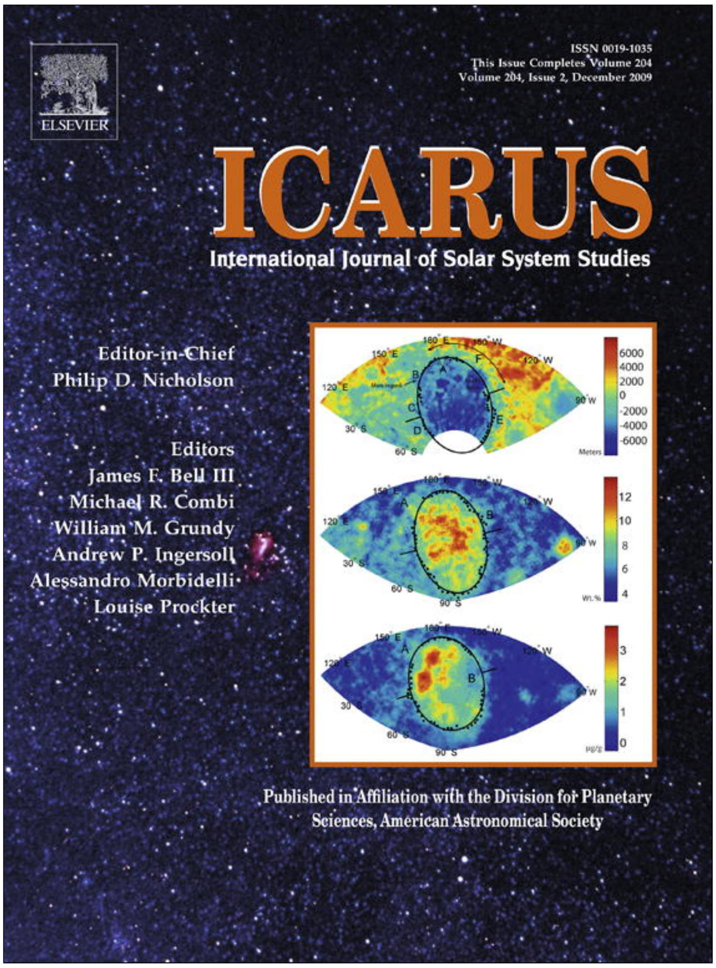
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Significance analysis of asteroid pairs

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ABSTRACT

We have studied statistical significance of asteroid pairs residing on similar heliocentric orbits with distances (approximately the current relative encounter velocity between orbits) up to $d = 36$ m/s in the five-dimensional space of osculating elements. We found candidate pairs from the Hungaria zone through the entire main belt as well as outside the main belt, one among Hildas and one in the Cybele zone. We first determined probability that the candidate pairs are just coincidental couples from the background asteroid population. Those with estimated probability <0.3 were further investigated. In particular we computed synthetic proper elements for the relevant asteroids and used them to determine the three-dimensional distance of the members in candidate pairs. We consider small separation in the proper-element space as a signature of a real asteroid pair; conversely, cases with large separation in the proper-element space were rejected as spurious. Finally, we provide a list of candidate pairs that appear real, genetically related, to facilitate targeted studies, such as photometric and spectroscopic observations. As a by-product, we discovered six new compact clusters of three or more asteroids. Initial backward orbit integrations suggest that they are young families with ages <2 Myr.

1. Introduction

Over the past 20 years there have been numerous attempts to identify streams or couples of asteroids residing on similar heliocentric orbits that would suggest they were of a common origin. In 1990s the searches were restricted to the population of near-Earth asteroids (see, e.g., Drummond, 1991, Steel, 1997). Confirmation of suggested candidate asteroidal couples or streams was hampered by a rapid and chaotic orbit evolution in the near-Earth space. With the boom of asteroid search programs in recent years, the population of main-belt asteroids has been sampled to a depth that allowed (Vokrouhlický and Nesvorný, 2008; hereafter VN08) to establish the existence of a significant population of pairs of small main-belt asteroids residing on closely similar orbits. According to VN08 their compactness cannot be coincidental and they proposed that members in each of the pairs have a common origin. They suggested a few possible mechanisms of pair formation, including rotational fission and collisional breakup of the parent body, or a gentle split of a binary pair. Working conservatively, VN08 identified 60 asteroid pairs that appeared statistically significant from their initial analysis.

The aim of this work is to estimate statistical significance of asteroid pairs in a more systematic way, in particular to establish pairs to greater relative distances than was done in VN08. The underlying scientific aim is to provide a list of them with information facilitating targeted studies such as photometric and spectroscopic observations.

2. Identification of candidate pairs and compact families

We searched for pairs among 342,444 orbits of asteroids from the AstDyS catalog (February 2009 release; Knežević et al., 2002; Knežević and Milani, 2003) with more than one observed opposition and the semimajor axis less than 10 AU. For each couple of neighboring orbits, we computed their distance d in five-dimensional space of osculating orbital elements (a, e, i, ϖ, Ω) defined as a positive-definite quadratic form:

$$\left(\frac{d}{na}\right)^2 = k_a\left(\frac{\delta a}{a}\right)^2 + k_e(\delta e)^2 + k_i(\delta \sin i)^2 + k_\Omega(\delta\Omega)^2 + k_\varpi(\delta\varpi)^2, \qquad (1)$$

where n and a is the mean motion and semimajor axis of either of the two asteroids and $(\delta a,\ \delta e,\ \delta \sin i,\ \delta\varpi,\ \delta\Omega)$ is the separation vector of their orbital elements.[1]

Following Zappalà et al. (1990), we used $k_a = 5/4$ and $k_e = k_i = 2$. The values of k_ϖ and k_Ω were estimated from a magnitude of differential precession that members of a pair could accumulate during a presumably young pair's age (assuming the

[1] Note that due to the singularity of Keplerian orbital elements at zero eccentricity and inclination, the metrics (1) would have to be modified to use non-singular elements for a search for pairs in orbits with eccentricity or inclination nearly zero.

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initial dispersion in the secular angles was small). For an asteroid pair at $a \simeq 2.5$ AU and with age $\leqslant 250$ ky, we note that $k_\varpi = k_\Omega \simeq 3.5 \times 10^{-5}$ imply that $k_\Omega(\delta\Omega)^2$ and $k_\varpi(\delta\varpi)^2$ terms have about the same contribution in d as the corresponding terms in (a, e, i) (assuming pair-components dispersion in these elements corresponds to the escape velocity of the typical multi-kilometer parent body).

We performed our analyses presented in the following sections for three different values for k_ϖ and k_Ω, namely 10^{-5}, 3×10^{-5}, and 10^{-4}. Larger values of these coefficients mean a greater weight given to differences in the secular elements, therefore effectively a higher preference for younger pairs. The analyses showed that we obtain a good consistency between estimated probabilities and long-term dynamics of candidate pairs for the largest of the tested values, $k_\varpi = k_\Omega = 10^{-4}$. Therefore we adopted these values for the analyses presented in this paper.

