Structure Of Be-9-Lambda With $\alpha\alpha\Lambda$ Three-Body Model

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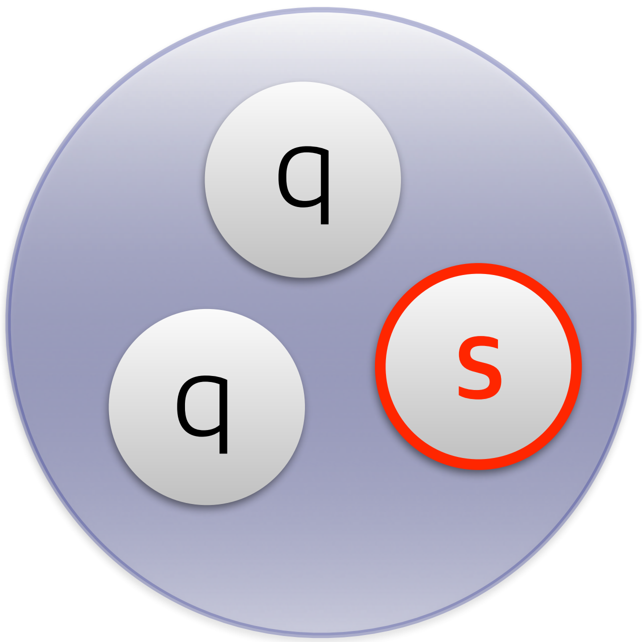
HYPERNUCLEI

- Hypernuclei = Nuclei + Hyperon
- Hyperon is baryon with one or more strangeness quarks.

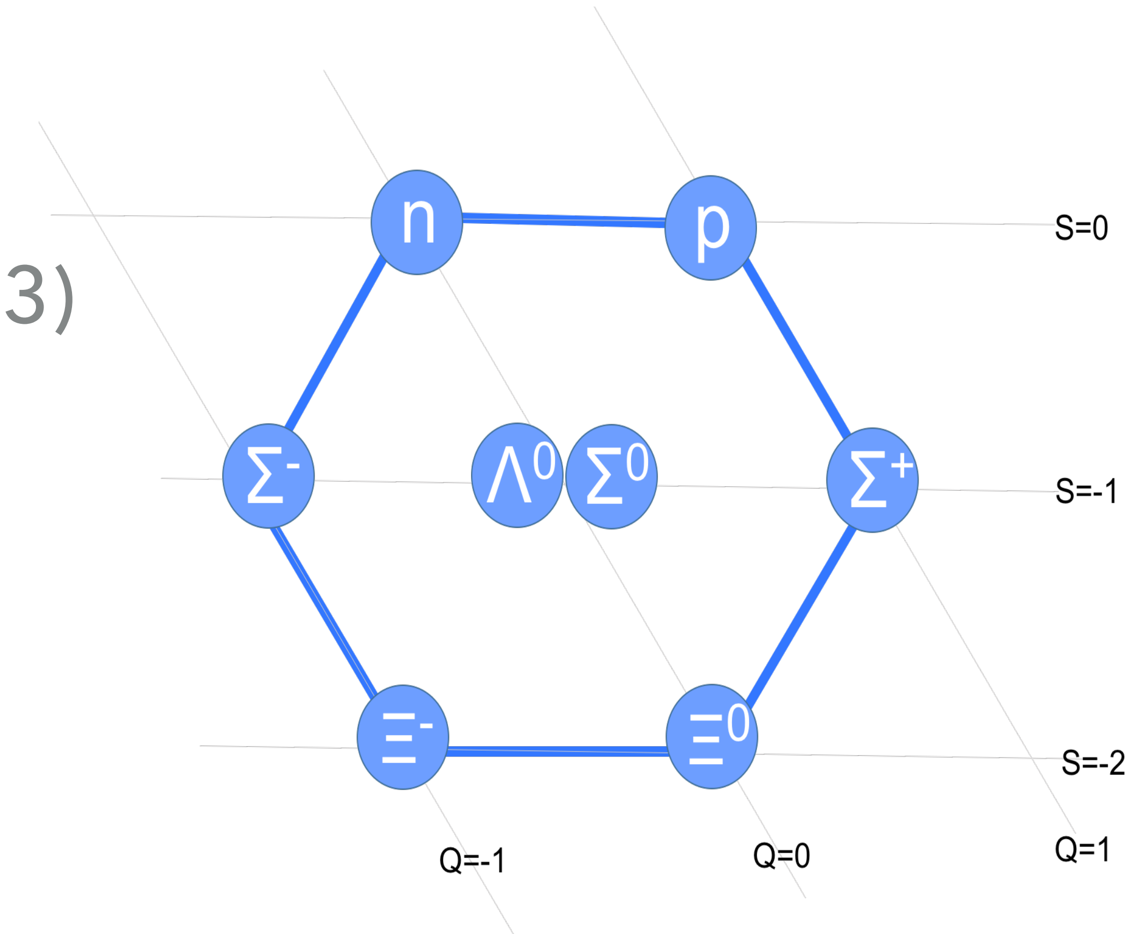


- In 1952, M. Danysz and J. Pniewski first discovered it.

