Structure of light hypernuclei in the framework of Fermionic Molecular Dynamics

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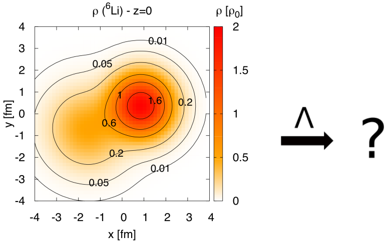
Introduction

Main goal

Study of light hypernuclei

- information about the ΛN (BB) interaction
- modification of the nuclear core
- cluster vs. shell nuclear structure
- Charge Symmetry Breaking (CSB) effects
- $\Lambda N - \Sigma N$ mixing
- 3-body YNN forces (neutron star structure)



The present work

- study of light hypernuclei within Fermionic Molecular Dynamics
- calculations of the ground and excited states of s- and p-shell Λ hypernuclei
- $\mathrm{V}_{\Lambda\mathrm{N}}$ and V_{NN} potential model dependence
- cluster structure

Fermionic Molecular Dynamics

(H. Feldmeier, Nucl. Phys. **A 515** (1990) 147)
(T. Neff, H. Feldmeier, Nucl. Phys. **A 738** (2004) 367)

system of interacting fermions described by an antisymmetrized many-body state $|Q\rangle$

Antisymmetrization

- many-body wave function approximated by a **Slater determinant**

spatial part of a single-particle state represented by **Gaussian wave packets**

$$\langle\vec{x}|\, q_k\rangle = \sum_i \exp\left(-\frac{(\vec{x}-\vec{b}_{k,i})^2}{2a_{k,i}}\right) \otimes \left|\chi^{\uparrow}_{k,i}, \chi^{\downarrow}_{k,i}\right\rangle \otimes |t_k\rangle$$

- complex width $a_{k,i}$, complex $\vec{b}_{k,i}$, complex $\chi^{\uparrow}_{k,i}$ and $\chi^{\downarrow}_{k,i}$ spin parameters (12 real parameters for each Gaussian)

Minimization

Hamiltonian

$$\hat{H} = \hat{T}_N + \hat{T}_\Lambda + \hat{V}_{NN} + \hat{V}_C + \hat{V}_{\Lambda N} - \hat{T}_{\mathrm{cm}}$$

Binding energy

$$E_{\mathrm{B}} = \min_{q_1,\ldots,q_n} \frac{\langle Q|\, \hat{H} \,|Q\rangle}{\langle Q|\, Q\rangle}$$

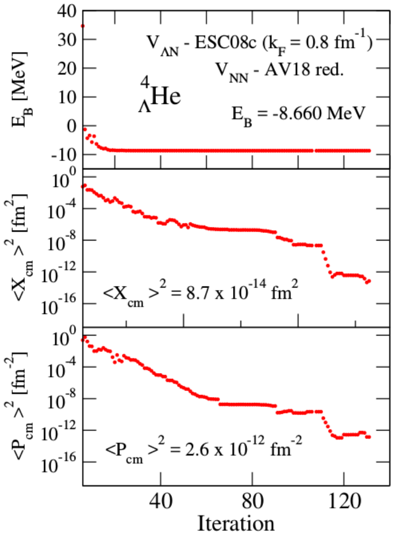
under conditions

$$< \hat{\mathbf{X}}_{\mathrm{cm}} >^2 = 0, \quad < \hat{\mathbf{P}}_{\mathrm{cm}} >^2 = 0, \quad Re(a_k) > 0$$

- single-particle state parameters $q_k = \{a_k, \vec{b}_k, \chi_k^{\uparrow}, \chi_k^{\downarrow}\}$

Result

- minimization yields an **intrinsic state** which is not parity and total angular momentum eigenstate J^π
- **broken symmetries** have to be **restored**



Projection techniques (T. Neff, H. Feldmeier, Eur. Phys. **J 156** (2008) 69)

Projections

Parity projection

$$\hat{P}^{\pi} = \frac{1}{2}(\hat{1} + \pi\hat{\Pi})$$

Total angular momentum projection

$$\hat{P}^{J}_{MK} = \frac{2J+1}{8\pi^2}\int d\Omega D^{J\,\star}_{MK}(\Omega)\hat{R}(\Omega)$$

Eigenstates

- total angular momentum and parity eigenstates are projected out of the minimized intrinsic state

$$|Q; J^{\pi}MK\rangle = \hat{P}^{J}_{MK}\hat{P}^{\pi}|Q\rangle$$

K-mixing

Orthogonal eigenstates

$$|Q; J^{\pi}M\kappa\rangle = \sum_{K}|Q; J^{\pi}MK\rangle\, C_K^{J^{\pi}\kappa}$$