We point out that the metrics with $k_\varpi = k_\Omega = 10^{-4}$ differs from the one used in VN08 where the value of 10^{-5} was used for the coefficients in the secular angles. While distances d between members in real pairs changed only moderately (which is due to their typically small separations in the secular angles and therefore a lower sensitivity to a choice of the value of the coefficients), distances between members of coincidental pairs usually increased substantially as their secular angles typically differed more than in real pairs; the distances for coincidental pairs in the modified metrics are greater by a factor between 1 and $\sqrt{10}$ than their distances in the metrics of VN08.

VN08 used a conservative limit $d = 10$ m/s for the pair identification. Their choice was based on behavior of the cumulative distribution $N(<d)$ function in the asteroid belt population, notably a change between two distinct regimes $N(<d) \propto d^5$ for large d values to $N(<d) \propto d^2$ for small d values. While the former corresponds to a uniform distribution of points in a five-dimensional space, the latter reveals an excess of nearby orbits. While setting a general scheme, VN08 did not attempt searching putative asteroid pairs beyond their 10 m/s limit of distance.

In a search for wider pairs, we extended the analysis to higher limits on d in this paper. Indeed, we found numerous significant pairs with $d > 10$ m/s, and the widest pairs which we have found to be remarkable candidates have $d \sim 35$ m/s. After the test runs, we used an upper limit on d of 36 m/s for analyses presented below.

We found 558 candidate pairs with $d < 36$ m/s among the multiple-opposition orbits. We exclude poorly constrained single-opposition orbits from our analysis because the formal uncertainty of distance d in pairs involving single-opposition asteroids were often comparable to or not much less than the distance itself. For pairs involving multiple-opposition asteroids, the uncertainty $\sigma(d)$ (the $1-\sigma$ uncertainty of d propagated from the covariance matrices of the orbits of the pair asteroids) was in a vast majority of cases much less than the distance itself, therefore allowing a reliable analysis of their significance.

Among the candidate pairs, there is a mixture of real, genetically related couples of asteroids and coincidental pairs of genetically unrelated asteroids from background population that were transported to similar orbits through their dynamical or collisional evolution. To discriminate asteroid couples in these two categories, we developed the following approach. First, we analysed a population of asteroids surrounding each candidate pair, specifically, we determined whether the surrounding population distribution is consistent with uniform distribution in the five-dimensional space of osculating elements. Second, for pairs that passed the uniform background population distribution test, we estimated probability that the given pair of orbits is coincidental or not. Mathematical details of our procedure are described in Sections 3 and 4, and results are presented in Section 5.

Table 1
Osculating elements ranges of populous asteroid families excluded from pair significance analysis in this work. Note that the Koronis family harbors the moderately young Karin family.

Family	a (AU)	e	i (deg)
Aeolia	2.738–2.744	0.12–0.21	2.2–4.7
Eos	2.96–3.08	0.02–0.13	8.4–12.0
Koronis	2.83–2.95	0.00–0.11	0.8–3.5
Massalia	2.38–2.44	0.11–0.21	0.5–2.6
Nysa	2.34–2.47	0.11–0.24	1.1–3.9
Veritas	3.16–3.18	0.02–0.10	8.0–10.6

2.1. Pairs in families excluded

Some of the candidate pairs were located in very young or populated asteroid families. VN08 showed that pairs identified in recently formed families Datura, Emilkowalski, Iannini, Lucascavin, and 1992 YC2 (e.g., Nesvorný et al., 2003, 2006, Nesvorný and Vokrouhlický, 2006), are probably fragments launched by impacts onto very similar orbits and thus they do not require a special formation mechanism. As such we drop them from our analysis in Section 3 and following. (These recent families have been, however, clearly detected as clusters of three or more asteroids by our pairs finding program; see below also for comments on a detection of other, new asteroid clusters.) Pairs found in large and old families may be real and formed long after the family origin. However, a statistical estimate of significance of a candidate pair located in a populated family would require a detailed model of the orbital distribution of asteroids in the family which is often non-uniform. We do not have available such models for the families.[2] We thus dropped from our present work analysis of asteroid candidate pairs in six large families that have big local deviations from a uniform orbit distribution. As guidance to localize borders of the six families, we first downloaded family-identification files from the PDS website[3] (Mothé-Diniz et al., 2005) and translated the families' locations in the proper-element space to the space of osculating orbital elements. Table 1 gives the derived ranges of the six asteroid families in osculating (a, e, i).

We also tested cases where one asteroid was involved in more than one apparent pair, i.e., being a cluster of three or more asteroids rather than a solitary pair. We found seven such clusters. Five of them were identified as the known compact families Datura, Emilkowalski, Iannini, Lucascavin, and 1992 YC2 that were found earlier by Nesvorný et al. (2003, 2006), and Nesvorný and Vokrouhlický (2006). Two clusters are new.

The first one has the brightest member (6825) Irvine, and we list identified members of the cluster in Table 2. In our procedure, we first identified the three smaller members that form a very compact group with distances ~15 m/s one from each other. Being interested in whether there are some more members of the cluster at slightly greater distances than the limit for d of 36 m/s we used in the nominal search for pairs, we extended our search around the cluster to somewhat greater distances and found one more candidate member of the cluster, (6825) Irvine that is somewhat displaced with $d \sim 60$ m/s from the three smaller members. To confirm that the asteroids in the cluster originated together and that (6825) Irvine really belongs to it, we did a full-fledged integration of the first three orbits – (6825) Irvine, (143797) 2003 WA112

[2] Even for well-studied families, such as Veritas (e.g., Tsiganis et al., 2007), previous works provided analysis of their member distribution in the proper-element space only. What we would need here though is a distribution model of osculating elements in the five-dimensional space that extends semimajor axis, eccentricity and inclination values by those of the secular angles. Inhomogeneities of the number distribution in the latter parameters need to be described.

[3] http://www.psi.edu/pds/resource/mothefam.html.

Table 2
Members of the Irvine cluster. Absolute magnitude *H* estimates and five osculating orbital elements for the epoch MJD 54900 from the AstOrb catalog are given.

Designation		*H*	*a* (AU)	*e*	*i* (deg)	*Ω* (deg)	*ω* (deg)
6825	Irvine	13.9	2.16766795	0.01449400	5.406520	6.856330	310.272305
143797	2003 WA112	16.4	2.16718955	0.01473914	5.429350	8.163460	328.810703
180233	2003 UU192	16.9	2.16700477	0.01487181	5.418685	8.891104	323.610355
	2005 UL291	17.3	2.16699552	0.01456923	5.417964	11.025187	324.850882

Table 3
Members of the 2000 GP36 cluster. Absolute magnitude *H* estimates and five osculating orbital elements for the epoch MJD 54900 from the AstOrb catalog are given. Object denoted with a star is single-opposition only.

Designation		*H*	*a* (AU)	*e*	*i* (deg)	*Ω* (deg)	*ω* (deg)
81337	2000 GP36	15.1	2.60883193	0.11883012	13.649054	10.369383	198.348565
	2001 UR193	15.6	2.61185823	0.11821001	13.604727	9.573822	200.805972
	2003 FK6	16.2	2.60987513	0.11761935	13.663285	11.953908	197.455703
	2008 GW33*	18.1	2.61163693	0.11680772	13.711007	13.121635	196.014329

Table 4
Members of the Rampo cluster. Absolute magnitude *H* estimates and five osculating orbital elements for the epoch MJD 54900 from the AstOrb catalog are given. Object denoted with a star is single-opposition only.

