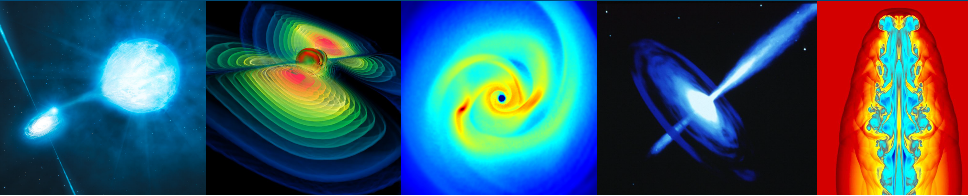
Hydrodynamics in X-ray binaries

A new hydrocode for wind simulation

Jan Čechura

Faculty of Mathematics and Physics of the Charles University in Prague

Astronomical Institute of the Academy of Sciences of the Czech Republic



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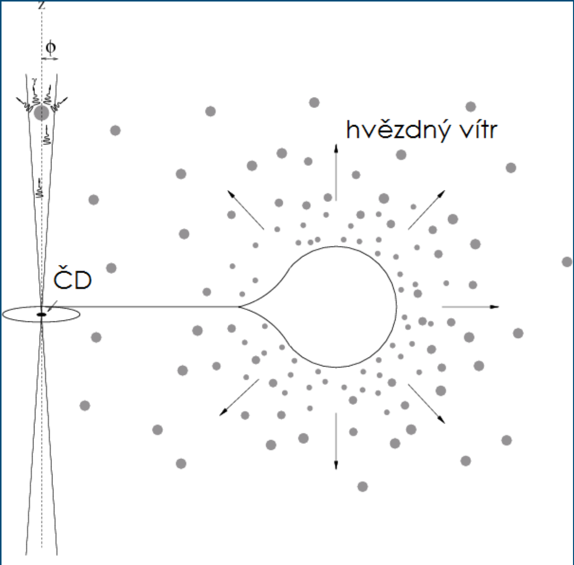
R(M)H Seminar, Mar 31, 2011, Ondřejov

X-ray binaries



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- Numerical model calculated in 3D-Eulerian coordinate grid.

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- Under the assumption of the Sobolev approximation, line force $f_{\rm L}$ could be approximated by:

$$f_{\rm L} = \frac{\sigma_{\rm e} L_*}{4\pi c r^2} k t^{-\alpha} \qquad t = \sigma_{\rm e}\rho v_{\rm th}\left(\frac{{\rm d}v}{{\rm d}r}\right)^{-1}$$

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- We use the continuity equation in form of

$$\frac{\partial \rho}{\partial t} = \nabla \cdot (\rho\vec{v}) \,.$$

Roche potential

$$\Phi_{\rm eff}(r,\vartheta,\varphi) = -\frac{GM_*}{D}\left\{\frac{D(1-\Gamma_*)}{r} + \frac{q(1-\Gamma_{\rm x})}{\left[1-2(r/D)\lambda+(r/D)^2\right]^{1/2}} - \left(\frac{r}{D}\right)\lambda + \frac{1}{2}(1+q)\left(\frac{r}{D}\right)^2\left(1-\mu^2\right)\right\}$$

- Mass rate of the components of the binary

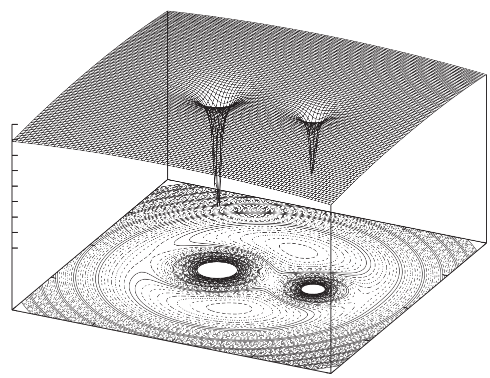
$$q = M_{\rm x}/M_*$$

- Source luminosity relative to the Eddington luminosity

$$\Gamma = \frac{\sigma_{\rm e}L_*}{4\pi GMc}$$

- Substitutions

$$\lambda = \cos\varphi\sin\vartheta$$
$$\mu = \cos\vartheta$$



Radial approximation

- Supposing the velocity field in form of: $\vec{v} = v\,(r)\,\frac{\vec{r}}{r}$
- For partial derivatives hold $\frac{\partial}{\partial\vartheta} = \frac{\partial}{\partial\varphi} = 0$

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$$v\frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{\mathrm{d}\Psi_{\mathrm{eff}}}{\mathrm{d}r} + f_{\mathrm{L}} - \frac{1}{\rho}\frac{\mathrm{d}P_{\mathrm{g}}}{\mathrm{d}r}$$

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- We presume a spherically non-symmetrical case. Therefore the continuity equation for mass reads

$$\frac{\mathrm{d}\dot{M}}{\mathrm{d}\Omega} = \rho v r^2$$

- The streamlines of the wind are strictly radial and the material is confined within a selected cone.

