The 9th international microconference

Analytic and algebraic methods in physics

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THE BOOK OF ABSTRACTS (the version of December 5, 2011)

(alphabetical ordering)

Fabio Bagarello

Linear and non-linear pseudo-bosons

After a short introduction on linear pseudo-bosons we will consider a non-linear extension of these "particles" and we will discuss the relations between them and the cryptohermiticity in the sense of Znojil.

Dorje Brody

Information geometry of density matrices and state estimation

Given a pure state vector $|x>$ and a density matrix ρ, the function $p(x|\rho) =< x|\rho|x >$ defines a probability density on the space of pure states parameterised by density matrices. The associated Fisher-Rao information measure is used to define a unitary invariant Riemannian metric on the space of density matrices. An alternative derivation of the metric, based on square-root density matrices and trace norms, is provided. This is applied to the problem of quantum-state estimation. In the simplest case of unitary parameter estimation, new higher-order corrections to the uncertainty relations, applicable to general mixed states, are derived.

Giuseppe Luca Celardo

Superradiance transition in quantum transport

Using an energy-independent non-Hermitian Hamiltonian approach to open systems, we fully describe transport through a sequence of potential barriers as external barriers are varied. Analyzing the complex eigenvalues of the non-Hermitian Hamiltonian model, a transition to a superradiant regime is shown to occur. Transport properties undergo a strong change at the superradiance transition, specifically a drastic change in the structure of resonances is demonstrated. This opens exciting research perspectives: coherent quantum regime is very important in nanoscale applications and it has important consequences in basic energy problems: evidence of coherent energy transfer has been recently found in photosynthetic light-harvesting systems.

Jaroslav Dittrich

Integer topological charges for finite energy fields in the O(3) sigma-model

In the (2+1)-dimensional classical O(3) σ-model, all finite energy fields have integer topological charges regardless of their asymptotic behavior at infinity. Topological charge is conserved for the fields with finite Euclidean action, without further assumptions on the field equations or asymptotics. The fields with continuous first derivatives as well as fields in Sobolev-like spaces analogical to $W^{1,2}$ are considered.

Eva-Maria Graefe:

Quantum signatures of three coalescing eigenfunctions

Parameter dependent non-Hermitian quantum systems typically not only possess eigenvalue degeneracies, but also degeneracies of the corresponding eigenfunctions at exceptional points. While the effect of two coalescing eigenfunctions on cyclic parameter variation is well investigated, little attention has hitherto been paid to the effect of more than two coalescing eigenfunctions. Here a characterisation of behaviours of symmetric Hamiltonians with three coalescing eigenfunctions is presented, using perturbation theory for non-Hermitian operators. Two main types of parameter perturbations need to be distinguished, which lead to characteristic eigenvalue and eigenvector patterns under cyclic variation.

Vít Jakubský

Twisting of carbon nanotubes via supersymmetry

We construct exactly solvable models of twisted carbon nanotubes via supersymmetry, by applying the matrix Darboux transformation. We derive the Green's function for these systems and compute the local density of states (LDOS). Explicit examples of twisted carbon nanotubes are produced, where the back-scattering is suppressed and bound states are present.

Jiří Janda

Weakly ordered partial groups as a generalization of effect algebras for not only positive linear operators

The generalized effect algebra was presented as an algebraic description of the set of all positive linear operators (unbounded operators included) with the usual sum of operators. The structure of the set of not only positive operators can be described by the notion of weakly ordered partial commutative group. We are investigating a subset of all self-adjoint linear operators restricting the usual sum to special pairs of linear operators. This yields new examples of weakly ordered partial commutative subgroups.

Petr Jizba

The emergence of Special and Doubly Special Relativity

In papers [1,2] we introduced a method for obtaining the exact Feynman propagator of a relativistic particle (for both Klein–Gordon and Dirac case) from a superstatistical average over non-relativistic single-particle paths. In this talk I show that this method could offer new insights into the currently much debated issue of emergent relativity. In particular, I will demonstrate that a Brownian motion on a short scale originates a relativistic motion on scales larger than particle's Compton wavelength. Viewed in this way, special relativity is not a primitive concept, but rather it statistically emerges when a coarse graining average over distances of order, or longer than the Compton wavelength is taken. I will also discuss the modifications necessary to accommodate in our scheme the doubly special relativistic dynamics. In this way, an unsuspected, common statistical origin of the two frameworks is brought to light. Salient issues such as generalized canonical commutation relations, connection with Feynman chessboard model, and Hausforff dimensions of corresponding path-integral trajectories will be also discussed. Further details can be found in Ref.[3].