K + (nucleus) → π + (hypernucleus)

HYPERNUCLEI

$J^P=1/2^+$ Baryon octet in SU(3)



Name	Quark content	Mass [MeV]	$I(J^P)$	s	commonly decay to
$\boxed{\Lambda^0}$	uds	1115.683	$0(\frac{1}{2}^+)$	-1	$p^+ + \pi^-$ or $n^0 + \pi^0$
Σ^0	uds	1192.642	$1(\frac{1}{2}^+)$	-1	$\Lambda^0 + \gamma$

WHY DO WE STUDY HYPERNUCLEAR PHYSICS?

- Information on the hyperon-nucleon(YN) interaction is limited.
- By studying hypernuclear, we can understand hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions.

- There is lack of experimental data on Λ hypenuclei.
- There are various experimental programs at J-PARC to measure properties of hypernuclei (E22).

Gaussian Expansion Method (GEM)

- It is one way to calculate few-body problems.
- This method is introduced by M.Kamimura in 1988.

Advantages…

- The Gaussian basis function is suitable for the calculation of the matrix elements.
- It can calculate bound state of the energy very accurately.
- Also, it can calculate it with short computing time.
- It can describe the wave function very precisely.

M. Kamimura, Phys. Rev. A 38, 2, (1988)
E. Hiyama, Y. Kino, M. Kamimura, Prog. Part, Nucl, Phys. 51, 223, (2003)

Gaussian Expansion Method (GEM)

- We assume that objects are hadrons such as baryons and mesons.
- Our aim is calculating of Schrödinger equation accurately.

$$\hat{H}\Psi = E\Psi$$

$$\Psi_{lm}(\vec{r}) = \sum_{n=1}^{n_{max}} c_n \phi_{nlm}(\vec{r})$$ → Wave function

$$\phi_{nlm}(\vec{r}) = N_{nl} r^l \boxed{e^{-\nu_n r^2}} Y_{lm}(\hat{r})$$ → Gaussian basis function

Gaussian Expansion Method (GEM)

In the Gaussian basis function,

$$\phi_{nlm}(\vec{r}) = \boxed{N_{nl}} r^l e^{-\boxed{\nu_n} r^2} Y_{lm}(\hat{r})$$

N_{nl} is the normalization constant from $\langle \phi_{nlm} | \phi_{nlm} \rangle = 1$

The Gaussian size parameter ν_n

$$\nu_n = \left(\frac{1}{r_n}\right)^2$$

$$r_n = r_1 a^{n-1}$$

$\boxed{\text{Free parameters: } n,\ r_1,\ a}$

GAUSSIAN EXPANSION METHOD (GEM)

- By employing the Rayleigh-Ritz variational method, we can get the accuracy solutions of the Schrödinger equation.

$$\sum_{n'=1}^{n_{max}} [(T_{nn'} + V_{nn'}) - \underbrace{\boxed{E}}_{\text{Eigen energy}} N_{nn'}] \underbrace{\boxed{c_{n'l}}}_{\text{Coefficient}} = 0$$

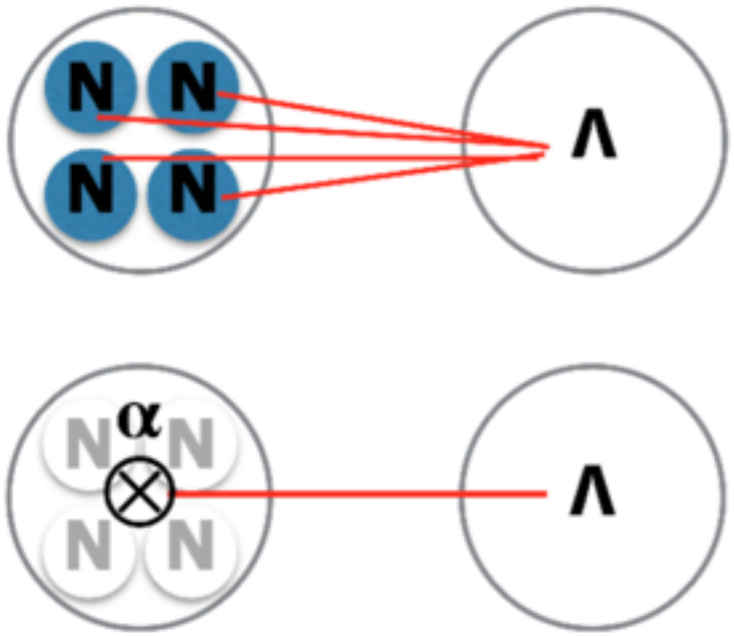
${}^{9}_{\Lambda}$Be with $\alpha\alpha\Lambda$ Model

- We consider ${}^{9}_{\Lambda}$Be employing the $\alpha\alpha\Lambda$ 3-body model.

