Hájek-style modalities in fuzzy intensional semantics

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Kripke-style fuzzy modal logic with a fuzzy accessibility relation notoriously invalidates the modal axiom K, due to the failure of the rule of contraction in most (t-norm based) fuzzy logics. While the failure of K may be desirable in some fuzzy modal logics (for example epistemic, where K entails logical omniscience), in others it is commonly seen as problematic.

In [3, §8.3], Hájek sketched a hierarchy of fuzzy Kripke-style modal operators $\Box_n, \Diamond_n$ for each $n \in \mathbb{N}$, defined w.r.t. n-times iterated self-intersection of the accessibility relation, i.e., with the following Tarski conditions in a fuzzy Kripke frame (W, R, L):

$$\|\Box_n \varphi\|_w = \bigwedge_{w' \in W} \big(R^n w w' \rightarrow_L \|\varphi\|_{w'}\big), \qquad \|\Diamond_n \varphi\|_w = \bigvee_{w' \in W} \big(R^n w w' \mathbin{\&_L} \|\varphi\|_{w'}\big),$$

where L is a BL-algebra, $R\colon W^2 \to L$, and $R^n w w' = Rww' \mathbin{\&} \ldots \mathbin{\&} Rww'$ (n times). Rather than to failure, this definition leads to a contraction-sensitive variant of K, namely $\Box_n(\varphi \to \psi) \to (\Box_m \varphi \to \Box_{n+m}\psi)$. Some other modal axioms that fail in simple Kripke-style fuzzy modal logics receive a multiplicity-sensitive variants, too.

In the talk we will elaborate Hájek's sketched idea in the systematic framework of fuzzy intensional semantics, developed by the present authors (full paper in progress). The formal semantics consists in a suitable translation of modal formulae into Russell-style higher-order fuzzy logic (a variant of which was introduced in [1]). As a case study demonstrating the applicability of the formal framework, we will slightly extend Hájek's results on $\Box_n, \Diamond_n$ in several directions and discuss the significance of his approach to fuzzy modalities. Furthermore, we will discuss the envisaged applications of the apparatus in further areas of intensional fuzzy logic (incl. probabilistic, epistemic, counterfactual [2], or non-monotonic reasoning in both classical and fuzzy settings).

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