Designation		*H*	*a* (AU)	*e*	*i* (deg)	*Ω* (deg)	*ω* (deg)
10321	Rampo	14.4	2.32952497	0.09429812	6.058556	53.984982	278.732560
	2006 UA169*	18.2	2.32822737	0.09472504	6.079391	58.541895	271.906463
	2007 UM101	17.2	2.32937162	0.09475521	6.050718	53.312986	280.347945

Table 5
Members of the 1998 HR37 cluster. Absolute magnitude *H* estimates and five osculating orbital elements for the epoch MJD 54900 from the AstOrb catalog are given. Objects denoted with a star are single-opposition only.

Designation		*H*	*a* (AU)	*e*	*i* (deg)	*Ω* (deg)	*ω* (deg)
39991	1998 HR37	14.1	2.44428599	0.16160551	3.430221	334.241401	98.231774
	2005 UU94*	18.5	2.44617318	0.15800130	3.423584	334.228120	97.903022
	2006 BR54	17.9	2.44493903	0.15923738	3.430668	334.528117	97.469210
	2006 YE19*	18.2	2.44646339	0.16075969	3.416523	334.480679	96.398549
	2008 YV80	17.3	2.44458581	0.16090795	3.430890	334.179068	98.445955

and (180233) 2003 UU192 – with their geometric and Yarkovsky clones using the methodology of Nesvorný and Vokrouhlický (2006). We found that the three orbits converge during the interval between 1.4 and 1.8 Myr. It suggests a formation of the cluster in a catastrophic event at that time. The observed offset between the three small members and the largest one (6825) Irvine may suggest that the three small asteroids are a cluster of fragments that were initially ejected with comparable velocity vectors.

The second new cluster has the brightest Asteroid (81337) 2000 GP36 and its members are listed in Table 3. Backtracking of their nominal orbits suggests an age about 0.7 Myr. The cluster is harbored in the Eunomia family.

During additional experiments analysing orbit similarities to greater distances and including single-opposition orbit asteroids, we have found four more clusters where our backward orbit integrations suggested that they are young families too. The first one with the brightest member (10321) Rampo appears to have an age between 0.5 and 1.1 Myr, from a backtracking of nominal orbits. Its members are listed in Table 4. The second one has the brightest member (39991) 1998 HR37 and it has a core of a compact group ($d < 20$ m/s) of three asteroids and two other slightly more distant members; they are listed in Table 5. The cluster lies in the Nysa family that we have excluded from our regular analyses (see above). The third cluster, around Asteroid (57738) 2001 UZ160, is adjacent to the J3/1 mean motion resonance and orbits of its members are affected by the $g + g_5 - 2g_6$ secular resonance. A backtracking of the nominal orbits suggests an age less than 0.5 Myr. Members of the cluster are given in Table 6. There is also a nearby Asteroid (18777) Hobson, another possible member of the cluster, but its association needs to be verified with a more detailed

Table 6
Members of the 2001 UZ160 cluster. Absolute magnitude *H* estimates and five osculating orbital elements for the epoch MJD 54900 from the AstOrb catalog are given. Objects denoted with a star are single-opposition only.

Designation		*H*	*a* (AU)	*e*	*i* (deg)	*Ω* (deg)	*ω* (deg)
57738	2001 UZ160	15.3	2.56350387	0.18300953	4.323400	104.967993	181.295028
	2001 NH14	17.1	2.56498589	0.17944925	4.312641	105.256929	181.961396
	2008 HQ46*	17.7	2.56344052	0.18457031	4.321620	105.338168	182.335050
	2008 JK37*	17.5	2.56348204	0.18273519	4.326637	104.339889	181.623919
18777	Hobson?	15.1	2.56097740	0.18508555	4.323817	105.509410	179.721618

backward integration of the multi-opposition orbits. The 2001 UZ160 cluster was not found in our regular analysis runs as it contains no pair of >1-opposition asteroids with d less than the limit of 36 m/s in the current orbit catalog. The last cluster, around Asteroid (119401) 2001 TY50, is a possible young family too. Initial backward integration of their nominal orbits shows a convergence of the three members of the cluster in Ω, but not in ϖ, so a more detailed analysis or more accurate orbits are needed to confirm the cluster as being a real family.

3. Background asteroid population density

To estimate a probability whether a small distance between two orbits from the asteroid population is coincidental or not, we first need to determine density of asteroids in the background population around a position of a candidate pair in the five-dimensional space of osculating elements. To do so, we constructed three boxes centered around each of the candidate pairs. The largest box had a width of $\Delta\varpi = \Delta\Omega = 2\pi$ in ϖ and Ω (reflecting the assumption that no variation of asteroid density in the two angular elements was expected). The width of the box in a, e, and $\sin i$ was set as $\Delta a = \sqrt{10^{-5}/k_a}\, a\Delta\Omega$, $\Delta e = \sqrt{10^{-5}/k_e}\,\Delta\Omega$, and $\Delta \sin i = \sqrt{10^{-5}/k_i}\,\Delta\Omega$. For the adopted coefficients (see Section 2), the widths were $\Delta a \doteq 0.0178\, a$ and $\Delta e = \Delta \sin i \doteq 0.014$. The other two nested boxes, a middle and a small one, respectively, had widths 1/2 and 1/4 of the largest box in each of the five elements. Volume of these two smaller boxes was $V_{1/2} \doteq V_1/2^5 = V_1/32$ and $V_{1/4} \doteq V_1/4^5 = V_1/1024$, where V_1 is the volume of the largest box. All boxes were centered on the orbit of the candidate pair.

Next we determined number of orbits N_1, $N_{1/2}$ and $N_{1/4}$ in the three nested boxes around each candidate pair. Only asteroids with absolute magnitudes $H < H_2 + 0.5$, where H_2 is the absolute magnitude of the fainter member of the asteroid pair, were counted. This is because only orbits of asteroids of size (brightness) similar to or greater than the members of the pair are relevant for statistical estimates below. The +0.5 mag buffer with respect to H_2 was chosen because an error of ±0.5 mag is about a typical uncertainty of H estimates from astrometric measurements.

Using the data in the largest box, the local number density of orbits around a candidate pair was estimated as $\eta = N_1/V_1$. Our working hypothesis, necessary for analysis in Section 4, is that of a uniform (random) distribution of orbits in the pair neighborhood. A basic test of this assumption was performed by comparing the observed asteroid counts $N_{1/2}$ and $N_{1/4}$ in the smaller boxes with the expected values $\eta V_{1/2}$ and $\eta V_{1/4}$. This is done as follows.