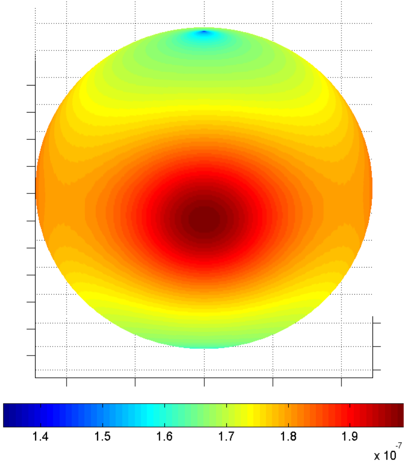
HDE 226868 – Cygnus X-1

- RE: 19h 58min 21.6756s
  DE: + 35 12’ 5.775”
- Spectral class: **O9.7Iab**
- Apparent visual magnitude of the optical component: **8.95 mag**
- Absolute visual magnitude of the optical component: **-6.5 mag**

Table: Parameters of Cyg X-1

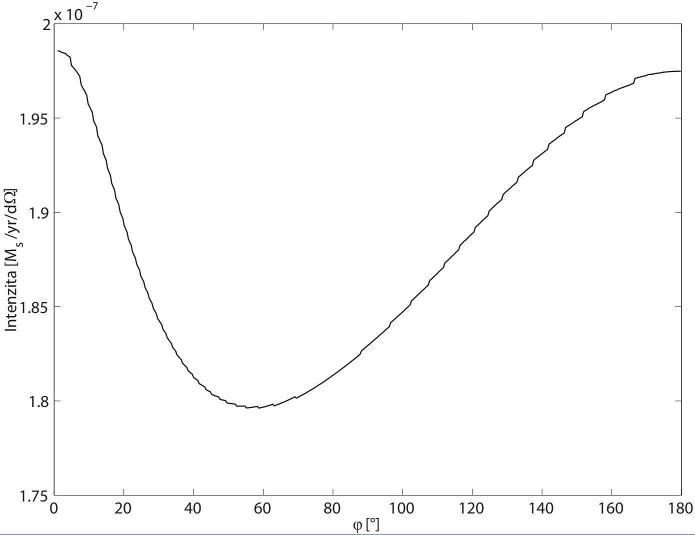
T_{eff}	[K]	effective temperature	28000 - 31000
$log(g)$	$[\log{(m \cdot s^{-2})}]$	surface acceleration	3.31 ± 0.07
R_1	$[R_\odot]$	radius of the supergiant	18
L_1	$[L_\odot]$	luminosity	2.3×10^5
Γ_1		rate to L_{edd}	0.26
m_1	$[M_\odot]$	mass of the supergiant	24 ± 5
m_2	$[M_\odot]$	mass of the black hole	8.7 ± 0.8
P_{orb}	[days]	orbital period	5.599829 ± 0.000016
D	$[R_\odot]$	distance of the components	42 ± 9
i	[°]	inclination	48 ± 7
d	[kpc]	distance	2.0 ± 0.1

