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The 11th international microconference
Analytic and algebraic methods in physics

October 30 - November 1, 2013, Villa Lanna, Prague

$\rightarrow$ THE BOOK OF ABSTRACTS $\leftarrow$

(the version of October 28, 2013)

(alphabetical ordering)

Philipp Ambichl

Breaking of PT-symmetry in bounded and unbounded scattering systems

While bounded PT-symmetric systems exhibit transitions from real values to complex conjugate pairs in the eigenspectra of the respective Hamiltonians [1], unbounded scattering systems can undergo similar transitions in the eigenvalues of the associated scattering matrix. These S-matrix eigenvalues make a transition from being on the unit circle to being away from it [2]. In my talk, I will demonstrate that a close relationship exists between the PT-breaking points in the unbounded system and the symmetry breaking in corresponding bounded systems [3]. Based on this relation I will show that the PT-transitions in the scattering matrix are, under very general conditions, entirely insensitive to a variable coupling strength between the bounded region and the unbounded asymptotic region. Testing this insensitivity is within reach of current experimental setups.

[1] C. M. Bender and S. Boettcher, Physical Review Letters 80, 5243 (1998).

[2] Y. D. Chong, L. Ge, and A. D. Stone, Physical Review Letters 106, 093902 (2011);

H. Schomerus, Physical Review Letters 104, 233601 (2010).

[3] P. Ambichl, K.G. Makris, L. Ge, Y. Chong, A.D. Stone, and S. Rotter, arXiv:1307.0149 (2013).

Fabio Bagarello

Mathematical aspects of non self-adjoint Hamiltonians

We discuss how loosing self-adjointness of the Hamiltonian could give rise to biorthogonal sets which are not bases but still produce some "weak" resolution of the identity. This is related to the fact that the metric operator, as well as its inverse, is unbounded.

Note added:
the speaker did not exclude the possibility of his submission of the written form of the talk to the special issue of Acta Polytechnica

Bijan Bagchi

Lienard oscillator and MDM/NHQM correspondence

We consider a nonlinear Lienard oscillator and examine its analytical structure to show it is a bi-Hamiltonian system depending upon the choice of the coupling parameters. Of the two, one has been recently studied in the context of a quantized momentum-dependent mass (MDM) system. We demonstrate that the other also reflects a similar feature in the mass function and depicts an isotonic type. Further, we point out interesting connections to a generalized non-Hermitian oscillator system and the relevance of the MDM in it.

Note added:

the speaker intends to submit the written form of the talk to Acta Polytechnica (the special issue is expected to appear, in print, in June, 2014)

Holger Cartarius

Stability of Bose-Einstein condensates in PT-symmetric traps close to branch points

It has been shown that the nonlinear Gross-Pitaevskii equation for Bose-Einstein condensates in a $\mathcal{PT}$-symmetric trap with gain and loss effects exhibits both $\mathcal{PT}$-symmetric and $\mathcal{PT}$-broken stationary solutions with similar relations as in linear quantum mechanics. A crucial difference appears for the emergence of the $\mathcal{PT}$-broken states. If the energy eigenvalues are plotted as a function of the strength of the gain and loss effects, i.e. the imaginary part of the external trap potential, the $\mathcal{PT}$-broken states of the nonlinear system bifurcate from *one* of the $\mathcal{PT}$-symmetric solutions in a third-order exceptional point. The $\mathcal{PT}$-symmetric states vanish in a second-order branch point, at which no $\mathcal{PT}$-broken states are born. In an appropriate analytic extension these states can be continued beyond the branch point.

At both branch points a change in the stability of the wave functions against small perturbations is expected. However, precise calculations reveal that this change in stability does not occur exactly at the branch points but somewhere in their vicinity. This "gap" between the bifurcation and the stability change can directly be related to the norm-dependence of the nonlinearity $\sim g|\psi|^2$ in the Gross-Pitaevskii equation. We demonstrate this relation for a Bose-Einstein condensate in a $\mathcal{PT}$-symmetric double-well trap and investigate its influence on the condensate's dynamics.