For a uniform distribution of orbits in the box, the probability P_b that there are N_b or more orbits in the box is given by the binominal distribution,[4]

$$P_b = 1 - \sum_{i=0}^{(N_b-2)-1} \binom{N_1-2}{i} p_v^i (1-p_v)^{N_1-2-i}, \tag{2}$$

where $b = 1/2$ and $1/4$ for the two nested boxes, and $p_v = b^5$ is their volume ratio to the largest box. In the above formula, the numbers entering the probability computation are $(N_b - 2)$ and $(N_1 - 2)$; two are subtracted from the total numbers because we estimate the probability of a number of orbits surrounding the specific pair. If the probability P_b is low, the observed number N_b in the box is inconsistent with the assumed uniform distribution of orbits in the largest box from which the number density η was estimated. In this case, the smaller box contains an excess number of orbits.

The above test of consistency of the orbit counts in the pair-surrounding boxes with assumed uniform number distribution of orbits in the largest box was performed for each candidate pair. When $P_{1/2}$ was found to be less than a certain level, the candidate pair did not pass the test for uniform distribution of orbits in the surrounding population and we dropped it from further considerations. The excess number of orbits observed in the smaller box around such candidate pair may be due to rapid changes of the density of orbits in the range around the position of the pair, but it may also be due to a concentration of orbits near the position of the pair. In the latter case, there may be a relation between the pair and the concentration of orbits around it, however, we did not study such possibilities.

We chose the following limiting levels for $P_{1/2}$ below which we considered that a candidate pair did not pass the test for uniform background asteroid distribution: (i) 0.01 for pairs with $d < 10$ m/s, and (ii) 0.05 for pairs with greater d. The choice of the threshold values was because probabilities lower than a threshold level between 1% and 5% is where the null hypothesis (of the uniform distribution, in our case) can be rejected. The reason for being less conservative with the closer pairs was that further analyses were typically less sensitive to non-uniformities in the distribution of background asteroids for the closer pairs. Adopting the above mentioned values, the sample of 277 candidate pairs (which were left from the initial set of 558 pairs after excluding those located in the families in the previous section) further decreased by 62 cases (i.e., 22%) that did not pass the test of uniform distribution of background asteroids.

We should point out that some candidate pairs that did not pass the test for uniform background distribution, and thus are not considered in the remaining part of this paper, may still be real pairs. We just cannot use the method presented in this paper to estimate a probability that the similarity of their orbits is a fluke.

An example of a real pair that did not pass the test for uniform background distribution is the pair 2002 PU155 and 2006 UT69 (backward integration of their orbits suggests good convergence of these two orbits in the past). For this case we have $N_1 = 174$ and $N_{1/2} = 18$, giving $P_{1/2} = 0.00015$. (An expected number of orbits in the nested box, $N_{1/2}$, is 7 for a uniform orbit distribution in the largest box.) In other words, it is unlikely that the observed number of 18 orbits (including the two orbits of the pair) in the middle box is just a random fluctuation of a uniform distribution of orbits in the largest box surrounding the pair. Whether the apparently non-uniform distribution of asteroids in the range around the pair is due to rapid changes of density unrelated to the pair, or if the pair and the apparent nearby concentration of orbits are related, remains to be seen from future studies.

4. Number of coincidental pairs

The probability of finding two orbits from a population with the number density η within a volume V is given by the Poisson statistics

$$p_2(V) = \frac{\nu^2}{2} e^{-\nu}, \tag{3}$$

where $\nu = \eta V$. If there are M cells of the volume V, the expected number of pairs found is

$$P_2(V) = M p_2(V) = M \frac{\nu^2}{2} e^{-\nu}. \tag{4}$$

Since

$$M = \frac{N}{\nu}, \tag{5}$$

where N is a number of orbits within the total volume of M cells, we obtain

[4] For large N_1 (i.e., for pairs residing in ranges with high density of orbits), the binominal distribution is approximated with the Poisson distribution. Formula (2) then becomes $P_b = 1 - \sum_{i=0}^{(N_b-2)-1} e^{-\lambda} \frac{\lambda^i}{i!}$, where $\lambda = (N_1 - 2)p_v$.

$$P_2(V) = \frac{N\nu}{2}e^{-\nu}. \quad (6)$$

Using the radius d of a hypersphere of volume V in the five-dimensional space given by following formula:

$$d = \left(\frac{8}{15}\pi^2\frac{1}{V}\right)^{-1/5}, \quad (7)$$

and the radius R_0 of a hypersphere with specific volume (note R_0 is a characteristic distance of objects for the observed density η)

$$R_0 = \left(\frac{8}{15}\pi^2\eta\right)^{-1/5}, \quad (8)$$

we obtain

$$\nu = \eta V = \left(\frac{d}{R_0}\right)^5. \quad (9)$$

Therefore the expected number of pairs with distances $\leqslant d$ in the population is given by

$$P_2(V) = P_2(d) = \frac{N}{2}\left(\frac{d}{R_0}\right)^5 e^{-(\frac{d}{R_0})^5}. \quad (10)$$

We call the ratio d/R_0 to be a normalized distance of orbits. In our analyses below, d/R_0 is always small, and the exponential term on the right side of Eq. (10) is close to unity and may be dropped in the first approximation. With the formalism, the above formula can be used to estimate the expected number of pairs with normalized distances even in a population where the number density η is not constant. (The assumption of locally uniform number density needed for the estimation of η in Section 3 does not need to be held globally in the whole asteroid population.)

5. Statistically significant pairs

A given pair with normalized distance d/R_0 is considered to be statistically significant if it has a low probability to be a chance coincidence of two independent orbits, i.e., if the expected number of pairs with normalized distances $\leqslant d/R_0$ is much less than an actual number of such pairs found in the population.

For each candidate pair with the distance d and the radius of specific volume R_0, we considered a number of pairs (N_p) in the volume V and the total number of asteroids in the population (N).[5] For the countings, we used the same criteria as in the pair selection procedure above. Specifically, we counted asteroids with absolute magnitudes less than $H_2 + 0.5$ that are not members of families listed in Table 1. The population and pairs counts were restricted to an orbital zone of the pair, to suppress biases caused by different observational completeness levels in different zones. Specifically, we divided the orbital space into six zones (where at least one pair was found) as follows:

- Hungaria, $1.78 < a < 2.00$ AU, $16^\circ < i < 34^\circ,\ e < 0.18$,
- Inner main belt, $2.00 < a < 2.50$ AU,
- Central main belt, $2.50 < a < 2.82$ AU,
- Outer main belt, $2.82 < a < 3.27$ AU,
- Cybele, $3.27 < a < 3.70$ AU, $i < 25^\circ,\ e < 0.30$,
- Hilda, $3.74 < a < 4.02$ AU.

In an electronic file available on http://www.asu.cas.cz/~asteroid/astpairs.htm, we list for each pair the number of pairs with normalized distances $\leqslant d/R_0$ found (N_p) and the expected number of pairs P_2 in the population of the pair's zone. When the ratio P_2/N_p is much less than one, we consider the pair to be statistically significant, i.e., having low probability to be a chance orbital coincidence of two genetically unrelated asteroids.