➜ The cluster-model aspects composing the light *P*-shell hypernuclei.

α cluster model

- Considering average potential of ΛN potential $\frac{1}{4}\sum_{i}^{4} V_{\Lambda N}(i)$.



$\alpha\alpha\Lambda$ Model

	Mass [MeV]	Spin
$\boldsymbol{\alpha}$ = 2p + 2n	3755.68	0
Λ	1115	1/2

- Spin-spin part of the ΛN interaction vanishes.
- Tensor term dose not work.

→ **Due to the α-cluster model.**

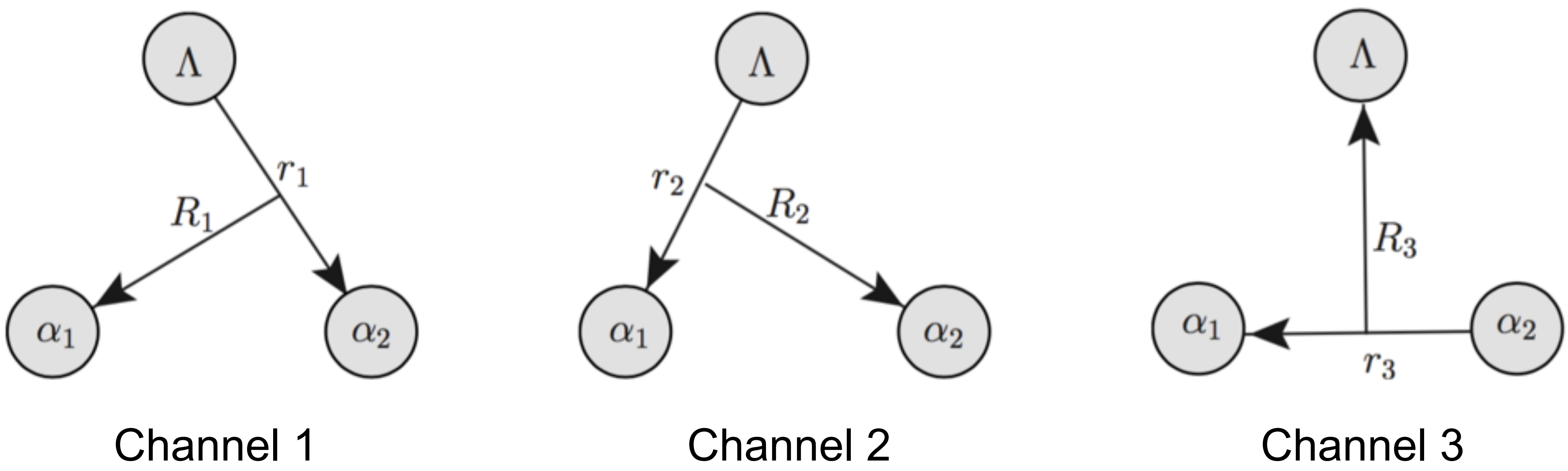
- Spin-orbit term can be negligible in the ground state.

→ **Due to the 1/2 spin of Λ particle**

⇨ **Spin-independent of the Λα interaction.**

ααΛ Model

- We introduce Jacobi coordinate.
- We can write down the potential energy simply.
- It is easy to calculate the matrix elements.



$\alpha\alpha\Lambda$ Model

Wave functions

$$\phi_{nlm}(\vec{r}) = N_{nl} r^l e^{-\boxed{\nu_n} r^2} Y_{lm}(\hat{r})$$

$$\varphi_{NLM}(\vec{R}) = N_{NL} R^L e^{-\boxed{\lambda_N} R^2} Y_{LM}(\hat{R})$$

$$r_n = r_1 \, a_r^{n-1} \, (n = 1, \ldots, n_{\max})$$

$$R_N = R_1 \, A_R^{N-1} \, (N = 1, \ldots, N_{\max})$$

Free parameters: n, r_1, a, N, R_1, A or n, r_1, r_{max}, N, R_1, R_{max}

αΛ Model

- In the channel 3.
- We calculate αΛ model and αα model, respectively.