Distribution of Stellar Wind Intensity of HDE 226868



Intensity of SW $[M_\odot \cdot yr^{-1} \cdot d\Omega^{-1}]$

Stellar Wind Intensity in the Equatorial plane



0. approximation

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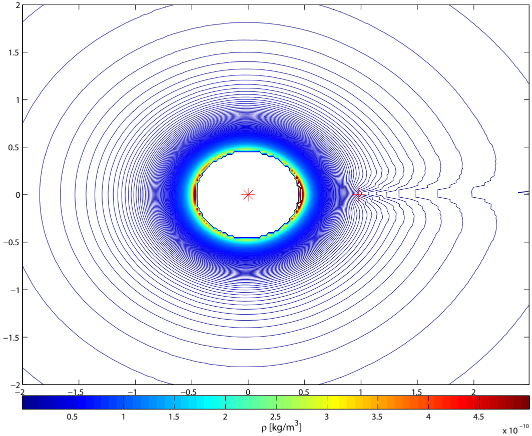
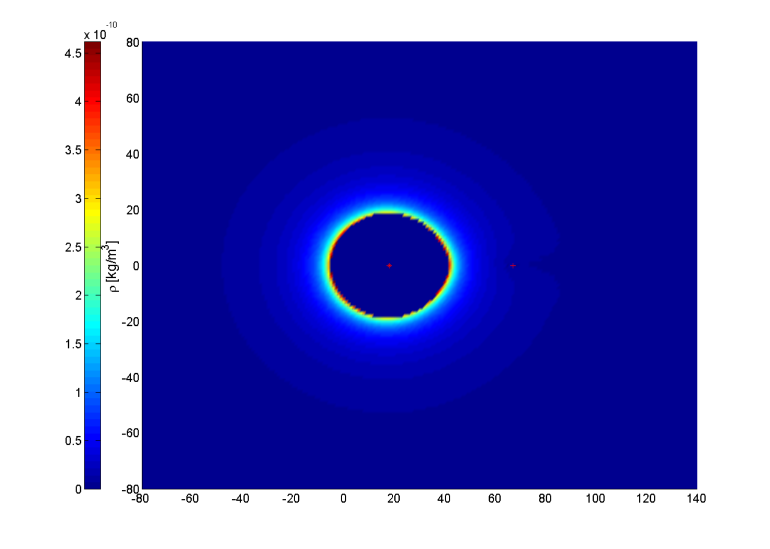
  - $\frac{\mathrm{d}\dot{M}}{\mathrm{d}\Omega} = \rho v r^2$
- $\rho\,(x, y, z)$ – spatial distribution of mass density



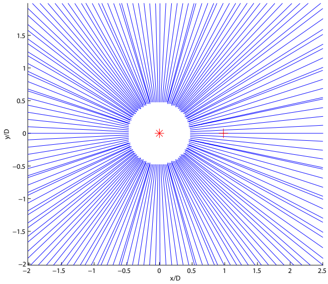
Evolution to a new Stationary Solution



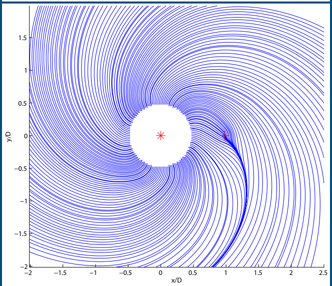
$$\rho \ \ [kg \cdot m^{-3}]$$

Streamlines

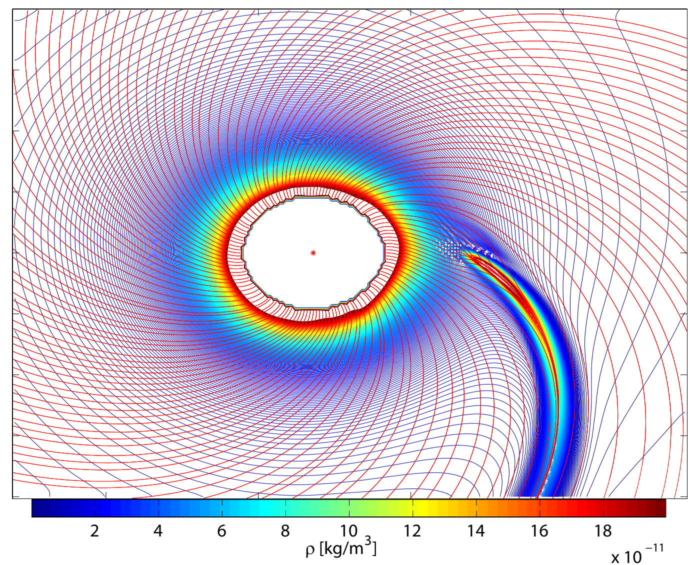
- Streamlines in the radial approximation