Obviously, the most statistically significant pairs are the closest ones (with small d) or those that lie in low-density regions of the orbital space (with large R_0). On the other hand, we expect a certain contamination by coincidental pairs among somewhat less significant pairs with $P_2/N_p \sim 0.1$ and larger.

In Table 7, we list pairs with $P_2/N_p < 0.3$. The first and second columns give designations of the brighter and fainter members of the asteroid pair. The third column gives the distance in the five-dimensional space of osculating orbital elements. The fourth column gives a distance in the space of proper orbital elements (see below). The next three columns contain the osculating semimajor axis, eccentricity, and inclination. The eighth and ninth columns give the absolute magnitude of the brighter member of the pair and the absolute magnitude difference between the two asteroids, respectively.[6] The tenth column gives the computed probability of that the pair is a coincidental pair of two unrelated background asteroids. The last column contains a remark on a long-term dynamics of the pair. It indicates whether the pair appears to be a real, genetically related pair ("ok" remark) based on its small distance in the space of proper elements or, in a case of somewhat larger d_{prop}, there is a dynamical cause (e.g., effect by a resonance) explaining it, or if there is neither a close proximity in the space of proper elements nor an obvious dynamical reason for the dissimilarity, thus suggesting that it is a spurious, coincidental pair.

5.1. Distances in proper elements

To further examine the relation between the pair orbits and to help discriminating between real and coincidental pairs, we determined proper orbital elements ($a_{prop},\ e_{prop},\ i_{prop}$) of the asteroids in the candidate pairs (one Hilda pair and one Cybele pair were not studied in this section). In the first step, we performed a numerical integration of the planetary system and selected asteroids (using their nominal, best-fit orbits) over a timespan of 5 Myr. We used asteroid initial data from AstDyS site for MJD54800 epoch and propagated JPL405 planetary orbits to the same epoch. Integration was performed with dense-enough output sampling, 100 yr in our case. We then applied methods similar to Knežević et al. (2002) and Knežević and Milani (2003) to determine synthetic proper orbital elements for the candidate asteroids. In particular, we Fourier-analysed secular frequencies in non-singular eccentricity and inclination vectors of planets. Variations with these frequencies were then filtered out from the corresponding non-singular elements of asteroid orbits; the remaining signal should very well correspond to the contribution of the proper term (either eccentricity or inclination). The numerically-determined mean value, and the corresponding standard deviation, of the amplitude of non-singular, post-filtered eccentricity and inclination vectors were thus considered as the best available proxy of the proper e_{prop} and i_{prop} values over the time interval of 5 Myr. The proper semimajor axis a_{prop} value was determined simply as numerically-determined mean value, and the corresponding standard deviation, of the osculating semimajor axis. In most cases of the numbered asteroids, for which the AstDyS site also provides synthetic proper orbital elements, our values and those of Knežević and Milani agreed very

[5] The volume between hyperspheres with radii $f_V^{-1}d/R_0$ and $f_V d/R_0$, where $f_V = \left[(1+\sqrt{5})/2\right]^{1/5}$ is equal to the volume V of the hypersphere with radius d/R_0. We determined the number of pairs in the equivalent volume between the two hyperspheres rather than in the hypersphere with radius d/R_0, to suppress a possible bias in the count resulting from different distributions of orbits of real and coincidental pairs that might occur at the smallest distances.

[6] The absolute magnitudes in the AstDyS catalog were derived from magnitude estimates from astrometric observations, and they are typically uncertain by ±0.5 mag.

Table 7
Asteroid pairs.

