UNIVERSITAT DE BARCELONA

Study of Strongly Interacting Matter

HadronPhysics3

Institut de Ciències del Cosmos

Light nuclei and hypernuclei from Lattice QCD (A=2,3,4)

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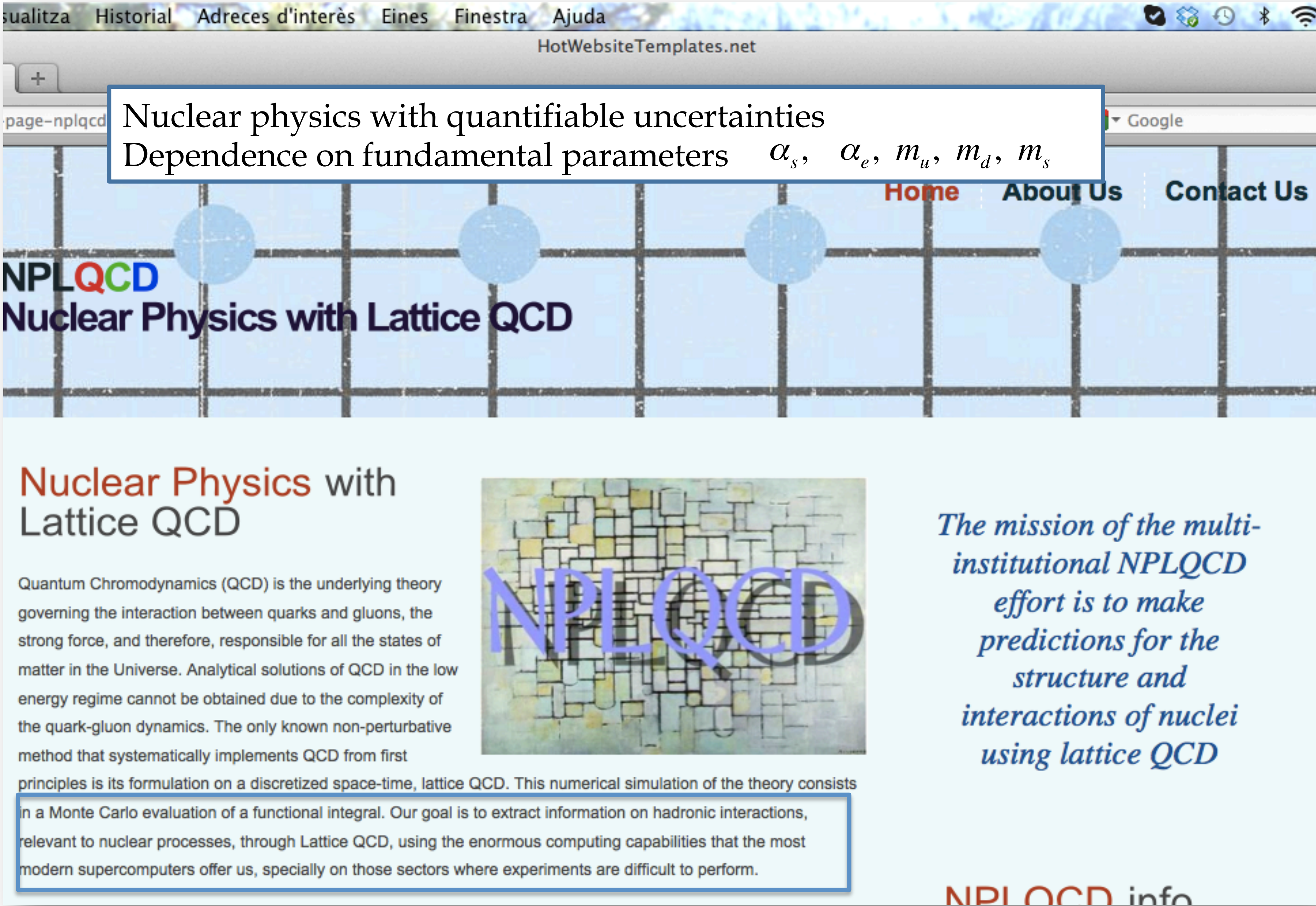
for the NPLQCD Collaboration