Generalized eigenvalue problem

$$\hat{H}|Q; J^{\pi}M\kappa\rangle = E^{J^{\pi}\kappa}|Q; J^{\pi}M\kappa\rangle$$

- diagonalization of the $\hat{H}$ in a subspace spanned by the projected states $|Q; J^{\pi}M\kappa\rangle$

$$\sum_{K'} H^{J^{\pi}}_{K,K'}\, C_K^{J^{\pi}\kappa} = E^{J^{\pi}\kappa}\sum_{K''} N^{J^{\pi}}_{K,K''}\, C_K^{J^{\pi}\kappa}$$

$$H^{J^{\pi}}_{K,K'} = \langle Q|\hat{H}\hat{P}^{J}_{KK'}\hat{P}^{\pi}|Q\rangle$$
$$N^{J^{\pi}}_{K,K'} = \langle Q|\hat{P}^{J}_{KK'}\hat{P}^{\pi}|Q\rangle$$

$\mathrm{V_{NN}}$ and $\mathrm{V_{\Lambda N}}$ potential input

NN two-body potential

AV18 reduced

(D. Weber, H. Feldmeier, H. Hergert, T. Neff, Phys. Rev. **C 89** (2014) 034002)

- minimal set of operators fitted to nucleon-nucleon matrix elements of AV18 UCOM transformed potential; realistic effective interaction

  (T. Neff, H. Feldmeier, Nucl. Phys. **A 713** (2003) 311)

ΛN two-body potential

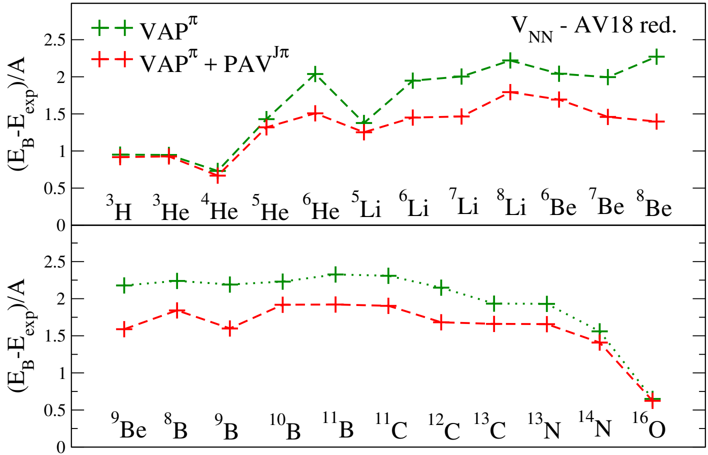
- G-matrix transformed YNG (Jülich - JA, JB, Nijmegen - ND, NF, NS)

  (Y.Yamamoto et. al, PTP Suppl. **117** (1994) 361)

- YNG ESC08c (M. Isaka et. al, Phys. Rev. **C 89** (2014) 024310); ALS and SLS terms

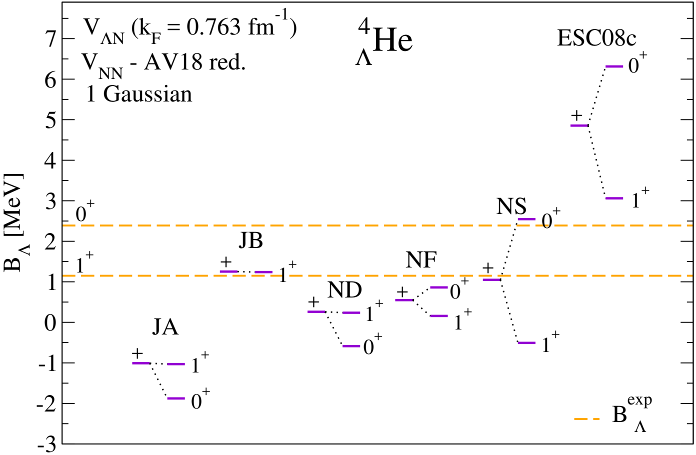
$$V_{\Lambda N}(r) = \sum_{i}^{3}(a_i + b_i k_F + c_i k_F^2)\exp\left\{-\frac{r^2}{\beta_i^2}\right\}$$

