THE $\Lambda_c NN$ CHARM NUCLEI WITH GAUSSIAN EXPANSION METHOD

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INTRODUCTION

➢ We have obtained many experimental data related to hypernuclei and hyperon-nucleon(YN) interactions. Theoretically, study of hyperon-hyperon interactions have advanced, too.

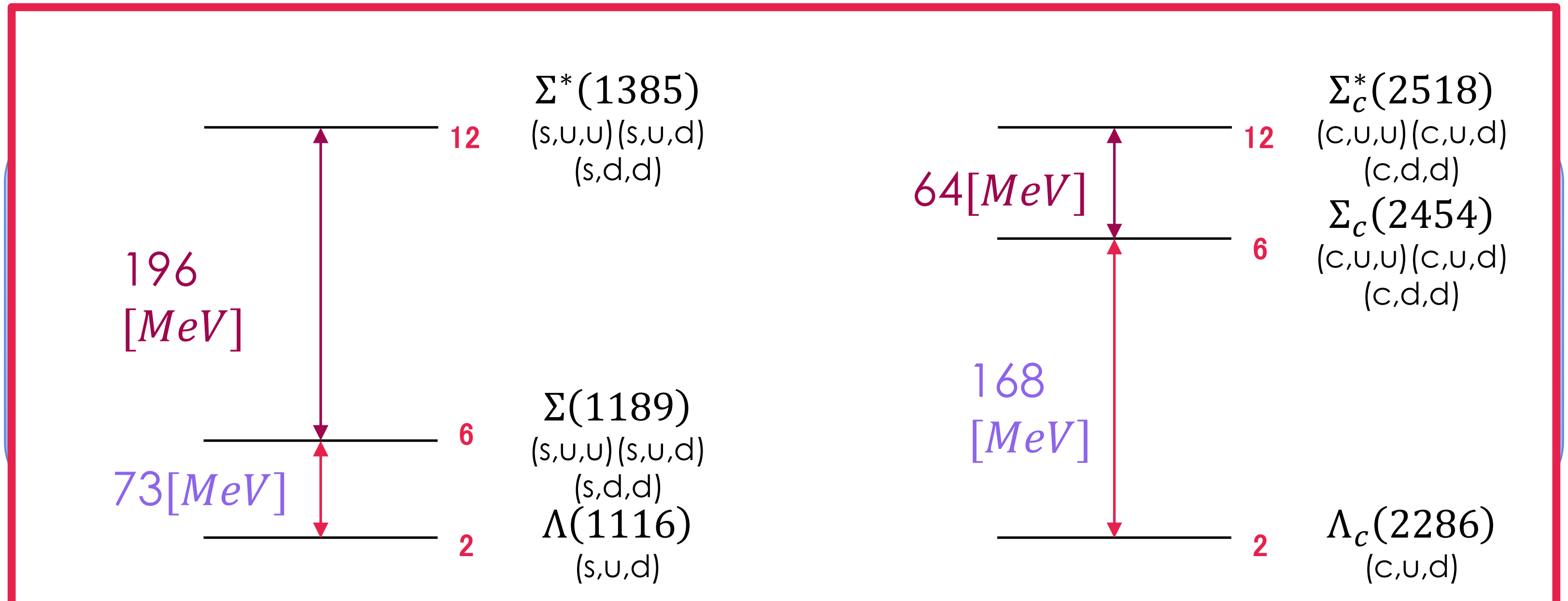
the next stage ➡ **We make theoretical study of charmed nuclei here.**

- According to recent studies, charmed baryons have properties which do not appear in light baryons and strange baryons.

・Heavy quark symmetry

・Channel coupling including higher state than strange sector.

INTRODUCTION



- Interesting properties of charm nuclei
  - Heavy quark symmetry
  - Channel coupling including higher state than strange sector.