Pair		d (m/s)	d_{prop} (m/s)	a (AU)	e	i (deg)	H_1	ΔH	P_2/N_p	l.t.d.
63440	2004TV14	0.52	0.21	1.9379	0.0885	19.987	14.90	2.33	0.0000	Ok
21436	2003YK39	0.98	5.88	2.1875	0.0844	3.736	15.02	3.27	0.0000	Ok
76111	2005JY103	1.64	0.50	2.7166	0.0483	6.925	14.54	1.84	0.0000	Ok
38707	32957	1.75	1.01	2.2782	0.1160	5.925	14.86	1.11	0.0000	Ok
106700	2007UV	4.44	0.61v	1.9095	0.0613	23.095	16.29	0.78	0.0000	Ok
88259	1999VA117	7.03	0.61v	1.9352	0.0647	20.182	14.82	2.16	0.0000	Ok
23998	205383	7.54	3.25	1.9346	0.0530	18.117	15.24	1.29	0.0000	Ok
2007AQ6	2002SF64	8.25	12.92v	1.9032	0.1076	19.034	17.34	1.07	0.0000	Ok (a)
84203	2000SS4	8.33	3.46	1.9611	0.0911	19.995	15.58	1.02	0.0000	Ok
195479	2008WK70	4.06	0.28	2.5556	0.0460	1.469	16.18	0.94	0.0001	Ok
25884	48527	11.88	7.49v	1.9542	0.0803	21.561	14.31	1.40	0.0001	Ok
9068	2002OP28	16.75	34.15v	1.8204	0.1498	20.298	13.45	4.10	0.0001	Ok (b)
4765	2001XO105	11.00	3.49v	1.9454	0.0603	23.707	13.54	3.80	0.0002	Ok
21930	22647	17.48	–	3.9545	0.2520	2.324	12.79	0.37	0.0002	
17198	2004FC126	4.91	0.93	2.2795	0.1027	3.288	14.91	2.57	0.0005	Ok
180906	2003YR67	4.98	0.25	2.2362	0.1159	3.843	17.37	0.24	0.0005	Ok
2002RJ126	2008TS51	6.56	9.46v	2.2047	0.2299	5.773	17.41	0.84	0.0005	Ok
38184	2008GR90	4.58	1.25	2.3269	0.2341	1.777	14.99	2.20	0.0006	Ok
52773	2001HU24	7.82	2.07	2.2184	0.2035	3.661	15.34	2.15	0.0013	Ok
56232	115978	9.85	40.31v	2.1926	0.2025	2.936	15.06	1.13	0.0013	Ok (c)
70511	2007TC334	7.48	1.60	2.4009	0.1943	4.404	15.13	3.31	0.0014	Ok
13653	113029	7.85	30.57v	2.1865	0.1670	4.599	15.38	0.99	0.0015	Ok (c)
184300	2001UU227	12.44	4.35v	2.1240	0.1809	2.739	17.62	1.19	0.0015	Ok
99052	2006KM53	5.37	10.20v	2.7313	0.0867	9.614	15.10	1.39	0.0016	Ok (c, d)
2110	44612	14.81	3.36	2.1982	0.1775	1.129	13.13	2.22	0.0019	Ok
189994	2004RJ294	8.17	2.16	2.2667	0.0981	4.239	16.69	1.50	0.0020	Ok
112249	2006GR49	8.29	1.10	2.2370	0.0863	5.599	16.02	0.75	0.0024	Ok
5026	2005WW113	8.97	13.99v	2.3769	0.2424	4.291	13.68	4.09	0.0029	Ok (c)
15107	2006AL54	7.71	2.07	2.2721	0.1761	4.591	14.27	2.55	0.0039	Ok
92336	143662	19.26	2.82v	1.9566	0.0879	22.438	15.30	1.10	0.0039	Ok
11842	2002VH3	9.76	0.71	2.2511	0.0940	3.691	13.89	2.56	0.0042	Ok
2000SP31	2007TN127	7.88	1.66	2.4054	0.1316	6.077	16.45	1.41	0.0047	Ok
10484	44645	12.97	0.43	2.3205	0.0791	5.726	13.72	0.92	0.0057	Ok
38395	141513	28.42	–	3.5082	0.0664	10.293	13.01	1.41	0.0061	
80218	2002ES90	19.27	1.34	2.2189	0.0274	1.796	16.50	0.03	0.0098	Ok
17288	203489	11.74	1.26	2.2870	0.1802	4.097	14.11	2.13	0.0104	Ok
2003AN55	2003UW156	11.91	11.23v	2.3613	0.2140	5.830	16.22	1.34	0.0114	Ok (c)
1979	13732	17.82	9.02	2.3740	0.1000	6.046	13.44	0.66	0.0118	Ok
26416	2007WO58	16.64	0.50	2.3434	0.0505	4.532	14.04	2.53	0.0119	Ok
40366	78024	13.94	27.22v	2.1760	0.1808	2.490	15.66	1.20	0.0158	Ok (c)
51609	1999TE221	11.72	1.49	2.3090	0.1540	5.641	15.01	1.52	0.0167	Ok
29358	40485	15.87	10.78v	2.2779	0.1450	2.666	15.01	0.43	0.0173	Ok (e)
69142	127502	30.12	4.81v	1.9640	0.0514	22.797	15.27	1.03	0.0174	Ok
117025	169718	10.84	53.53	2.6683	0.0909	15.638	15.01	0.69	0.0200	Spurious
6412	24903	28.99	65.54	2.3575	0.0479	6.835	13.38	0.21	0.0234	Spurious
34081	33560	25.30	72.78	2.3344	0.1056	7.109	14.22	0.05	0.0304	Spurious
138938	2003SL308	19.81	185.93v	2.1794	0.1105	5.592	16.03	1.07	0.0375	Ok (c)
54041	2002TO134	14.25	0.56	2.3234	0.1316	5.515	14.43	1.86	0.0425	Ok
19289	2006YY40	25.72	6.31v	2.1197	0.1262	1.632	15.25	2.33	0.0444	Ok
31569	21321	25.42	141.83	2.3143	0.1223	6.405	14.00	0.24	0.0510	Spurious
101065	2002PY103	22.53	1.34	2.3782	0.2224	8.513	15.73	1.80	0.0543	Ok
88604	60546	17.09	1.61	2.6667	0.1085	11.779	13.10	1.31	0.0548	Ok
90757	170236	19.49	30.93	2.3677	0.1542	4.606	15.84	0.39	0.0553	Spurious
2005UJ157	2001OT77	28.53	356.73v	2.2967	0.2801	5.908	16.29	1.37	0.0564	Spurious
92652	194083	24.88	0.15	2.3408	0.0744	2.231	15.11	1.53	0.0570	Ok
103483	179863	20.90	38.79v	2.3638	0.1502	4.088	15.76	0.01	0.0592	Spurious
7343	154634	18.95	19.91v	2.1921	0.1397	3.959	13.82	2.94	0.0593	Ok (c, f)
74043	111919	26.52	93.70v	2.2980	0.0599	8.291	15.30	0.69	0.0595	Spurious
22280	2005YN176	15.03	2.80v	3.0387	0.1970	0.565	13.70	2.80	0.0601	Ok
52852	2003SC7	19.44	1.51v	2.2629	0.1113	7.264	14.57	1.99	0.0601	Ok
60744	2002MH3	27.05	8.18	2.3084	0.2381	7.613	14.69	1.07	0.0620	Ok
150202	2007VC55	17.51	33.42v	2.3183	0.1591	4.370	16.46	0.57	0.0659	Spurious
157576	120638	19.51	34.68	2.3456	0.0880	4.614	16.55	0.19	0.0668	Spurious
106598	2000AH207	20.92	299.01v	2.2207	0.0598	6.031	15.24	1.37	0.0679	Ok (f)
10123	117306	23.92	20.04v	2.2696	0.2057	4.053	14.04	2.23	0.0705	Ok (g)
31507	98212	23.27	51.93	2.2765	0.1005	5.066	14.82	0.55	0.0716	Spurious
52478	2003QA18	15.17	4.51	2.2100	0.1577	3.747	15.38	1.71	0.0727	Ok
2000SM320	2008TN44	24.65	1.38	2.5993	0.1557	17.555	16.87	0.74	0.0752	Ok
139537	2001SR218	21.25	6.61v	2.6849	0.2791	7.040	14.84	1.52	0.0862	Ok
194561	2008VR13	21.97	15.41v	2.4035	0.2075	6.175	16.27	0.83	0.0866	Spurious
...										
2005QC62	2005VC55	19.53	11.65v	2.7644	0.1117	3.677	16.75	0.19	0.1462	Ok (a)
51161	2001BN16	27.36	391.66v	2.6017	0.1734	5.839	15.00	0.88	0.1559	Ok (d)
118645	2007DK79	17.80	6.87v	2.3316	0.1700	2.919	16.14	1.53	0.1735	Ok
13046	2003QS31	28.66	1.59v	2.5514	0.2629	2.956	14.71	2.45	0.1908	Ok

(*continued on next page*)

Table 7 (*continued*)

Pair		d (m/s)	d_{prop} (m/s)	a (AU)	e	i (deg)	H_1	ΔH	P_2/N_p	l.t.d.
2003UQ164	2003SB338	26.29	4.86	2.5890	0.1875	9.262	16.01	0.89	0.1974	Ok
76148	56048	32.27	0.98	2.3892	0.0764	7.992	14.82	0.17	0.2137	Ok
128637	2001WR54	35.10	1.65	2.6596	0.2169	1.913	15.71	1.05	0.2236	Ok
42946	165548	21.25	35.23v	2.5676	0.0734	4.686	13.52	1.95	0.2252	Ok (c)
64092	130179	35.26	9.27v	2.7068	0.1898	4.350	14.60	1.15	0.2318	Ok
101703	142694	33.90	13.82v	2.5427	0.1288	2.577	15.07	1.94	0.2381	Ok (d)
2003SG111	2001AV49	34.34	3.49	2.6445	0.2907	10.676	16.25	0.04	0.2394	Ok
40837	2005UL431	34.07	2.74	2.6770	0.0608	3.239	14.67	2.26	0.2626	Ok
34380	2006TE23	28.71	3.35	2.6582	0.1022	2.246	14.88	1.91	0.2661	Ok
165389	2001VN61	19.73	1.91	2.3087	0.1114	6.695	16.30	0.27	0.2872	Ok

Notes: (a) the computation of proper elements was corrupted by interaction with a weak secular resonance affecting e and i, (b) by perturbations from Mars, (c) by irregular jumps over a weak mean motion resonance, (d) by interaction with the $g + g_5 - 2g_6$ secular resonance, (e) weak convergence of some clones at times 100–150 ky before present, but case uncertain, (f) by interaction with the z_2 secular resonance, (g) by irregular jumps over the 7:2 mean motion resonance with Jupiter. Spurious pairs with $P_2/N_p > 0.1$ are not listed, they are available in the file mentioned in Section 5.

well. Small deviations were observed in cases of orbits residing very close or inside weak mean motion or secular resonances, for which the AstDyS site provides more stable values. These cases were, however, inspected individually and we decided how reliable is the proper-element computation for our purposes.

