Geometric aspects of MV-algebras

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The fact that every semisimple MV-algebra is isomorphic to a separating subalgebra of the algebra of continuous functions from X into $[0,1]$, for X a compact Hausdorff space, has long been known. Considerable information on the structure of MV-algebras follow. E.g., *every* compact Hausdorff space can be obtained as the maximal spectrum of a suitable MV-algebra. Nonetheless, the result leaves at least two questions open:

1. Can the functions forming the aforementioned separating subalgebra be characterised in an intrinsic way?

2. How can the above result be extended to the whole class of MV-algebras?

In this talk I will report on a series of works (some in collaboration with Leonardo Cabrer and some in collaboration with Vincenzo Marra) that tackle the above questions.

The main point in Question 1 is that the topological structure alone is not sufficient to reconstruct the MV-algebra. It seems inevitable to move from pure topological spaces to *geometric* spaces, i.e., topological spaces equipped with a system of coordinates. The coordinatisation is crucial to describe the separating functions, but it turns out to be useful also in the characterisation of special classes of MV-algebras. E.g., finitely presented MV-algebras correspond to rational polyhedra, and the latter concept cannot be formalised in pure topological terms.

To address Question 2, one needs to deal with non-semisimple MV-algebras, that is, the ones containing infinitesimal elements. It turns out that infinitesimal elements correspond to regions of the space in which not only the *value* of a function matters, but also the *rate* at which this value is attained. We deal with this differential phenomenon considering the *pro-completion* of the category of rational polyhedra. This leads to a duality between the whole category of MV-algebras and a category whose objects are systems of rational polyhedra approximating a compact Hausdorff space.