$Y_c N$ INTERACTION

- $Y_c N$ potential ($Y_c = \Lambda_c, \Sigma_c, {\Sigma_c}^*$)

First, we construct the YcN potential taking account of channel couplings.
In this study, we construct a hybrid potential using a hadron model and a quark model

▪One Boson Exchange potential [Y.R.Liu, M.Oka, Phys. Rev. D 85, 014015 (2012)]

▪Quark Cluster Model [M.Oka, Nuclear Physics A 881 (2012) 6–13]

$$V_{(Y_c N)} = V_{OBEP} + V_{QCM}$$

- Channel coupling

Channels	1	2	3	4	5	6	7
$J^\pi = 0^+$	$\Lambda_c N(^1S_0)$	$\Sigma_c N(^1S_0)$	$\Sigma_c^* N(^5D_0)$				
$J^\pi = 1^+$	$\Lambda_c N(^3S_1)$	$\Sigma_c N(^3S_1)$	$\Sigma_c^* N(^3S_1)$	$\Lambda_c N(^3D_1)$	$\Sigma_c N(^3D_1)$	$\Sigma_c^* N(^3D_1)$	$\Sigma_c^* N(^5D_1)$

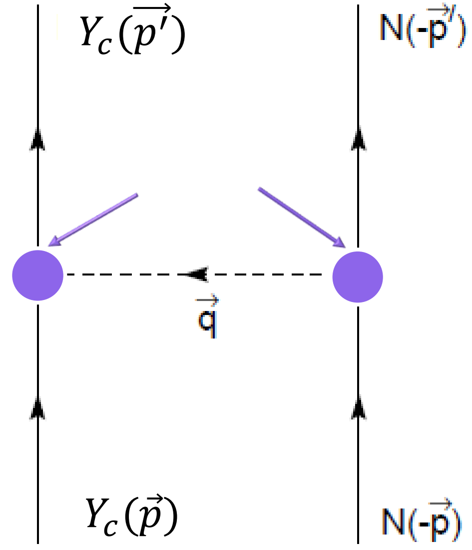
$Y_c N$ INTERACTION

- One Boson Exchange potential

We assume that the pion and the sigma meson exchange between the charm baryon and the nucleon.

At the vertices, we introduce the form factor $F(q)$ as follows

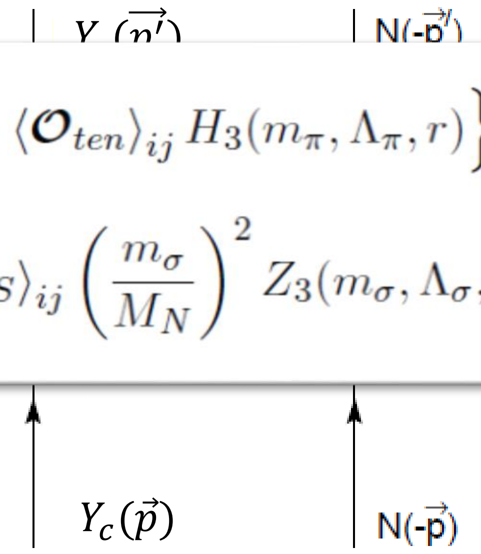
$$F(q) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}$$



$Y_c N$ INTERACTION

- One Boson Exchange potential

We assume that the pion and the sigma meson exchange



$$V_\pi(i,j) = C_\pi(i,j)\frac{m_\pi^3}{24\pi f_\pi^2}\left\{\langle\mathcal{O}_{spin}\rangle_{ij} Y_1(m_\pi,\Lambda_\pi,r) + \langle\mathcal{O}_{ten}\rangle_{ij} H_3(m_\pi,\Lambda_\pi,r)\right\}$$

$$V_\sigma(i,j) = C_\sigma(i,j)\frac{m_\sigma}{16\pi}\left\{\langle\mathbf{1}\rangle_{ij}\, 4Y_1(m_\sigma,\Lambda_\sigma,r) + \langle\mathcal{O}_{LS}\rangle_{ij}\left(\frac{m_\sigma}{M_N}\right)^2 Z_3(m_\sigma,\Lambda_\sigma,r)\right\}$$

$$F(q) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}$$

$Y_c N$ INTERACTION

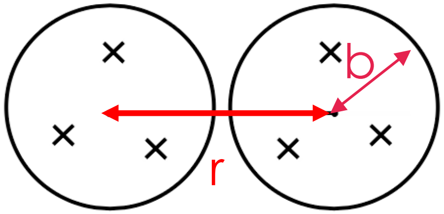
- Quark Cluster Model (QCM)

The QCM considers **two baryon clusters** each made of **three quarks**.