Note added:
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Francisco M. Fernández

On the eigenvalues of some nonhermitian oscillators

We consider a class of one-dimensional nonhermitian oscillators and discuss the relationship between the real eigenvalues of PT-symmetric oscillators and the resonances obtained by different authors. We also show the relationship between the strong-coupling expansions for the eigenvalues of those oscillators. Comparison of the results of the complex rotation and the Riccati-Padé methods reveals that the optimal rotation angle converts the oscillator into either a PT-symmetric or an Hermitian one. In addition to the real positive eigenvalues the PT-symmetric oscillators exhibit real positive resonances under different boundary conditions. They can be calculated by means of the straightforward diagonalization method. The Riccati-Padé method yields not only the resonances of the nonhermitian oscillators but also the eigenvalues of the PT-symmetric ones.

Work done with Javier Garcia

address:

INIFTA (UNLP, CCT La Plata-CONICET), División Química Teórica, Blvd. 113 S/N, Sucursal 4, Casilla de Correo 16, 1900 La Plata, Argentina

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Vladimir P. Gerdt:

Singular field theories, Lagrangean constraints and Thomas decomposition

In the talk we consider a field theory whose Lagrangian density L is a polynomial in the dependent (field) variables and their first-order derivatives in the spatial variables and in the temporal variable. The theory is singular or constrained if the Hessian matrix H is singular, and there are primary Lagrangean constraints [1]. The consistency of time evolution of the primary constraints with the Euler-Lagrange equations may lead to new constraints. Then, their consistency analysis may lead to further constraints, etc. We show that the differential Thomas decomposition [2] of the Euler-Lagrange equations is a universal algorithmic tool for computation of the full set of Lagrangean constraints.

References

[1] A.Wipf. Hamiltons Formalism for Systems with Constraints. In: "Canonical Gravity: From Classical to Quantum" (J. Ehlers and H. Friedrich, eds.), Springer-Verlag, New York, 1994, pp. 22-58. arXiv:hep-th/9312078

[2] T.Bächler, V.P.Gerdt, M.Lange-Hegermann and D.Robertz. Algorithmic Thomas Decomposition of Algebraic and Differential Systems. Journal of Symbolic Computation, 47 (2012), 1233.1266. arXiv:math.AC/1108.0817

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Martin Himmel

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Daniel Hook

Poles and Turning-points in complexified classical mechanics

Classical particle trajectories in the complex plane are studied in the context of Hamiltonians that have both poles and turning-points. The effects of the two features are compared and contracted.

Robin Hudson

Why quantum stochastic double products matter, in mathematics and physics

Product integrals are often dismissed as merely a notation for solutions of differential equations. But they originated with Volterra in the 19th century as multiplicative analogues of the additive Riemann integral and even when reduced to differential equations they suggest novel methods of solution especially in the quantum context. In this talk I will combine three kinds of generalization. The resulting objects have applications to deformation quantization and suggestive connections with Bernoulli and Euler numbers and classical mathematics.

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Vít Jakubský

Carbon nanotubes in an transverse magnetic field: exactly solvable model

Single-wall carbon nanotubes are considered in the presence of an external magnetic field with inhomogeneous transverse component. The continuum model is employed where the dynamics of the charge carriers is governed by the Dirac-Weyl equation. It is shown that a small fluctuation of the transverse field around a constant value represented by a finite-gap vector potential provides exact solutions of the stationary equation. An example is elaborated in detail.

Jiří Janda

On Bilinear Forms from the Point of View of Generalized Effect Algebras

We study positive bilinear forms on a Hilbert space which are not necessarily bounded nor induced by some positive operator. We show when different families of bilinear forms can be described as a generalized effect algebra. In addition, we present families which are or are not monotone downwards (Dedekind upwards) σ-complete generalized effect algebras.

Work done with Anatolij Dvurečenskij

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Hugh F. Jones

The Floquet Method for PT-symmetric Periodic Potentials

The abstract is under preparation.

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Boubakeur Khantoul:

Heisenbergs uncertainty relations and noncommutative space-time.