- Streamlines in a fully 3D radiation-hydrodynamic simulation



Streamlines and ρ



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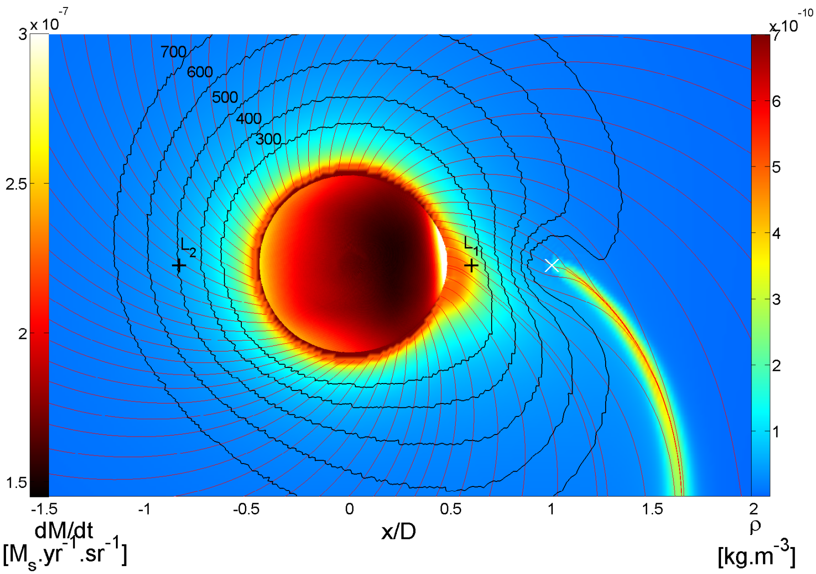
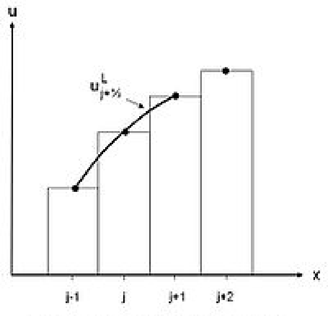


PPM advection scheme

This code is based on the PPM advection scheme. PPM stands for piecewise parabolic method presented by Collela & Woodward in 1984.



Piecewise parabolic reconstruction

- Higher-order extension of Godunov's method of a type used in van Leer's MUSCL algorithm.
- The scheme uses parabolae as the basic interpolation functions.
- $\Rightarrow$ a more accurate representation of smooth spatial gradients as well as a steeper representation of captured discontinuities, particulary contant discontinuities

Conservative Eulerian scheme

$$Q = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \epsilon \end{pmatrix}, \quad E = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ \rho u w \\ (\epsilon + p)\, u \end{pmatrix}, \quad F = \begin{pmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ \rho v w \\ (\epsilon + p)\, v \end{pmatrix}, \quad G = \begin{pmatrix} \rho w \\ \rho u w \\ \rho v w \\ \rho w^2 + p \\ (\epsilon + p)\, w \end{pmatrix}$$

$$Q_t + E_x + F_y + G_z + S = 0$$

- ϵ ... *total energy density*

$$\epsilon = \frac{1}{2}\rho v^2 + \rho e$$

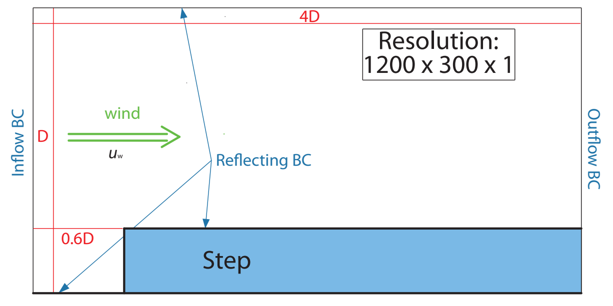
- e ... *specific internal energy*

3 Mach Wind Tunnel with a Step

Initial conditions:

$$\rho_{\text{int}} = 1.4$$
$$p_{\text{int}} = 1.0$$
$$T_{\text{int}} = 273.15$$

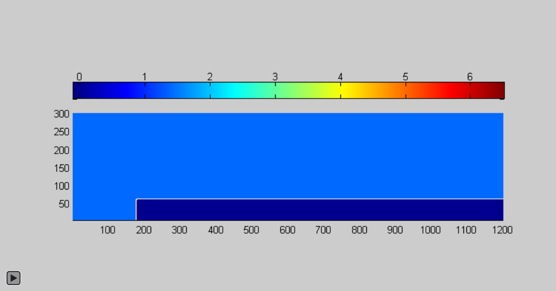
$$u_{\text{int}} = 2.8$$
$$v_{\text{int}} = 0$$

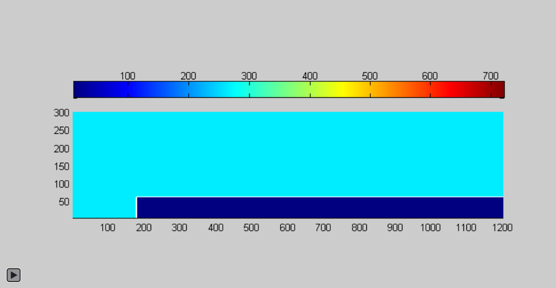


Physical model:

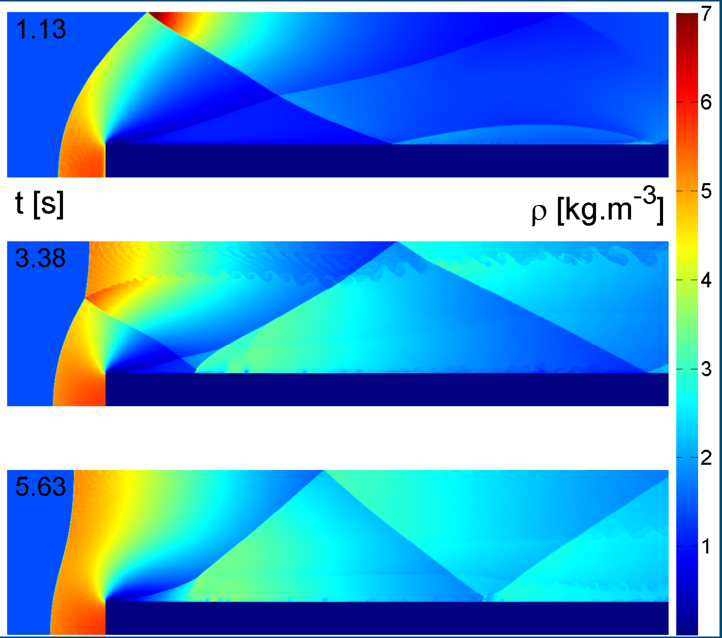
$$C = 1/2$$
$$\gamma = 1.4$$

Animation of ρ and T evolution

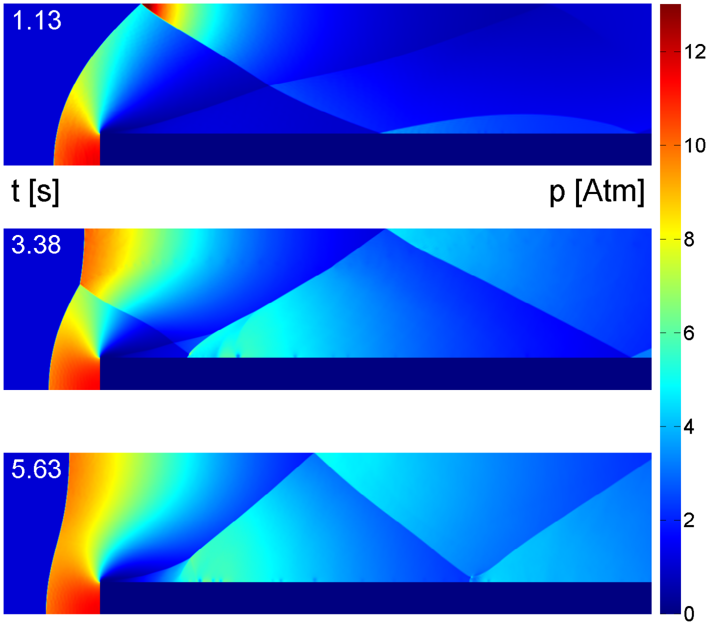




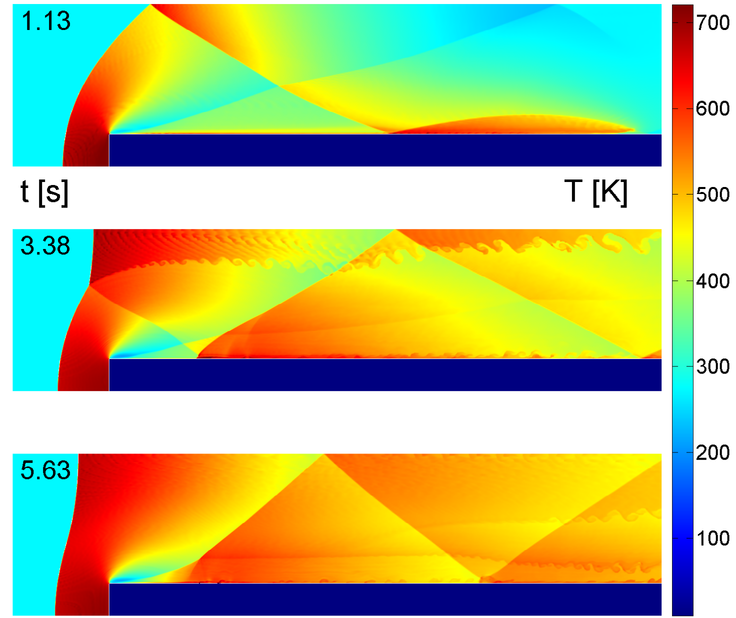
Evolution of ρ



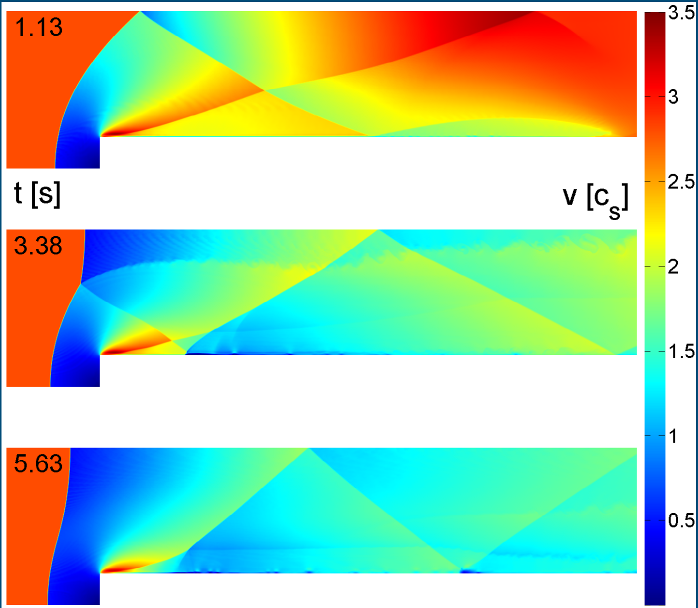
Evolution of p



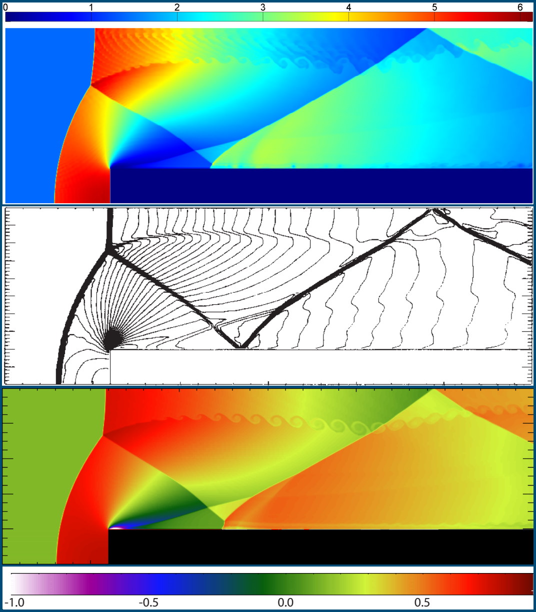
Evolution of T



Evolution of v



Comparison of ρ with the existing models



new hydrocode

- $t = 4.0$
- $\rho_{\text{max}} = 22.48$

C&W (1984)

- $t = 4.21$
- 30 contours
- $0.26 \rightarrow 6.93$

Flash code

- $t = 4.0$