When two baryons overlap completely, r=0, all the six quarks occupy the lowest energy orbit with a single center.

- Potential equation

$$V_{QCM} = V_0 e^{-\frac{r^2}{b^2}}$$



$Y_c N$ INTERACTION

- Parameter fix

In our previous study using the OBE potential, it is found that the results are very sensitive to the cutoff parameters.

To remedy this problem, in the present approach, we determine the parameters of the potential so as to reproduce the NN interaction data using the same model.

▪Fixed parameter

Pi-baryon coupling constants, Range parameter of QCM

▪Determined parameter

Cutoff parameter (Λ_π, Λ_σ), sigma-baryon coupling constants C_σ

Y_cN INTERACTION

- Parameter f

Y_cN-Corresponding To NN

In our previou ... found that the results are very sensitive to the cutoff parameters.

To remedy thi ... , we determine the ... to reproduce the NN intera

YcN-CTNN	C_σ	b[fm]
parameter a	-67.58	0.6
parameter b	-77.5	0.6
parameter c	-60.76	0.5
parameter d	-70.68	0.5

・Fixed param

Pi-baryon cou ... of QCM

・Determined parameter

Cutoff parameter (Λ_π, Λ_σ), sigma-baryon coupling constants C_σ

GAUSSIAN EXPANSION METHOD (GEM)

[E. Hiyama et al. Prog. Part. Nucl. Phys. **51**, 223 (2003)]

We solve the coupled channel Schrodinger equation using the Gaussian expansion method, GEM. The definition is as this slide. The GEM is the one of variational methods based the Rayleigh-Ritz principle.

- Definition

$$(H-E)\Psi_{JM}(\boldsymbol{r})=0,\quad \Psi_{JM}(\boldsymbol{r})=\sum_{n=1}^{n_{max}} C_n^{(J)}\phi^G_{JM,nlm}(\boldsymbol{r})$$

$$\phi^G_{JM,nlm}(\boldsymbol{r})=\phi^G_{nl}(r)Y_{lm}(\hat{r})$$

$$\phi^G_{nl}(r)=N_{nl}r^l e^{-\nu_n r^2}$$

$$\nu_n=\frac{1}{{r_n}^2},\qquad r_n=r_1 a^{n-1}$$

$$N_{nl}=\left(\frac{2^{l+2}(2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi}(2l+1)!!}\right)^{\frac{1}{2}}$$

$$\sum_{n=1}^{n_{max}}(H^{(J)}_{nn'}-EN^{(J)}_{nn'})C_n^{(J)}=0$$

GAUSSIAN EXPANSION METHOD (GEM)

[E. Hiyama et al. Prog. Part. Nucl. Phys. **51**, 223 (2003)]

- Matrix element

$$
\begin{aligned}
N_{n,n'} &= \langle \phi^G_{nlm} | \phi^G_{n'lm} \rangle \\
&= \left(\frac{2\sqrt{\nu_n \nu_{n'}}}{\nu_n + \nu_{n'}} \right)^{l+\frac{3}{2}}
\end{aligned}
$$

$$
\begin{aligned}
T_{n,n'} &= \langle \phi^G_{nlm} | -\frac{\hbar^2}{2\mu} \nabla^2 | \phi^G_{n'lm} \rangle \\
&= \frac{\hbar^2}{\mu} \frac{(2l+3)\nu_n \nu_{n'}}{\nu_n + \nu_{n'}} N_{nn'}
\end{aligned}
$$

$$
\begin{aligned}
V_{n,n'} &= \langle \phi^G_{nlm} | V(r) | \phi^G_{n'lm} \rangle \\
&= N_{nl} N_{n'l} \int_0^\infty r^{2l} e^{-(\nu_n + \nu_{n'})r^2} V(r) r^2 dr
\end{aligned}
$$

$Y_c N$ INTERACTION

- Result of binding energy and scattering length

$J^\pi = 0^+$	CTNN-a	CTNN-b	CTNN-c	CTNN-d
B.E. [MeV]	-	-	1.72×10^{-3}	1.37
(+ Coulomb)				(0.56)
scattering length [fm]	-3.64	-65.15	130.93	5.31