Having determined synthetic proper orbital elements for all asteroids of interest, we could then compute their traditional three-dimensional distance d_{prop} in the space of proper elements (e.g., Zappalà et al., 1990); d_{prop} is basically given by formula (1) with the first three terms on the right hand side and proper elements used instead of the osculating elements. We give d_{prop} for asteroids in each of the candidate pairs in the fourth column of Table 7.

Pairs with low chance of being random coincidence, as revealed by small probabilities P_2/N_p, have systematically small values of d_{prop}. This is expected and consistent with a separation at very low relative velocity at origin as suggested by VN08. As the value of probability P_2/N_p increases, reflecting higher chance of fluke in having close-by orbits, we obtain larger values of d_{prop} more frequently. The large d_{prop} may be a tracer of a coincidental asteroid pair, but before we can conclude so we need to check that proper elements have been determined reliably enough. Note that candidate pairs that happen to reside near (or inside) mean motion and/or secular resonances may have uncertain proper elements. In this case the nominal orbits used in our analysis may be incidentally distant, while the overlap of proper-element distributions determined from admissible asteroid clones may correspond to much smaller distance. Since we do not intend to perform an in-depth analysis of such cases, we took a simpler procedure.

We numerically integrated the planetary system with selected asteroids for another 5 Myr and determined synthetic proper elements of asteroid orbits anew from this second interval of time. If the pair distances d_{prop} in the space of proper orbital elements determined from the first and second interval of 5 Myr differed by more than 3 m/s, we flagged the pair as having apparently unstable orbits (the "v" flag in the fourth column of Table 7). An unflagged (apparently long-term stable orbits) pair was considered consistent with being a real pair with its components separated in a gentle way if its d_{prop} was less than a certain threshold value. We tested threshold values in the range 5–15 m/s (i.e., up to a few times the escape velocity from a few km-sized asteroid, which is the typical size of brighter members of asteroid pairs in the sample), checking with backward integrations all pairs with d_{prop} = 5–15 m/s. There was only little sensitivity of the resulting sample on the choice of the threshold value in the tested interval; most pairs with d_{prop} in the interval were found to show a good convergence in the past or there was a dynamical cause for their somewhat unstable (and elevated) d_{prop} values. The tests showed that a few pairs with d_{prop} between 8 and 11 m/s have the least distinctive orbital behaviors in comparison with other pairs in the tested interval, so it suggests that the threshold value should be choosen from this narrower interval. Therefore, we adopted the threshold value on d_{prop} of 10 m/s.

A pair with unflagged (stable) d_{prop} greater than 10 m/s was considered spurious, as high relative velocities of pair members are not expected from the separation mechanisms, and such pairs might be coincidental couples of unrelated asteroids. Pairs with $d_{prop} > 10$ m/s, but apparently unstable orbits (with the "v" flag), were set for a more detailed check of their orbit evolution over the total 10-Myr interval. Usually, we were able to recognize dynamical causes for the long-term orbital irregularities (usually jumps of the semimajor axis over a weak mean motion or secular resonance). Such pairs are considered as orbitally unproblematic despite their higher d_{prop} values and considered being real couples. A few examples are in order to see our procedure.

5.2. Examples

5.2.1. 9068 and 2002 OP28

The first pair with noticeably large d_{prop}, yet very small value of P_2/N_p probability, is the Hungaria population pair (9068) 1993 OD and 2002 OP28. Detailed inspection of their orbits reveals that both asteroids undergo frequent close approaches to Mars (well within its Hill sphere) that makes their semimajor axis values random walk with a typical sudden increment of $\sim 10^{-3}$ AU at each encounter. This explains the elevated d_{prop} for this pair[7] and we consider this couple a real pair of asteroids. We also note that the ability to closely approach Mars for this case is exceptional even for asteroid pairs in the Hungaria population. While direct tidal disruption of the parent body in Mars' gravity field is an unlikely formation mechanism for this pair, the tidal split of a weakly-bound binary would be a possible process additional to those mentioned in Section 1. However, a more detailed analysis of this interesting pair requires more astrometric data to improve orbit determination for both components.

5.2.2. 56232 and 115978

Another interesting situation is presented by the pair (56232) 1999 JM31 and (115978) 2003 WQ56. This case is again characterized by (i) small P_2/N_p probability and (ii) significantly elevated d_{prop} value. A difference between $d_{prop} \simeq 40.3$ m/s computed from the first 5-Myr interval with $d_{prop} \simeq 68.1$ m/s computed from the second 5-Myr interval suggests orbital instability. A detailed scrutiny shows that: (i) semimajor axes of both components in this candidate pair exhibit random jumps over a weak mean motion resonance of $\sim 10^{-3}$ AU width (most likely the M11/19 exterior resonance with Mars; e.g., Morbidelli and Nesvorný, 1999), and (ii) the eccentricity and inclination values are strongly affected by z_2

[7] In fact, d_{prop} computed from the second 5-Myr long time interval increases to $\sim$2.6 km/s. This witnesses significant divergence of the two orbits triggered by continuous Mars encounters.

secular resonance (e.g., Knežević et al., 2002). This latter resonance causes inclination and eccentricity values to oscillate with a very long period (comparable to 10 Myr). The 5-Myr time intervals that serve for computation of the proper e_{prop} and i_{prop} are thus insuficiently long and result in large uncertainty in their values. Therefore we propose the pair 56232 and 115978 is real with a common origin. To further probe this conclusion we numerically backtracked orbits of 2000 geometric and Yarkovsky clones of both asteroids using technique described in Vokrouhlický and Nesvorný (2009). We note that the closest clones can approach to 750 km (about the Hill sphere of mutual gravitational influence of the two components) for epochs earlier than ∼60 ky before present, confirming thus a possibility of their common origin. However, both orbits are much too uncertain to draw more conclusions.

5.2.3. 117025 and 169718

Similarly to the previous example, this candidate pair has (i) small P_2/N_p probability and (ii) significantly elevated d_{prop} value. The difference though is that here we do not see any orbital instability on the monitored 10-Myr interval. Indeed, the d_{prop} values computed from the two consecutive 5-My long intervals of time differ by less than 1.5 m/s. As a result we are thus led to conclude that this pair is a random fluke in the orbital distribution of asteroids (despite its probability of ∼2%, we note flukes exist).

5.2.4. 2000 SM320 and 2008 TN44

This example illustrates that real asteroid pairs can be found at significantly larger d distances than considered by VN08. In this case $d \simeq 24.7$ m/s, yet $d_{prop} \simeq 1.38$ m/s a very small value. While the two orbits are presently too uncertain to explore their past evolution, the small d_{prop} value is suggestive of a real pair.