[1] P.J. and H.Kleiert, Phys.Rev.D 82,085016 (2010)

[2] P.J. and H.Kleiert, Phys.Rev.E 78,031122 (2008) [3] P.J. and F.Scardigli, arXiv:1105.3930

Martin Kalina

Atomic lattices equipped with several effect-algebraic operations

Recently, Z. Riečanová [1] published an open problem: 'Assume that $(E, +, 0, 1)$ is an Archimedean atomic lattice effect algebra such that an effect algebraic operation onto MC(E) (the MacNeille completion of E) exists. An unanswered question is whether there exists also such operation on MC(E) that extends the operation $+$.' In our talk we will deal with this problem.

[1] Z. Riečanová: Lattice effect algebras densely embeddable into complete ones. Kybernetika 47 (2011), 100–109.

Maurice Kibler:

Coherent states for a generalized Weyl-Heisenberg algebra

It is the aim of this talk to show how to construct à la Perelomov and à la Barut-Girardello coherent states for a polynomial Weyl-Heisenberg algebra. This algebra depends on r parameters. For some special values of the parameter corresponding to r = 1, the algebra covers the cases of the su(1,1) algebra, the su(2) algebra and the ordinary Weyl-Heisenberg or oscillator algebra. For r arbitrary, the generalized Weyl-Heisenberg algebra admits finite or infinite-dimensional representations depending on the values of the r parameters. Coherent states of the Perelomov type are derived in finite and infinite dimensions through a Fock-Bargmann approach based on the use of complex variables. The same approach is applied for deriving coherent states of the Barut-Girardello type in infinite dimension. In contrast, the construction of à la Barut-Girardello coherent states in finite dimension can be achieved solely at the price to replace complex variables by Grassmann variables.

References

M. Daoud and M.R. Kibler, J. Phys. A: Math. Theor. 43 (2010) 115303

N.M. Atakishiyev, M.R. Kibler and K.B. Wolf, Symmetry 2 (2010) 1461

M. Daoud and M.R. Kibler, J. Math. Phys. 52 (2011) 082101

M. Daoud and M.R. Kibler, arXiv:1110.4799v1[quant-ph]

Sergii Kuzhel:

On scattering theory for PT -symmetric operators

Nowadays, the scattering theory problems for PT -symmetric operators attract a lot of interests due to intensive development of Pseudo Hermitian Quantum Mechanics. However, the relevant mathematically study of the subject is not yet developed well. The present talk contains a contribution to this field, where the scattering of PT-symmetric operators is investigated with the use of an operator-theoretical interpretation of the Lax-Phillips scattering theory [1] developed in [2, 3].

References

[1] P. Lax and R. Phillips, *Scattering Theory*, Second edition, Pure and Applied Mathematics, 26 Academic Press, New York, 1989.

[2] S. Kuzhel, *On the inverse problem in the Lax-Phillips scattering theory method for a class of operator-differential equations*, St. Petersburg Math. J. 13 (2002), 41-56.

[3] S. Kuzhel and Yu. Moskalyova, *The Lax - Phillips scattering approach and singular perturbations of Schroedinger operator homogeneous with respect to scaling transformations*, J. Math. Kyoto Univ. 45 (2005), 265286.

M. Howard Lee:

Dynamics of a harmonic oscillator chain of finite length and the thermodynamic limit.