(to be presented in poster session)

We investigate four different types of representations of deformed canonical variables leading to generalized versions of Heisenbergs uncertainty relations resulting from noncommutative spacetime structures.We demonstrate explicitly how the representations are related to each other and study three characteristically different solvable models on these spaces, the harmonic oscillator, the manifestly non-Hermitian Swanson model and an intrinsically noncommutative model with PoeschlTeller type potential. We provide an analytical expression for the metric in terms of quantities specific to the generic solution procedure and show that when it is appropriately implemented expectation values are independent of the particular representation. A recently proposed inequivalent representation resulting from Jordan twists is shown to lead to unphysical models. We suggest an anti-PT -symmetric modification to overcome this shortcoming.

Frieder Kleefeld

Selected Foundations of non-Hermitian Quantum Theory

Several corner stones of the gradually developing formalism of non-Hermitian Quantum Theory are presented according to our understanding. In particular are addressed retarded and respective advanced wave-equations, resulting scalar products with probabilistic interpretation, completeness relations for the spectral, spatial and the Fourier-representation, non-Hermitian Quantum Theory in matrix representation, the resulting concept of the S-matrix, density matrix and non-Hermitian Quantum-Field Theory. The presented formalism is underlined by suitable examples.

Peter G. L. Leach:

Dirac and Hamilton.

Dirac devised his theory of Quantum Mechanics and recognised that his operators resembled the canonical coordinates of Hamiltonian Mechanics. This gave the latter a new lease of life. We look at what happens to Dirac's Quantum Mechanics if one starts from Hamiltonian Mechanics.

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Ali Mostafazadeh:

A Dynamical Theory of One-Dimensional Scattering

A two-component formulation of one-dimensional scattering theory is used to develop a dynamical scattering theory where the transfer matrix and consequently the transmission and reflection amplitudes are given as solutions of certain dynamical equations. We use this approach to outline an inverse-scattering method for constructing finite-range complex potentials with desirable scattering properties. In particular, we construct optical potentials displaying threshold lasing, anti-lasing, and unidirectional invisibility. Our results reveal a unique role played by a certain non-Hermitian matrix Hamiltonian in the standard one-dimensional scattering theory. For a real scattering potential this Hamiltonian is pseudo-Hermitian and has exceptional points at the classical turning points.

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Anatoly G. Nikitin:

Laplace-Runge-Lenz vector with spin

Superintegrable quantum mechanical systems with arbitrary spin are presented. These systems admit generalized Laplace-Runge-Lenz vector and posses a dynamical symmetry with respect to group SO(4) for bound states. In the main 2d and 3d systems are discussed. However, a special class of arbitrary dimension systems is presented also.

Radek Novák:

Bound states in $\mathcal{PT}-$symmetric layers

We consider the Laplacian in a tubular neighborhood of a hyperplane subject to non-Hermitean $\mathcal{PT}-$symmetric Robin-type boundary conditions. They bring the non-self-adjointness into the problem as the probability current does not vanish on either component of the boundary and the layer therefore behaves as an open system. We analyze the influence of the perturbation in the boundary conditions on the threshold of the essential spectrum using the Birman-Schwinger principle. Our aim is to derive a sufficient condition for existence, uniqueness and reality of discrete eigenvalues. We show that discrete spectrum exists when the perturbation acts in the mean against the unperturbed boundary conditions and we are able to obtain the first term in its asymptotic expansion in the weak coupling regime.

Satoshi Ohya

Recurrence Relations for CFT Correlators: A Potential Algebra Approach

In this talk I would like to discuss a new algebraic approach to momentum-space correlators in conformal field theory by using the AdS/CFT correspondence. As an illustration we present a Lie-algebraic method to compute frequency-space two-point functions for charged scalar operators of one-dimensional conformal field theory (CFT_1) dual to 2-dimensional anti-de Sitter (AdS_2) black hole with constant background electric field. Our method is much inspired by the potential algebra approach to S-matrix in quantum mechanics with dynamical symmetry. Making use of unitary representations of the Lie algebra of the isometry group of Euclidean AdS_2 black hole, we derive novel recurrence relations for Euclidean CFT_1 two-point functions, which are exactly solvable and completely determine the frequency- and charge-dependences of two-point functions. Wick-rotating back to Lorentzian signature, we obtain retarded and advanced CFT_1 two-point functions that are consistent with the known results.