5.2.5. 34380 and 2006 TE23

Our final example has again, as in the previous case, a somewhat elevated d value and a small d_{prop} value, hinting a real pair. In this case though, we have also moderate $P_2/N_p \simeq 0.27$ probability of a coincidental association. To see the case, we thus integrated backward orbits of the two asteroids together with their clones and noticed a possibility of close approaches at the mutual Hill sphere distance for epochs larger than 100 ky before present. Indeed, the formation mechanism of the asteroid pairs operates uniformly in time and space such that we should expect pairs with a continuum of ages. The older ones, likely the case of 34380 and 2006 TE23 pair, would thus show larger separations in d. Moreover, some of them are also expected to be located in denser locations of the asteroid belt where P_2/N_p is found higher.

Performing similar analyses where needed we give remarks on the long-term dynamics of the pairs in the last column of Table 7. In particular the note "ok" specifies pairs with $d_{prop} < 10$ m/s and/or those for which our checks of long-term orbital evolutions showed dynamical causes for their increased d_{prop} values. We mention there specific dynamical causes of the apparent long-term orbital instabilities, such as (weak) mean motion resonance for interaction with a mean motion resonance, z_2 for the appropriate weak secular resonance, or close encounters with Mars in the above discussed case of 9068 – 2002 OP28 pair. These cases with the "v" flags are obviously less reliable than the cases with apparently stable orbits and more investigation of their dynamical evolution will be needed in future studies. Pairs that have $d_{prop} > 10$ m/s and that appear stable are marked as spurious and they may be random flukes in the asteroid orbital distribution.

An interesting statistic indicating different physical properties of the groups of ok and spurious pairs is a distribution of absolute magnitude differences between pair members, see Fig. 1. The ΔH distribution for real pairs is wide, apparently reflecting a wide size distribution of real pairs, modified by selection effects (incompleteness of the observed asteroids population) of surveys as well as our pair significance estimation method, that both are probably responsible for most or all of the decrease of the number with increasing ΔH in bins >1.5 mag. The distribution for spurious pairs is quite different, with a majority of spurious pairs having $\Delta H < 1$ (median ΔH in the "spurious pairs" sample is 0.69), i.e., coincidental pairs mostly consist of nearly equal brightness asteroids. It is consistent with expectation; coincidental pairs mostly occur between asteroids that are most numerous in the sample population, i.e., between asteroids just slightly brighter (larger) than a value where the completeness level of the observed population drops down significantly, thus they tend to be similar size.

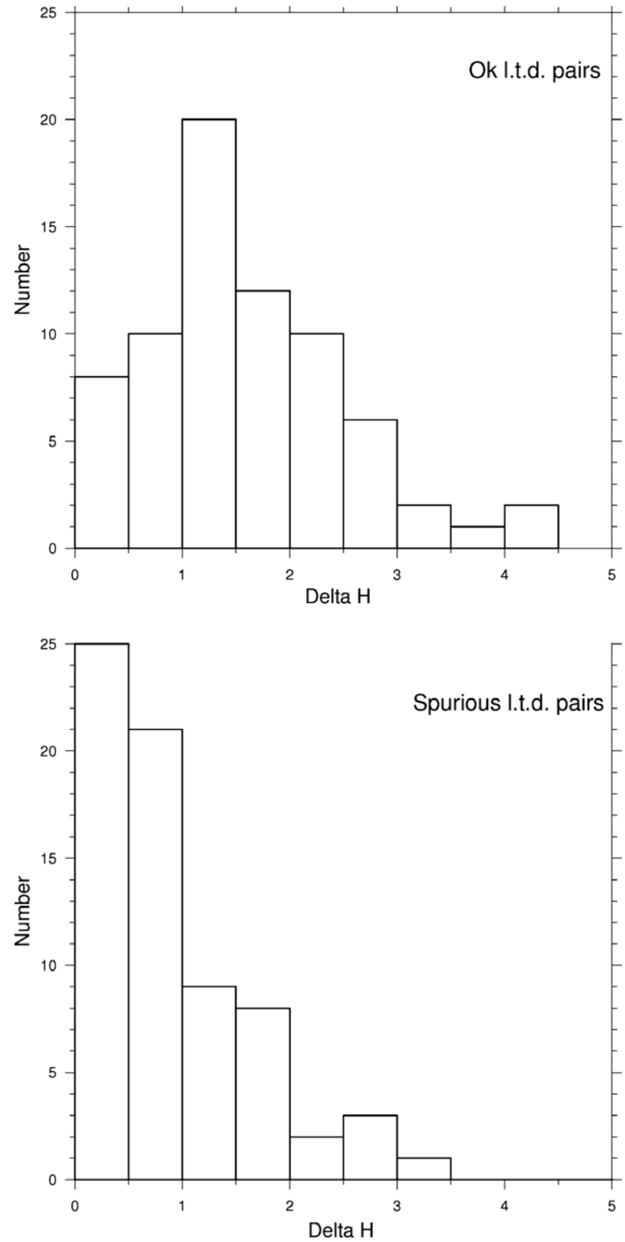

Fig. 1. Distribution of differences of absolute magnitude values $\Delta H = H_2 - H_1$ of ok pairs from Table 7 (upper panel) and spurious pairs (lower panel).

6. Concluding remarks

Studies of most significant pairs (with lowest P_2/N_p, low d_{prop} and/or an "ok" long-term dynamics in Table 7) should bring the best data sample for understanding their characteristics. This is because we believe this sample is the least contaminated by spurious coincidental pairs. We plan to use this information in a photometric survey of the paired asteroids.

Two additional considerations are noteworthy. Working conservatively, we discarded all candidate pairs for which the asteroid density in the five-dimensional space of osculating orbital elements was not uniform enough (Section 3) and those located in certain asteroid families (Section 2). With that, our analysis certainly misses a number of interesting pairs for which we simply cannot estimate probabilites of being coincidental pairs. For instance, we had to discard the best studied case of (6070) Rheinland and (54827) 2001 NQ8 (VN08, Vokrouhlický and Nesvorný, 2009) that resides at the outskirts of the Nysa–Plana clan. The best way for these situations, and in general, is to perform backward integrations of the asteroids in the pair to see if their orbits converge enough. Most often though the orbits are uncertain and a huge number of clones of the nominal orbits needs to be propagated along with the nominal solutions to sample the uncertainty range in the orbital elements space. This is computationally expensive and anyway does not reveal deterministic conclusions. Some of these candidate cases should be, however, studied individually elsewhere.

A question also remains whether real, genetically related asteroid pairs were all formed with low relative velocities. If some of them formed by more energetic mechanism(s) that gave them a significant impulse, then their d_{prop} may not be as low as proposed in VN08. If true, some pairs dubbed "spurious" in Table 7 may still be real ones, just formed or affected by another mechanism than the low-d_{prop} pairs (for instance, medium-size asteroid families formed by super-catastrophic collisions). Studies of some pairs with low P_2/N_p but the "spurious" remark could be therefore useful as well.

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