$J^\pi = 1^+$	CTNN-a	CTNN-b	CTNN-c	CTNN-d
B.E. [MeV]	-	2.62×10^{-4}	1.97×10^{-2}	1.57
(+ Coulomb)				(0.72)
scattering length [fm]	-4.11	337.53	39.27	5.01

$\Lambda_c NN$ CHARM NUCLEI

- Effective potential

For simplicity, we first calculate without channel coupling. We choose a 2-range Gaussian potential for the single $\Lambda_c N$ S-wave channel so that it represents the full coupled channel potential. Now, we consider only the YcN-CTNN-d potential to construct an effective potential.

$$V_{\Lambda_c N} = \underbrace{V_1 e^{-\frac{r^2}{{b_1}^2}}}_{\text{OBEP like}} + \underbrace{V_2 e^{-\frac{r^2}{{b_2}^2}}}_{\text{QCM like}}$$

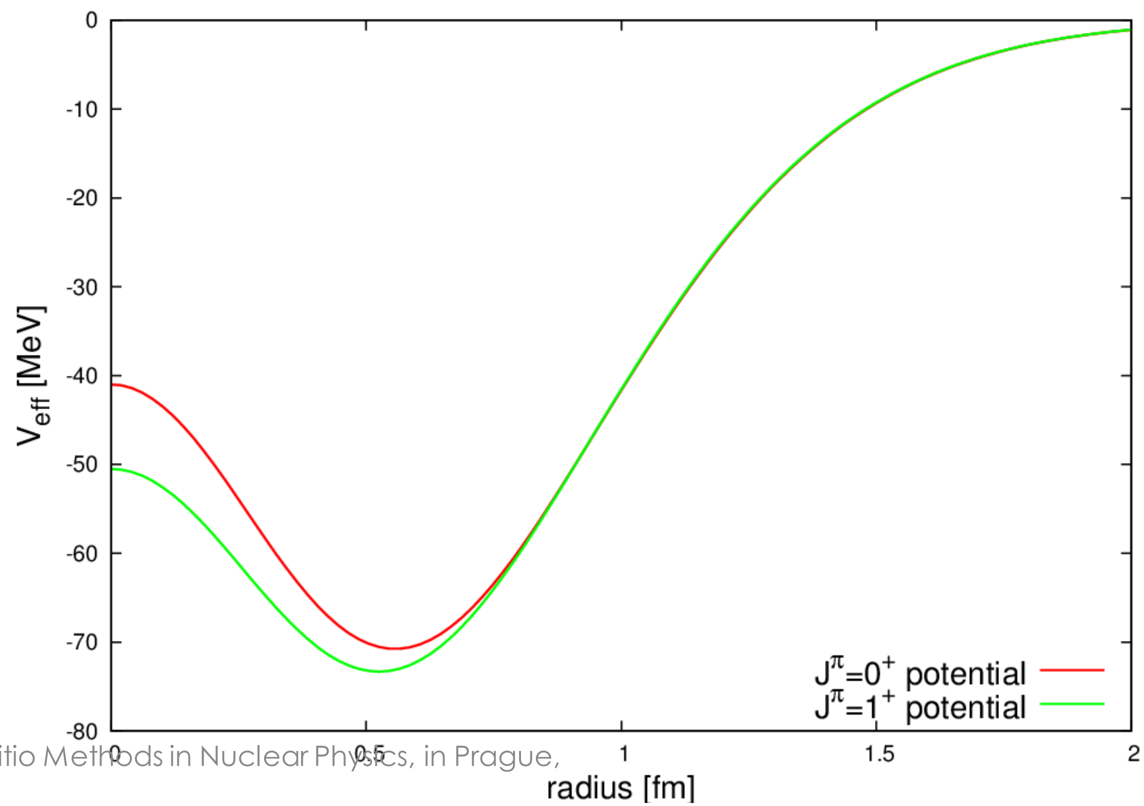
OBEP like QCM like

Parameter fix: $b_1 = 0.9$ fm, $b_2 = 0.5$ fm

$\Lambda_c NN$ CHARM NUCLEI

- Effective potential

$$V_1^{0+} = -150.0[\mathrm{MeV}],$$
$$\mathrm{V}_2^{0+} = 109.0[\mathrm{MeV}],$$
$$\mathrm{V}_1^{1+} = -149.0[\mathrm{MeV}],$$
$$\mathrm{V}_2^{1+} = 98.5[\mathrm{MeV}].$$