Potential energy

$$V_{\Lambda\alpha} = 4 \sum_i V_D^i \left(\frac{8\nu_n}{8\nu_n + 3\eta_i} \right)^{3/2} \exp\left[-\frac{8\nu_n \eta_i}{8\nu_n + 3\eta_i} r^2 \right]$$

$$\nu_D^i = \frac{(\nu_{0,even} + \nu_{0,odd})}{2}, \quad \nu_N = \frac{1}{2b_N^2}, \quad \eta_i = \frac{1}{\beta_i^2}$$

$$b_N = 1.358 \text{ fm}, \quad \nu_{\Lambda N} = -38.19 \text{ MeV}, \quad \beta_1 = \beta_{\Lambda N} = 1.034 \text{ fm}.$$

E. Hiyama, M. Kamimura, T. Motoba, T. Yamada and Y. Yamamoto, Prog. Theor. Phys. **97**, 881 (1997).
T. Motoba, H. Bando, K. Ikeda, and T. Yamada, Prog. Theor. Phys. (1985).
S. Ali, A.R. Bodmer, Nucl. Phys. 80, 99 (1966).

αΛ Model

- By applying the GEM, we can obtain the eigenvalue.

n_{max}	r_1 [fm]	r_{max} [fm]	B [MeV]	$\sqrt{\langle r^2 \rangle}_{\Lambda-\alpha}$ [fm]
15	0.1	10	-3.119	2.75

B_Λ^{exp} = 3.12 MeV

αα Model

- We calculate αα model as a two-body problem.
- In this system, there is not only Ali-Bodmer potential with two range gaussian but also coulomb potential from protons.

Potential energy

$$V_{\alpha\alpha} = v^1_{\alpha\alpha} e^{-\nu^1_{\alpha\alpha} r^2} + v^2_{\alpha\alpha} e^{-\nu^2_{\alpha\alpha} r^2}$$

$$V_c = \frac{4e^2}{r} = \frac{4e^2}{r} \mathrm{erf}\left(\frac{\sqrt{3}r}{2R_a}\right)$$

$$v^1_{\alpha\alpha} = -150 \text{ MeV}, \quad \nu^1_{\alpha\alpha} = 0.25 \text{ fm}^{-2},$$
$$v^2_{\alpha\alpha} = 1050 \text{ MeV}, \quad \nu^2_{\alpha\alpha} = 0.64 \text{ fm}^{-2}.$$

E. Hiyama, M. Kamimura, T. Motoba, T. Yamada and Y. Yamamoto, Prog. Theor. Phys. **97**, 881 (1997).
T. Motoba, H. Bando, K. Ikeda, and T. Yamada, Prog. Theor. Phys. (1985).
S. Ali, A.R. Bodmer, Nucl. Phys. 80, 99 (1966).

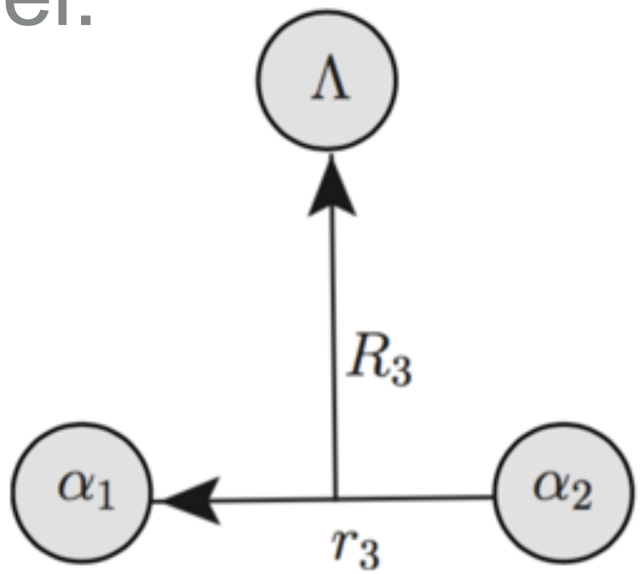
αα MODEL

$n_{\max}$	r_1 [fm]	$r_{\max}$ [fm]	B [MeV]	$\sqrt{\langle r^2 \rangle}_{\alpha-\alpha}$ [fm]
15	0.95	7.4	-5.55×10^{-2}	5.0306

5.6×10^{-2} MeV

ααΛ Model

- We calculate this system only for one channel.



Channel 3

n_{MAX}	r_1 [fm]	a	B [MeV]
15	0.1	1.52	
N_{MAX}	R_1 [fm]	A	7.0
15	0.2	1.13	

Summary and Outlook

- I try to understand the concept of the Gaussian Expansion Method.
- I check that the GEM works very well by reproducing the two-body problems.
- I expand into three-body problems, alpha-alpha-Lambda model.
- Moreover, I will investigate a structure of ${}^4{}_\Lambda H$ and ${}^4{}_\Lambda He$, by taking into account the NNNΛ single channel and the NNNΛ +NNNΣ coupled channel.

THANK YOU