Dynamical properties such as the autocorrelation function in many body systems whether at the ground state or in the classical domain may be calculated by the techniques of the recurrence relations method. For this talk we shall consider a system of a classical harmonic oscillator chain of finite length in 1d and calculate the velocity autocorrelation function analytically. In the Laplace transformed space, this function is expressed in a finite continued fraction in an explicit form, which translates into a periodic solution in time space. Such a form would represent a special example in dynamics of a mesoscopic system. But if one takes the thermodynamic limit, the continued fraction becomes an infinite one, but summable. As a result, one can obtain the dispersion relation and the density of frequencies or the spectrum. What is perhaps most remarkable is that the spectrum has an invariant measure isomorphic to what one finds in a standard 1d chaotic map. The chaotic behavior emerges only at the thermodynamic limit, thus absent at a mesoscopic scale.

Géza Lévai:

Solvable $\mathcal{PT}$-symmetric potentials with more flexible energy spectra

One of the most characteristic feature of $\mathcal{PT}$-symmetric potentials is their energy spectrum, which can be partly or fully real or complex. Complex conjugate energy eigenvalues typically appear when the non-Hermitian (i.e. imaginary) potential component reaches a specific limit. This can be interpreted as the spontaneous breakdown on $\mathcal{PT}$ symmetry. This process can be gradual as for the $V(x) = \mathrm{i}x^3$ and related potentials, or the $\mathcal{PT}$-symmetric square well, but it can also occur suddenly, i.e. when all the energy eigenvalues turn into complex conjugare pairs at the same parameter value. This was found to be the case for practically all the shape-invariant potentials [1], but also for the $\mathcal{PT}$-symmetric version of the generalized Ginocchio potential [2], which belongs to the more general Natanzon potential class. The only known exception was the DKV potential [3], which was also identified as a Natanzon-class potential.

Here we discuss the reason why the energy spectrum of most exactly solvable potentials exhibit such a rigid nature and construct solvable potentials with more general spectral patterns. The potential considered belongs to the Natanzon hypergeometric class and its specific feature is that depending on two of its parameters, it contains pairs of shape-invariant potentials as special limits that can be transformed into each other continuously (Scarf II and Rosen–Morse I; Scarf I and Rosen–Morse II; and generalized Pöschl–Teller, Eckart) as well as the DKV potential [3]. The energy spectrum of this potential is defined implicitly, as is the case for the general Natanzon potential class with the

$$2n + 1 = (f + 1 - aE_n)^{1/2} - (h_0 + 1 - c_0E_n)^{1/2} - (h_1 + 1 - c_1E_n)^{1/2} \ ,$$

prescription, where a, c_0, c_1, f, h_0 and h_1 are the six parameters of the Natanzon class. This equation can be solved explicitly for shape-invariant potentials, because then several parameters are zero. Somewhat more involved, but explicit expression can be obtained for the generalized Ginocchio potential [2] too. However, for the potential considered here

the energy eigenvalues have to be determined solving a a quartic algebraic equation, which obviously leads to a more complicated spectral structure. (In the case of the DKV potential this reduces to solving a cubic algebraic equation.) We analyze the consequences of this result with special attention to the known limiting cases represented by thoroughly studied shape-invariant $\mathcal{PT}$-symmetric potentials.

[1] G. Lévai, M. Znojil: *J. Phys. A* **33** (2000) 7165.

[2] G. Lévai, A. Sinha, P. Roy: *J. Phys. A* **36** (2003) 7611.

[3] M. Znojil, G. Lévai, P. Roy, R. Roychoudhury: *Phys. Lett. A* **290** (2001) 249.

Matthias Liertzer:

Scattering and lasing in coupled systems with gain and loss.

I will present our recent insights into the relation between the eigenstates of a closed PT-system and the scattering states in the corresponding open system. These scattering states can be strongly affected by exceptional points at which the eigenvalues of the non-hermitian system Hamiltonian coalesce. As we demonstrate explicitly, such exceptional points do also affect the above-threshold behavior of a coupled laser with gain and loss, leading to a non-uniform pump-dependence of the emitted laser light intensity. Possible experimental realizations of these effects will be discussed.

Jiří Lipovský:

Resonances on quantum graphs and hedgehog manifolds.