$\Lambda_c NN$ CHARM NUCLEI

$$V_{\mathrm{eff}_{YcN}} = \left[V_r^1 + \boldsymbol{\sigma}_{\Lambda_c}\cdot\boldsymbol{\sigma} V_s^1\right] e^{-\frac{r^2}{b_1^2}} + \left[V_r^2 + \boldsymbol{\sigma}_{\Lambda_c}\cdot\boldsymbol{\sigma} V_s^2\right] e^{-\frac{r^2}{b_2^2}},$$

$$\begin{aligned} V_r^i &= \tfrac{1}{4}(V_i^{0+} + 3V_i^{1+}), \\ V_s^i &= \tfrac{1}{4}(V_i^{1+} - V_i^{0+}). \end{aligned}$$

V

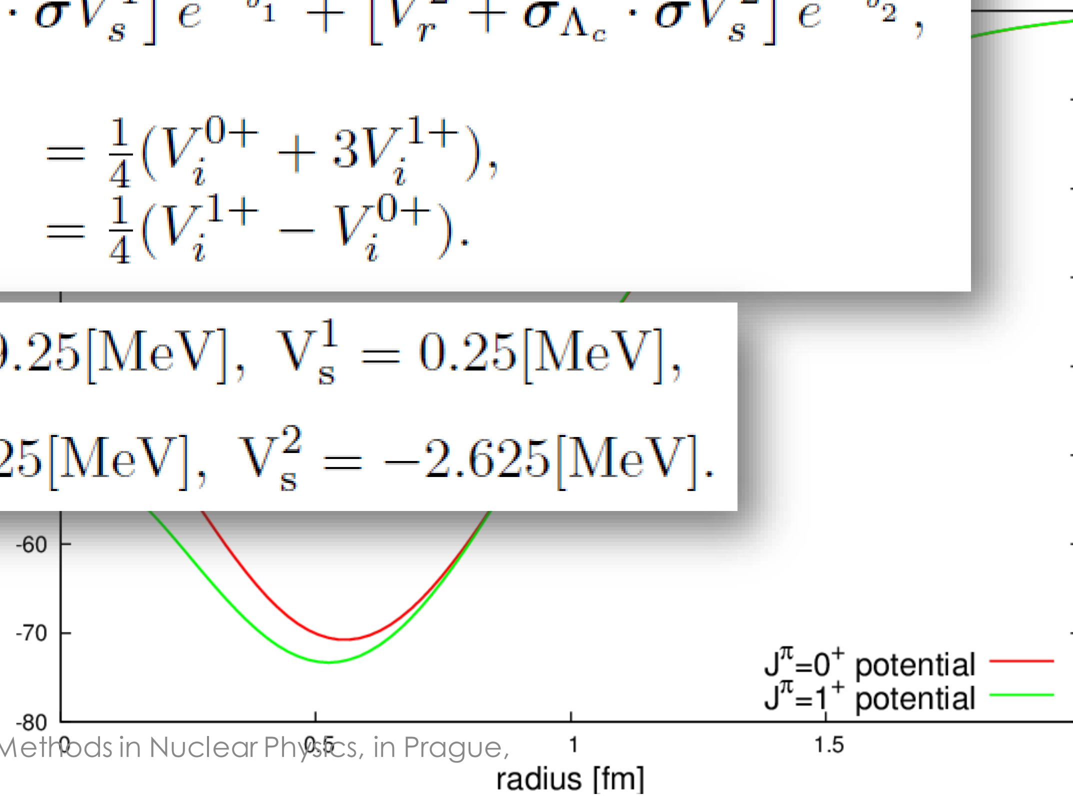
$\mathrm{V}_2^{0+} = 109.0$

$\mathrm{V}_1^{1+} = -149$

$\mathrm{V}_2^{1+} = 98.5$

$$V_r^1 = -149.25[\mathrm{MeV}],\ \mathrm{V}_s^1 = 0.25[\mathrm{MeV}],$$

$$\mathrm{V}_r^2 = 101.125[\mathrm{MeV}],\ \mathrm{V}_s^2 = -2.625[\mathrm{MeV}].$$