- colorbar [$\log \rho$]

Double Mach Reflection of a Strong Shock

Pre-shock :

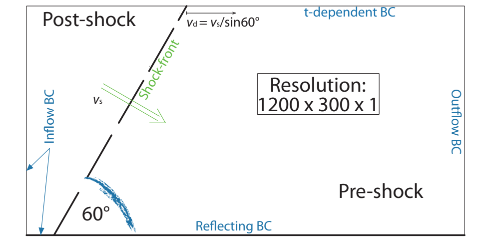
$$\rho_{\mathrm{pre}} = 1.4$$
$$p_{\mathrm{pre}} = 1.0$$
$$T_{\mathrm{pre}} = 273.15$$
$$u_{\mathrm{pre}} = v_{\mathrm{pre}} = 0$$

Shock wave & Grid:

$$v_{\mathrm{s}} = 10.0$$
$$\alpha = 60^\circ$$
$$C = 1/3$$
$$\gamma = 1.4$$

Post-shock:

$$\rho_{\mathrm{post}} = 8.0$$
$$p_{\mathrm{post}} = 116.5$$
$$T_{\mathrm{post}} = 2273.15$$
$$u_{\mathrm{post}} = 8.25 \times \sin 60^\circ$$
$$v_{\mathrm{post}} = 8.25 \times \cos 60^\circ$$