We study resonance asymptotics in the system of a scattering center with semiinfinite leads attached. First we introduce the one-dimensional case – quantum graphs. By an observation by Davies and Pushnitski the coefficient by the leading term of Weyl's asymptotics can be in some cases smaller than expected. We find a general criterion for non-Weyl graphs and explain this behaviour. Furthermore, we study resonances in hedgehog manifolds (two or three dimensional Riemannian manifolds with halflines attached) and their asymptotics. In the case of a hedgehog manifold non-Weyl behaviour probably does not appear due to different dimensionality of the center and leads. We prove it in the case of one halfline and show simple examples.

Anatoly G. Nikitin:

Dual shape invariance and superintegrable models for arbitrary spin

Shape invariant matrix potentials are classified. It is demonstrated that some of such potentials can be generated by two non-equivalent superpotentials. We call this property dual shape invariance.Example of physical models admitting dual shape invariance are discussed, and they are the superintegrable Pronko models for arbitrary spin. We prove that, in addition to the dynamical symmetry w.r.t. algebra 0(1,2) these models are supersymmetric and dually shape invariant, and use these properties to construct their exact solutions.

Jan Paseka

Triple Representation Theorem for homogeneous effect algebras

The aim of our lecture is to prove the Triple Representation Theorem, which was established by Jenča in the setting of complete lattice effect algebras, for a special class of homogeneous effect algebras, namely TRT-effect algebras. This class includes complete lattice effect algebras, sharply dominating Archimedean atomic lattice effect algebras and homogeneous orthocomplete effect algebras.

Silvia Pulmannová:

Spin factors and generalized hermitian algebras

We relate so-called spin factors and generalized Hermitian (GH) algebras, both of which are partially ordered special Jordan algebras. Our main theorem states that positive-definite spin factors of dimension greater than one are mathematically equivalent to generalized Hermitian algebras of rank two.

Zdena Riečanová

The set of positive linear operators densely defined in an infinite-dimensional complex Hilbert space

The set of all positive linear operators (including unbounded operators) with the ususal sum of operators form a generalized effect algebra. Thus the effect algebraic partial order coincides with the usual partial order of positive operators. Moreover, intervals in this set are operator effect algebras. We show algebraic and topological properties of these (generalized) effect algebras (e.g., convexity, compatibility, continuity of operations with respect to uniform topology induced by physical states and the faithful states existing on every such interval).

Ray Rivers

Path Integrals approaches for (Complex) Classical Physics

We consider ways for extending the tactics of Koopman and von Neumann (KvN) for the Hilbert space construction of classical mechanics to a complex extension of phase space. We make extensive use of the path integral realisation of the KvN construction by Gozzi, and examine its relationship to the approach of 't Hooft, particularly as realised by Jizba and coworkers.

Frederik G. Scholtz

Geometrical Representations of Lie Algebras

We show that representations of Lie algebras can be constructed in terms of functions defined over some Riemann manifold and equipped with an appropriate star product. The information on the representation gets encoded in the geometry of the manifold. We also discuss the link to the geometric interpretation of quantum mechanics introduced some time ago by Provost et al and Ashtekar et al.

Günter Wunner:

Investigation of PT symmetry in Bose-Einstein condensates

The observation of PT symmetry in a coupled optical wave guide system that involves a complex refractive index potential has been demonstrated impressively in the recent experiment by Rüter el al. (Nature Physics 6, 192, 2010). This is, however, only an optical analogue of a quantum system, and it would be highly desirable to observe the manifestation of PT symmetry and its resulting properties also in a real, experimentally accessible, quantum system. Following a suggestion by Klaiman et al. (Phys. Rev. Lett. 101, 080402, 2008), we investigate a PT symmetric arrangement of a Bose-Einstein condensate in a double well potential, where in one well cold atoms are injected while in the other particles are extracted from the condensate. We investigate, in particular, the effects of the nonlinearity of the PT symmetric Gross-Pitaevskii equation, describing the condensate, on the PT properties of the condensate. This nonlinearity represents a major difference to the optical analogues. To study these effects we first analyze in detail a simple one-dimensional model system in which the condensate is placed into two PT symmetric δ traps. The analysis serves as a useful guide for understanding the behaviour of Bose-Einstein condensates in realistic PT symmetric double well potentials.