GAUSSIAN EXPANSION METHOD (GEM)

[E. Hiyama et al. Prog. Part. Nucl. Phys. **51**, 223 (2003)]

- For 3-body problem

$$\Psi_{JM} = \Phi_{JM}^{(c=1)}(\boldsymbol{r}_1, \boldsymbol{R}_1) + \Phi_{JM}^{(c=2)}(\boldsymbol{r}_2, \boldsymbol{R}_2) + \Phi_{JM}^{(c=3)}(\boldsymbol{r}_3, \boldsymbol{R}_3)$$

$$\Phi_{JM}^{(c)}(\boldsymbol{r}_c, \boldsymbol{R}_C) = \sum_{n_c, l_c, N_c, L_c} A_{n_c, l_c, N_c, L_c}^{(c)} \left[\phi_{n_c, l_c}^{G}(r_c)\varphi_{N_c, L_c}^{G}(R_c)\right]_{JM}$$

$$\phi_{nlm}^{G}(\boldsymbol{r}) = \phi_{nl}^{G}(r) Y_{lm}(\hat{r})$$

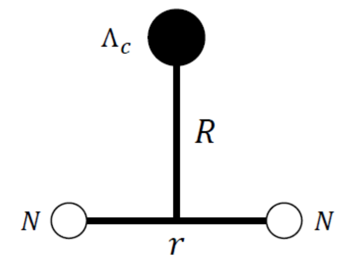
$$\phi_{nl}^{G}(r) = N_{nl} r^{l} e^{-\nu_n r^2}$$

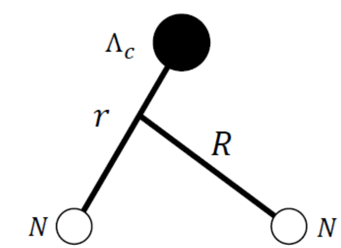
$$\nu_n = \frac{1}{{r_n}^2}, \qquad r_n = r_1 a^{n-1}$$

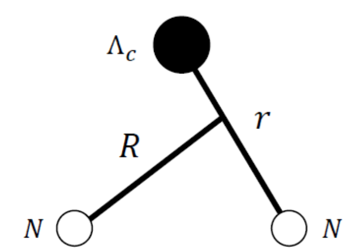
$$\varphi_{NLM}^{G}(\boldsymbol{R}) = \varphi_{NL}^{G}(R) Y_{LM}(\hat{R})$$

$$\phi_{NL}^{G}(R) = N_{NL} R^{L} e^{-\lambda_n R^2}$$

$$\lambda_n = \frac{1}{{R_n}^2}, \qquad R_n = R_1 A^{n-1}$$







$\Lambda_c NN$ CHARM NUCLEI

- Charm 3-body calculation (only S-wave)

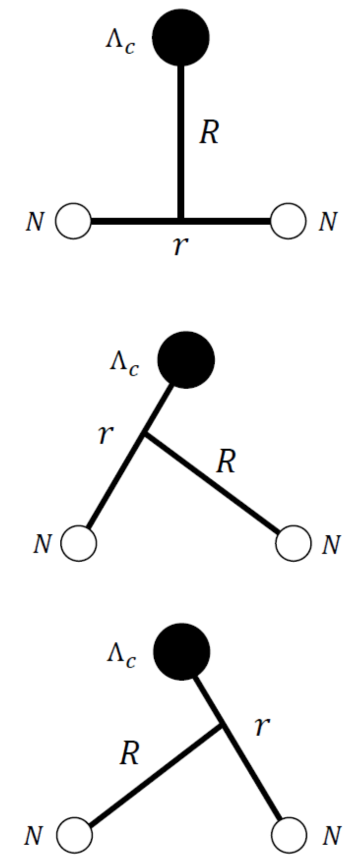
$$I=0 \quad \cdots \quad S_{NN}=1, \text{and } \mathrm{J}^\pi=\tfrac{1}{2} \text{ and } \tfrac{3}{2},$$
$$I=1 \quad \cdots \quad S_{NN}=0, \text{and } \mathrm{J}^\pi=\tfrac{1}{2}.$$

