Towards a Topological Theory of Knowledge, Inquiry and Correlations

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I accordance to the recent trend towards an interrogative epistemology, "To know is to know the answer to a question" (Schaffer). But this begs the question: *what is a question?*

At a first approximation, an *'exact' propositional question* (i.e. always having a proposition as its unique correct answer) *is a partition* of the state space. This representation suffers from three problems, none of which is addressed by its recent generalizations. I claim that a topological theory can address all of them

First, an answer to a non-propositional question (of the form *what, where, who*, etc) is not a set of worlds. Informationally, *a non-propositional exact question can be encoded as a variable*, taking various values ("answers") in different possible worlds. Reinterpreting the above quote in this way, we are lead to a paraphrase of Quine: *To know is to know the value of a variable.*

Second, questions are never investigated in isolation: we answer questions by reducing them to other questions. This means that the proper object of knowledge is uncovering correlations between questions. *To know is to know a functional dependence between variables.*

Third, when talking about *inexact* (or *empirical*) questions/variables, the exact value/answer might not be knowable, and instead only "feasible answers" can be known. It is reasonable to assume that the conjunction of two (correct) feasible answers is a (correct) feasible answer: this suggests modelling propositional questions as *topological bases* on the state space. I investigate this conception and show the importance of topological notions for understanding propositional knowledge and questions.

Combining further the three issues, we arrive at a conception of *an inexact (not necessarily propositional) question as a map from the state space into a topological space.* Here, the exact value of the variable (exact answer) is represented by the output of the map, while the open neighborhoods of this value represent the feasible answers (knowable approximations of the exact answer). A question Q epistemically solves question Q' if every feasible answer to Q' can be known if given some good enough feasible answer to Q. I argue that knowability in such an empirical context amounts to the *continuity* of the functional correlation. *To know is to know a continuous dependence between variables.*

I investigate a logic of *epistemic dependency*, that can express knowledge of functional dependencies between (the values of) variables, as well as dynamic modalities for learning new such dependencies. This dynamics captures the widespread view of knowledge acquisition as a process of *learning correlations* (with the goal of eventually tracking causal relationships in the actual world).