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Jan Paseka

On Realization of Partially Ordered Abelian Groups

The lecture is devoted to algebraic structures connected with the logic of quantum mechanics. Since every (generalized) effect algebra with an order determining set of (generalized) states can be represented by means of an abelian partially ordered group and events in quantum mechanics can be described by positive operators in a suitable Hilbert space, we are focused in a representation of partially ordered abelian groups by means of sets of suitable linear operators.

We show that there is a set of points separating R-maps on a given partially ordered abelian group G if and only if there is an injective non-trivial homomorphism of G to the symmetric operators on a dense set in a complex Hilbert space H which is equivalent to an existence of an injective non-trivial homomorphism of G into a certain power of R. A similar characterization is derived for an order determining set of R-maps and symmetric operators on a dense set in a complex Hilbert space H. We also characterize effect algebras with an order determining set of states as interval operator effect algebras in groups of self-adjoint bounded linear operators.

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Peter Prešnajder

Laplace-Runge-Lenz Vector in Noncommutative Quantum Mechanics

We shall examine the "hidden" dynamical symmetry connected with the conservation of Laplace-Runge-Lenz (LRL) vector in the hydrogen problem formulated in quantum mechanics in noncommutative quantum mechanics (NCQM). We introduce basic facts about NCQM and finally we construct analog of the LRL vector in the new scheme.

Silvia Pulmannová

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Note added:

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Ingrid Rotter

Exceptional points and nonlinear Schrödinger equation

Open quantum systems are described by a Schrödinger equation with source term which contains the coupling of the different states via the environment. Due to the biorthogonality of the eigenfunctions, the Schrödinger equation with source term is nonlinear in the neighborhood of an exceptional point. In the talk, different examples for nonlinear effects caused by 2, 3 and more crossing states, are considered. Of special interest is the influence of a third state near by an exceptional point onto the dynamics of an open quantum system.

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Andrei Shafarevich

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Petr Siegl

Basis properties of perturbations of the harmonic oscillator type operators

We analyze perturbations of the harmonic oscillator type operator in a Hilbert space, i.e. of the self-adjoint operator with simple positive eigenvalues μ_k satisfying $\mu_{k+1} - \mu_k \geq \Delta > 0$. Perturbations are considered in the sense of quadratic forms. Under a "local subordination assumption", the eigenvalues of the perturbed operator become eventually simple and the root system forms a Riesz basis. The application of the abstract results to the perturbations of one-dimensional harmonic oscillator by both functional and distributional potentials will be presented.

The talk is based on paper

[1] B. Mityagin, P. Siegl: *Root system of singular perturbations of the harmonic oscillator type operators*, arXiv:1307.6245.

Teoman Turgut:

Bose Einstein condensation on curved manifolds via Bogoliubov theory

We will discuss the free gas on a manifold with positive Ricci tensor via heat kernel techniques. Subsequently, we develop the Bogoliubov approach to a weakly interacting hard core repulsive Bose gas. We find upper bounds on the finite temperature depletion of the condensate. Moreover, we show that the assumption of condensate is not consistent in two dimensions which hints an analog of Hohenberg theorem for manifolds.

[This talk is based on joint work with L. Akant, F. Tapramaz and E. Ertugrul]

Günter Wunner:

A Bose-Einstein condensate with $\mathcal{PT}$ symmetric double-delta function loss and gain in a harmonic trap

We solve the linear and nonlinear Schrödinger equation for a Bose-Einstein condensate in a harmonic trap with $\mathcal{PT}$ symmetric double-delta function loss and gain terms, and try to establish a link to recent analytical results by Mityagin and Siegl on singular perturbations of harmonic oscillator type self-adjoint operators (see talk by P. Siegl).

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