- Minnesota potential

$$V=\left(V_R+\frac{1}{2}\left(1+P_{ij}^{\sigma}\right)V_t+\frac{1}{2}\left(1+P_{ij}^{\sigma}\right)V_s\right)\left(\frac{1}{2}u+\frac{1}{2}(2-u)P_{ij}^{r}\right)$$

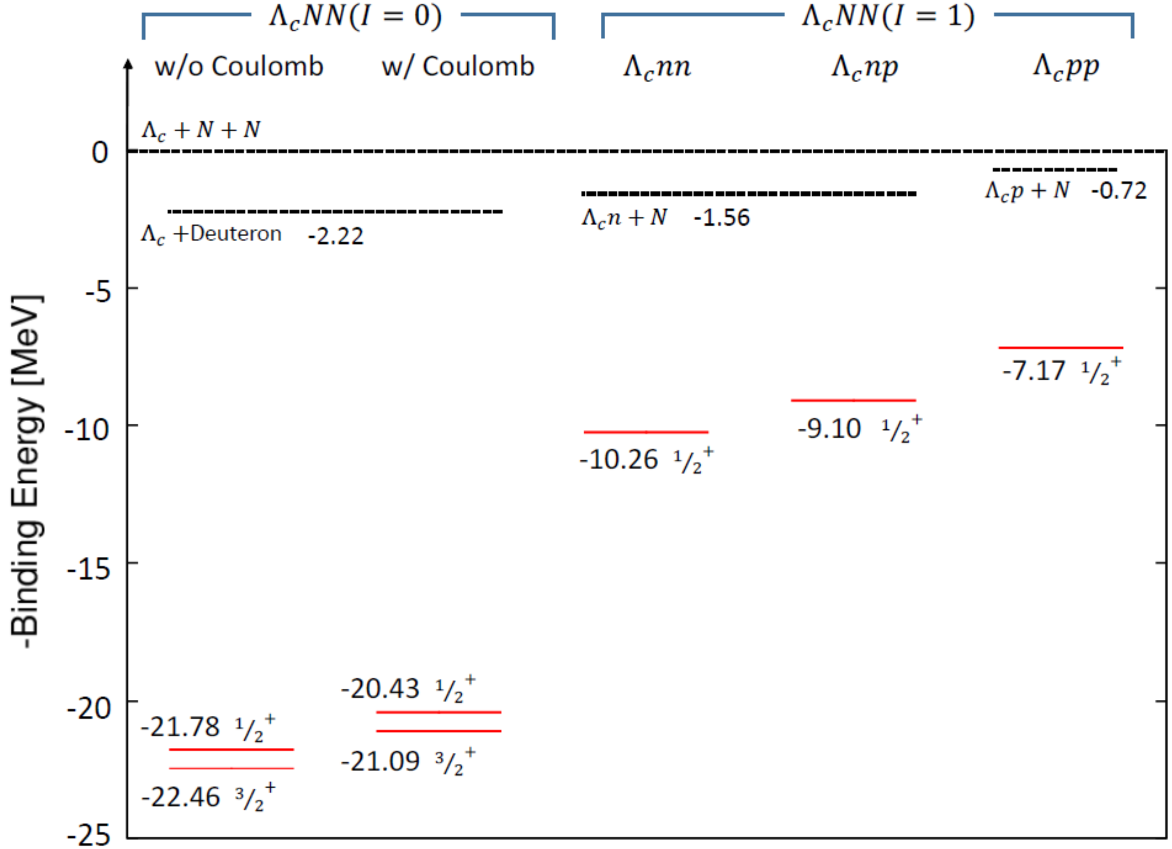
$$P_{ij}^{\sigma}=\frac{1+(\vec{\sigma_i}\cdot\vec{\sigma_j})}{2}$$

[D. R. Thompson, M. Lemere, and Y. C. Tang, Nuci. Phys. A **286**, 53 (1977)]



$\Lambda_c NN$ CHARM NUCLEI

- Binding Energy



SUMMARY

- We propose the $Y_c N$ potential model based on the hadron model and the quark model, and find four parameter set to reproduce experimental data of NN system.
- Calculating the $Y_c N$ 2-body system with Coulomb potential, we get the shallow bound state.
- Using the effective single-channel potential, we found that the $\Lambda_c NN$ 3-body system has a deeply bound state.