Nuclear chart



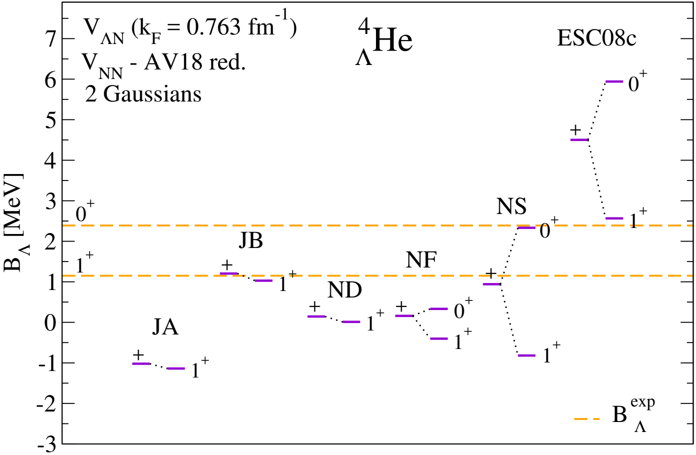
VAP^{π} - variation after parity projection
$\mathrm{VAP}^{\pi} + \mathrm{PAV}^{J^{\pi}}$ - variation after parity projection + projection on J^{π} quantum numbers after variation

$V_{\Lambda N}$ potential model dependence



Substantial difference between Λ separation energies as well as $|B_\Lambda(0^+) - B_\Lambda(1^+)|$ for various $V_{\Lambda N}$

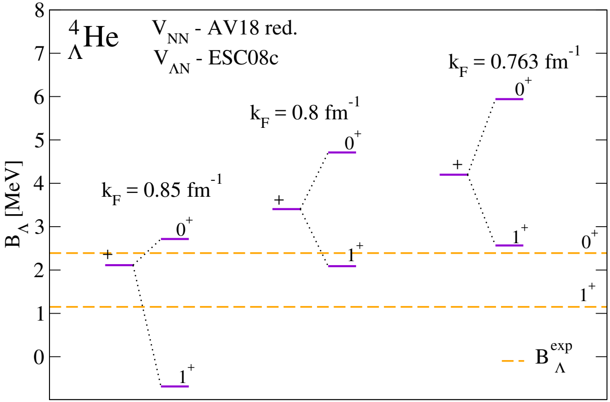
$V_{\Lambda N}$ potential model dependence



Substantial difference between Λ separation energies as well as $|B_\Lambda(0^+) - B_\Lambda(1^+)|$ for various $V_{\Lambda N}$

Fermi momentum k_F dependence in $V_{\Lambda N}$

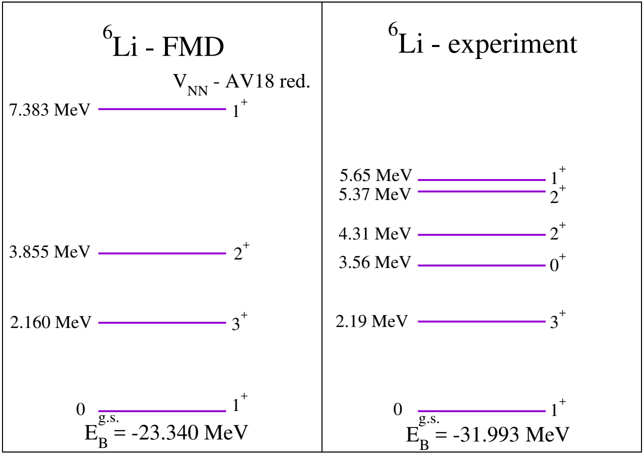
- value of k_F reflects the nuclear medium surrounding the Λ hyperon



$k_F = 0.8$ fm^{-1} (Y.Yamamoto et al, PTP Suppl. **117** (1994) 361), $k_F = 0.763$ fm^{-1} (^{3}He rms radius approximation), and $k_F = 0.85$ fm^{-1}(test value)

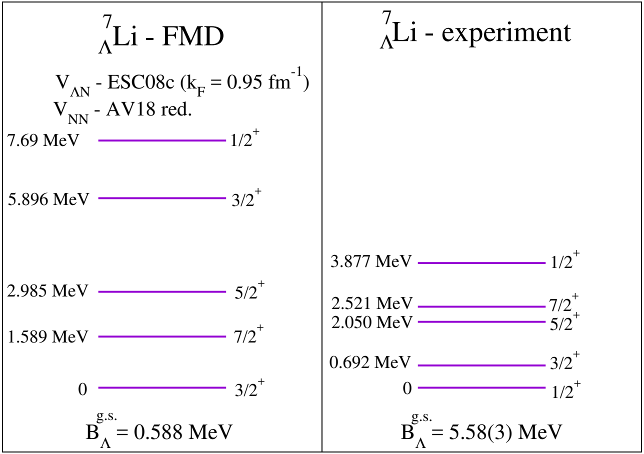
Strong Fermi momentum dependence in the $V_{\Lambda N}$ part (k_F acts as a scaling factor)

