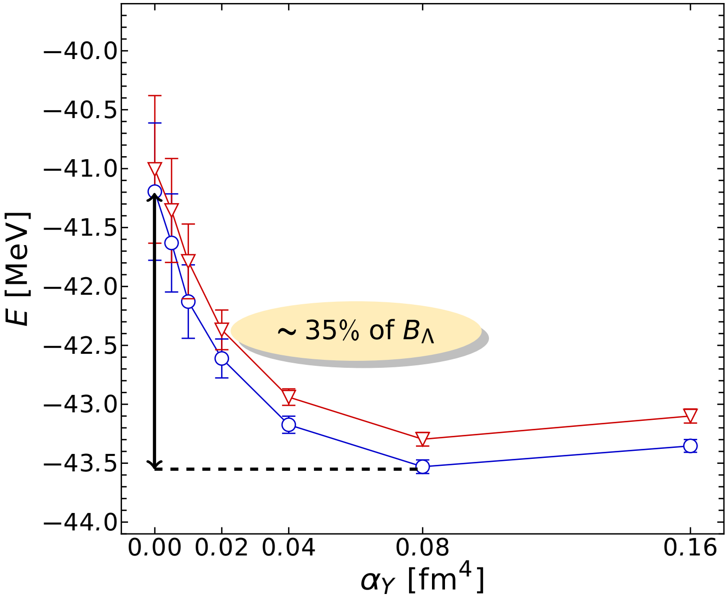

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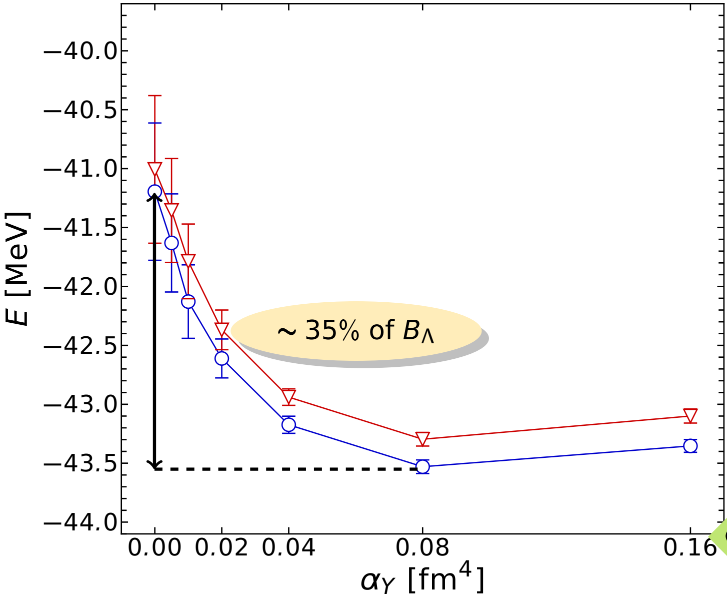

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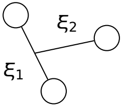
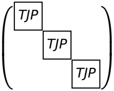
Capture induced YNN terms!

Evolution in Three-Body Space

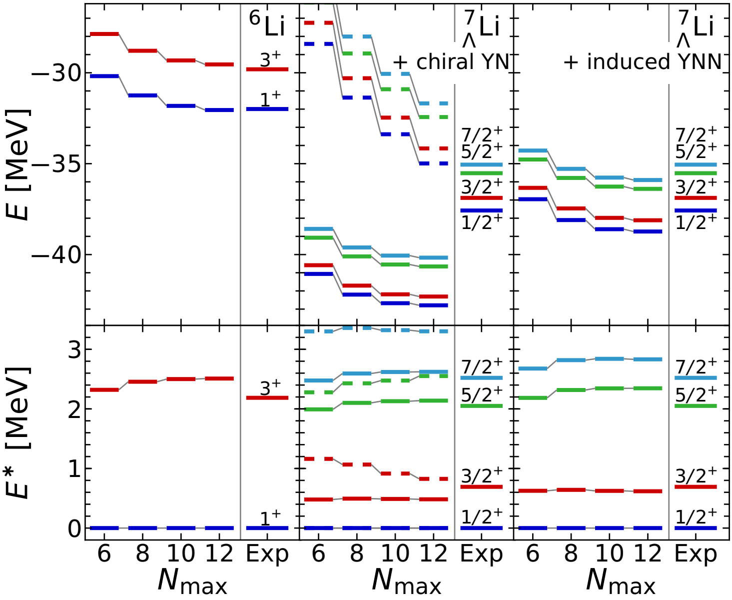




- Introduce Jacobi coordinates and partially antisymmetrized states $|\alpha\rangle$
- Diagonalize antisymmetrizer $\langle\alpha|\mathcal{A}|\alpha'\rangle$
  $\Rightarrow$ Basis $|EiXJT\rangle_a$ (X = ΛNN, ΣNN)
- Hamiltonian decouples into *TJP* blocks
- For each block: Compute matrix elements and solve SRG flow equation
- Transform back to single-particle coordinates (*JT*-coupled matrix elements)

Like 3N, but 3 isospin channels and 2 particle combinations

${}^{7}_{\Lambda}$Li — Energies and Spectra



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Evolved YN overbinds

Induced YNN improves agreement

Summary & Outlook

- *Ab initio* calculations for hypernuclei feasible
- SRG transformation allows for converged results
- SRG-induced YNN must be accounted for
  $\Rightarrow$ Good reproduction of data

