# Logic

## An introduction to Abstract Algebraic Logic

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Abstract algebraic logic is a theory which aims to provide tools and techniques for the uniform study of arbitrary propositional logics, understood as substitution invariant consequence relations defined on the set of formulas of arbitrary algebraic language.

One of the main achievements of the theory is the discovery that, even at this high level of generality, it is possible to individuate some recurrent patterns in the behaviour of the relation that logics enjoy with their natural matrix-based semantics. This observation led to the construction of the so-called Leibniz and Frege hierarchies, in which logics are classified according to some of these patterns. More precisely, logics are classified in the Leibniz hierarchy according to properties related to the definability of their truth predicates and of logical equivalence, while the Frege hierarchy classifies logics according to general replacement principles. A remarkable feature of the classes of logics in the Leibniz hierarchy is that they are characterized by means of the order-theoretic behaviour of the Leibniz operator, which is a map defined for every algebra and independent from any logic.

In this series of talks we will present an introduction to the basic ideas of abstract algebraic logic with particular attention to the theories of the Leibniz and Frege hierarchies. The prerequisites of the lectures consist in a minimal knowledge of universal algebra and algebraic logic.

The course is tentatively organized in the following four blocks, of which the last three are mutually independent.

### Block 1

General matrix-semantics for arbitrary logics, basics of the theory of algebraizable logics, characterization of algebraizability in terms of the Leibniz operator.

### Block 2

Generalization of algebraizability to $M$-sets (where $M$ is a monoid), examples of different algebraizations (e.g. order algebraizability and algebraization of Gentzen systems), examples of transfer theorems (e.g. Beth befinability vs epimorphism surjectivity).

### Block 3

Overview of the Leibniz hierarchy, characterization of classes in the Leibniz hierarchy by properties of the Leibniz operator, characterization of classes in the Leibniz hierarchy by means of closure under class-operators.

### Block 4

Overview of the Frege hierarchy, finitary selfextensional logics with a conjunction or the (uniterm) deduction theorem, relations with Kripke semantics.