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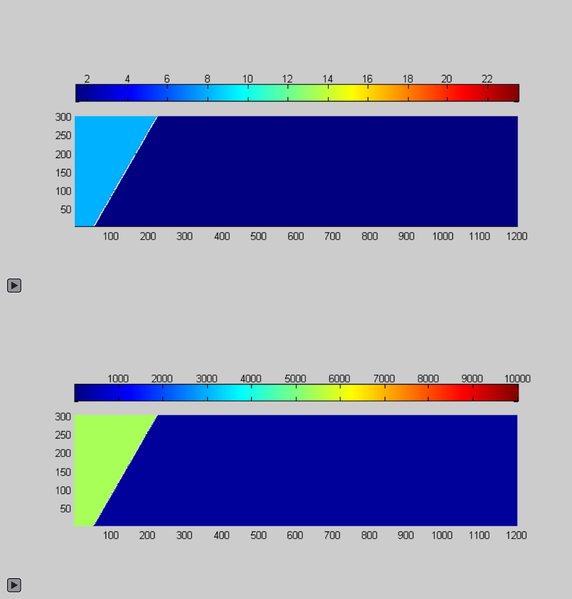
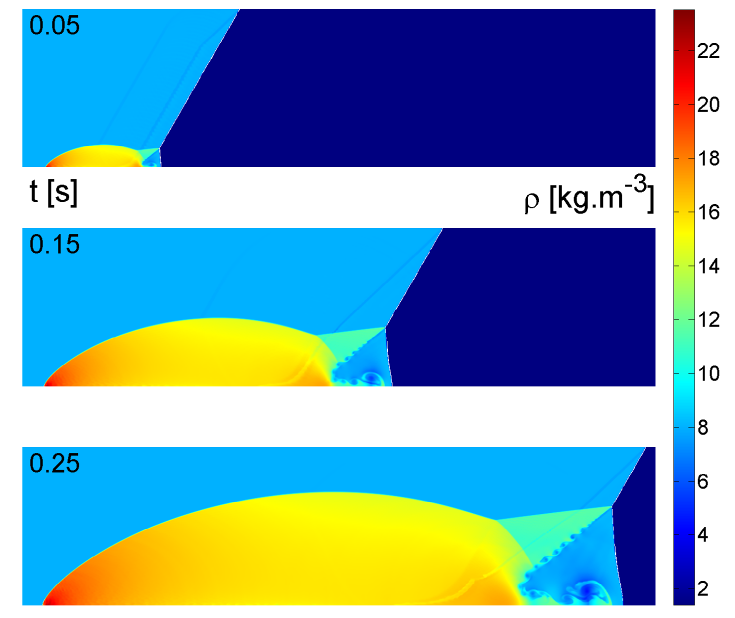
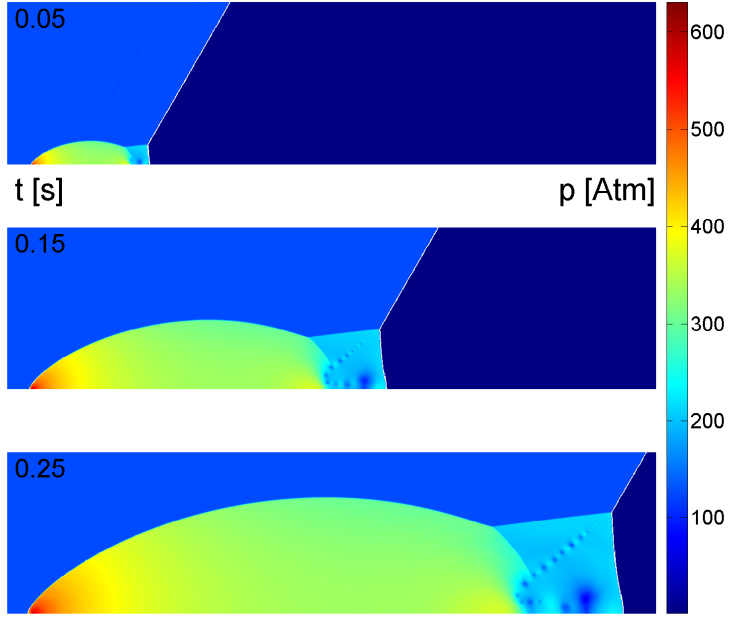