- Analyze interaction: NLO, LEC variation at LO
- Build systematics: more *p*-shell hypernuclei
- Other observables: radii, electromagnetic moments & transitions

Epilog

- **Thanks to my group & collaborators**
  - S. Alexa, S. Dentinger, E. Gebrerufael, T. Hüther, L. Kreher, L. Mertes, **R. Roth**, S. Schulz, H. Spielvogel, H. Spiess, C. Stumpf, A. Tichai, R. Trippel, K. Vobig, T. Wolfgruber
    Institut für Kernphysik, TU Darmstadt
  - **P. Navrátil**
    TRIUMF, Canada
  - **D. Gazda**
    Chalmers, Sweden

- **Thank you for your attention!**

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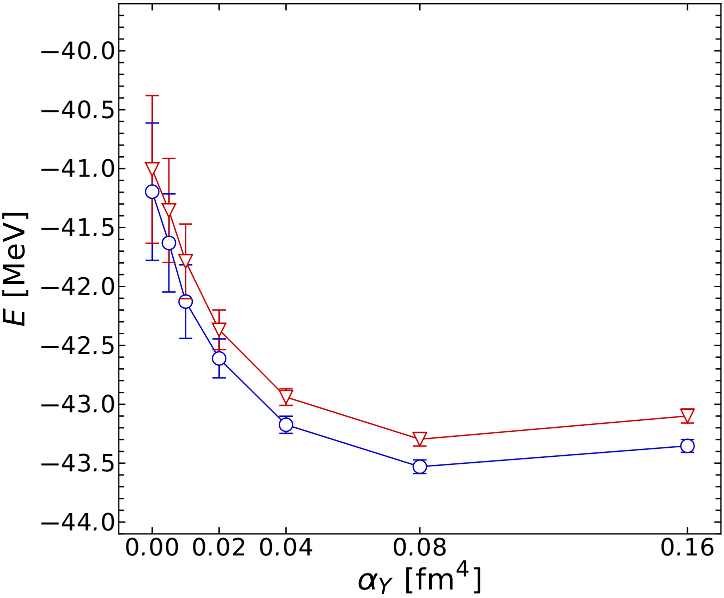
Backup Slides

Flow Parameter Dependence



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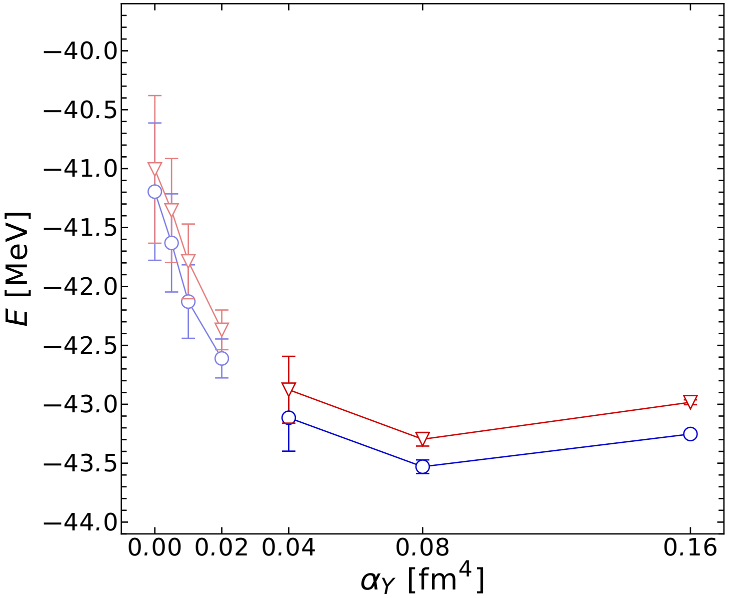

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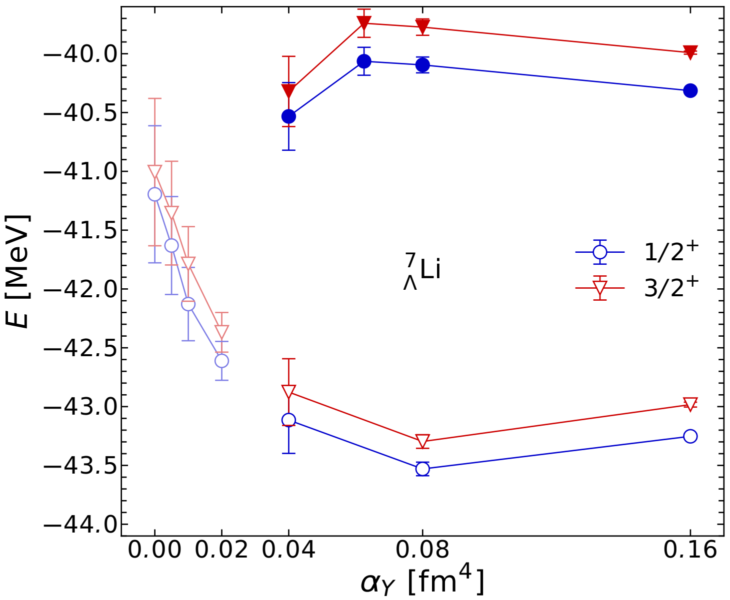

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Induced YNN reduces α-dependence