Energy levels in ^{6}Li



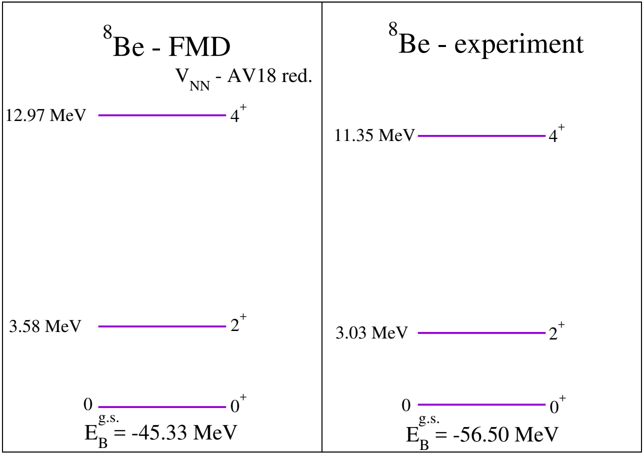
Inconsistency between calculated and experimentally measured total binding energy of the 1^+ ground state – attributed to the insufficient VAP^{π} minimization scheme

Energy levels in $^{7}_{\Lambda}$Li - preliminary



Considerable inconsistency between calculated and experimentally measured excitation spectra – may be attributed to the wrong choice of the Fermi momentum k_F parameter

Energy levels in ^{8}Be



Inconsistency between calculated and experimentally measured total binding energy of the 1^+ ground state – attributed to the insufficient VAP^{π} minimization scheme

Conclusions

In this work :

- realistic effective $\mathrm{V_{NN}}$ interaction included
  - calculations of s- and p-shell nuclei up to $^{16}\mathrm{O}$ using VAP^{π} minimization scheme
  - VAP^{π} does not yield sufficient binding of ground states of calculated nuclei
- YNG ESC08c $\mathrm{V_{\Lambda N}}$ interaction with SLS and ALS terms included
- substantial difference between various $\mathrm{V_{\Lambda N}}$ potential models
- strong $\mathrm{k_F}$ dependence
  ($\mathrm{k_F}$ acts as a scaling parameter of YNG $\mathrm{V_{\Lambda N}}$ potentials)

Next steps :

- $\mathrm{VAP}^{\mathrm{J}^{\pi}}$ minimization scheme, multiconfiguration, constraints on physical quantities
- selfconsistent calculation of the $\mathrm{k_F}$ parameter in each iteration step (Thomas-Fermi + ADA approximation)
- more sophisticated interactions ($\mathrm{V_{\Lambda N}}$ potentials with $\Lambda - \Sigma$ mixing, chiral $\mathrm{V_{NN}}$ and $\mathrm{V_{\Lambda N}}$ potentials)

Variational parameters

Single-particle wave function

$$\langle \vec{x} |\, q_k \rangle = \exp\left(-\frac{(\vec{x} - \vec{b}_k)^2}{2a_k} \right) \otimes \left| \chi_k^{\uparrow}, \chi_k^{\downarrow} \right\rangle \otimes |t\rangle$$

Spatial part

Complex width

- $a_k = \mathrm{Re}(a_k) + \mathrm{i}\mathrm{Im}(a_k)$

Complex vector parameter $\vec{b}_k$

- position and velocity
- $\vec{b}_k = (b_{k1}, b_{k2}, b_{k3})$
- **8** real parameters

Spin part parameters

- the most general form ensures a rotation of an arbitrary angle

$$\left| \chi_k^{\uparrow}, \chi_k^{\downarrow} \right\rangle = \begin{pmatrix} \mathrm{Re}(\chi_\mathrm{k}^{\uparrow}) + \mathrm{i}\mathrm{Im}(\chi_\mathrm{k}^{\uparrow}) \\ \mathrm{Re}(\chi_\mathrm{k}^{\downarrow}) + \mathrm{i}\mathrm{Im}(\chi_\mathrm{k}^{\downarrow}) \end{pmatrix}$$

- **4** real parameters

Position, momentum, and the spread

$$\vec{r} = \frac{\mathrm{Re}(a)\mathrm{Re}(\vec{b}) + \mathrm{Im}(a)\mathrm{Im}(\vec{b})}{\mathrm{Re}(a)} \qquad \vec{p} = \frac{\mathrm{Im}(\vec{b})}{\mathrm{Re}(a)} \qquad (\Delta r)^2 = 3\frac{\mathrm{Re}(a)^2 + \mathrm{Im}(a)^2}{2\mathrm{Re}(a)}$$