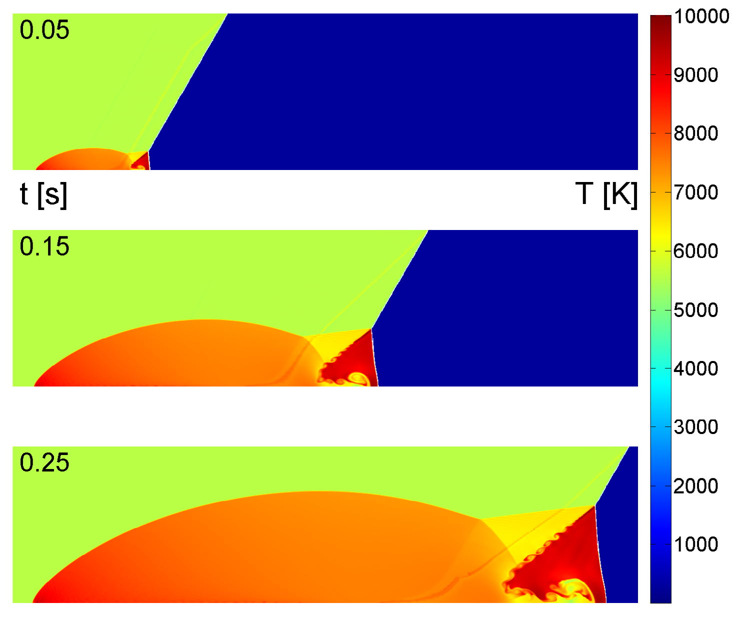
Evolution of ρ



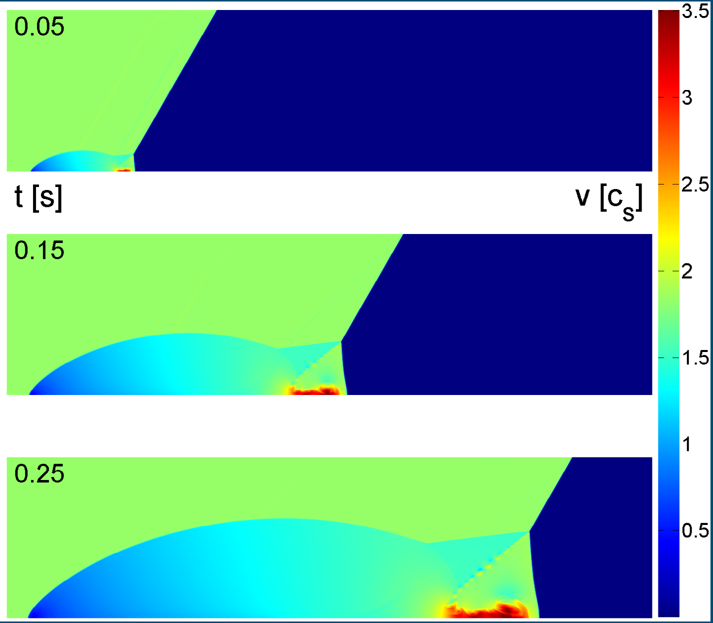
Evolution of p



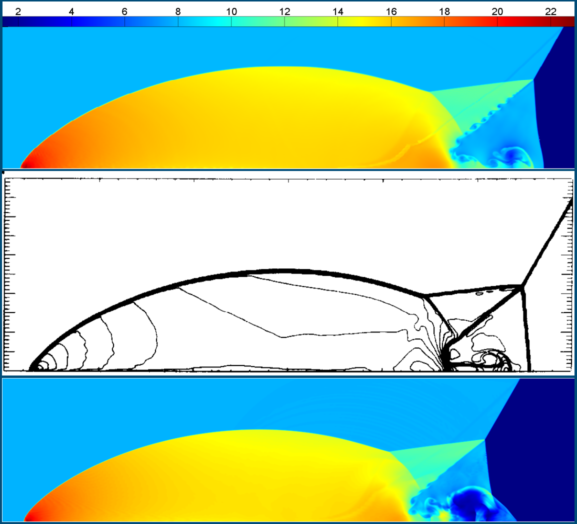
Evolution of T



Evolution of v



Comparison of ρ



new hydrocode

- $t = 0.25$
- $\rho_{\text{max}} = 22.48$

C&W (1984)

- $t = 0.20$
- 30 contours
- $1.73 \rightarrow 20.92$

Athena code

- $t = 0.25$
- $\rho_{\text{max}} = 22.74$

Future development

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  - Stellar wind in the vicinity of X-ray binary
  - modified Bondi-Hoyle-Lyttelton accretion
- Improvement of the physical model
  - Photo-ionization of the circumstellar medium
  - Inclusion of a non-radial component of the CAK mechanism

Thank you for your attention