SPHERE Meeting 2014

September 9 - 11, 2014 in Prague, Czech Republic

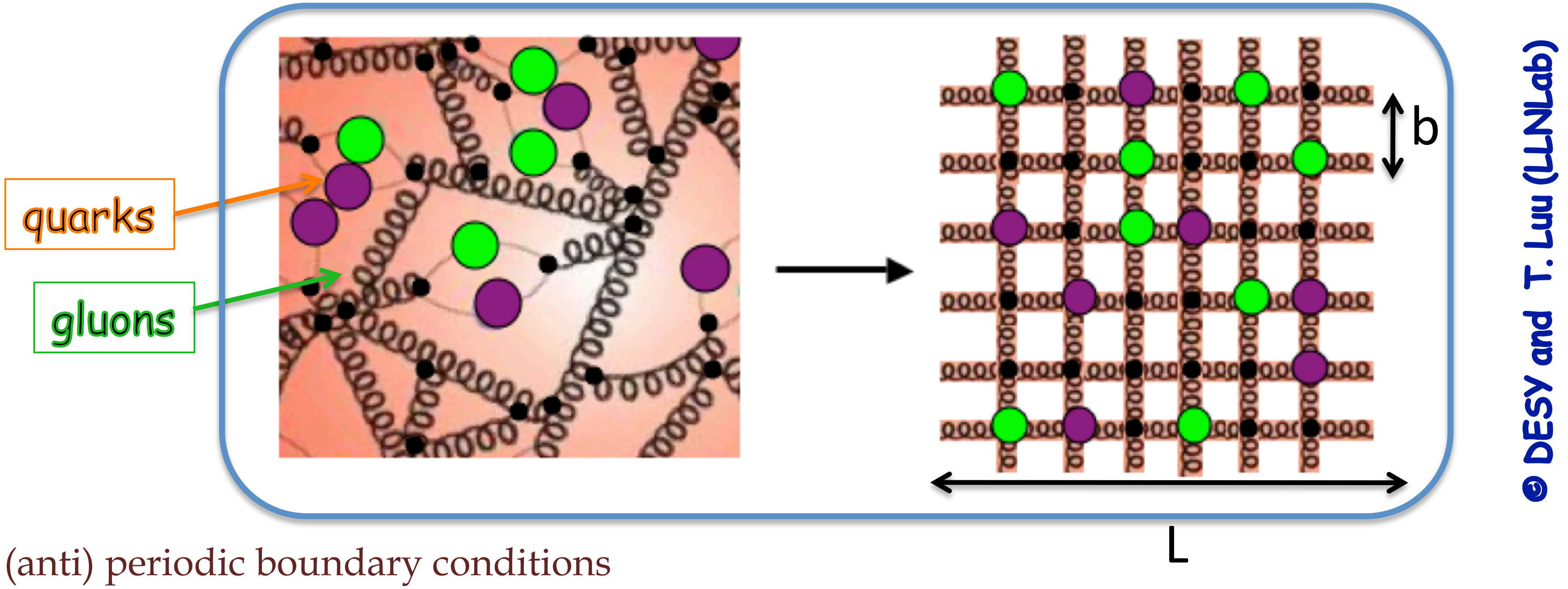
Nuclear physics with quantifiable uncertainties

Dependence on fundamental parameters $\alpha_s, \quad \alpha_e, \ m_u, \ m_d, \ m_s$



Lattice Quantum Chromo Dynamics

space-time lattice
$N_s \times N_s \times N_s \times N_t$



(anti) periodic boundary conditions

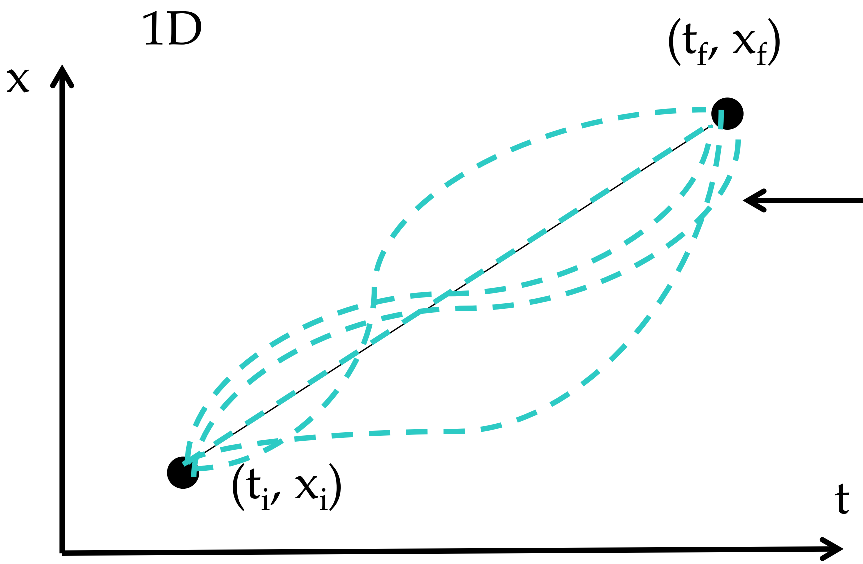
For numerical calculations in QCD, the theory is formulated on a (Euclidean) space-time lattice, as depicted in the right cartoon.

$$L >> \text{ relevant scales } >> b \quad \left(\frac{1}{L} << m_\pi << \Lambda_\chi << \frac{1}{b} \right)$$

$$\text{Cost} \approx \alpha \left[\frac{1}{m_q} \right] [V]^a \left[\frac{1}{\mathrm{b}} \right]^\gamma$$

finite volume L, discretization (finite spacing) b, value of the light quark masses

LQCD is a non-perturbative implementation of Field Theory, which uses the Feynman path-integral approach to evaluate transition matrix elements



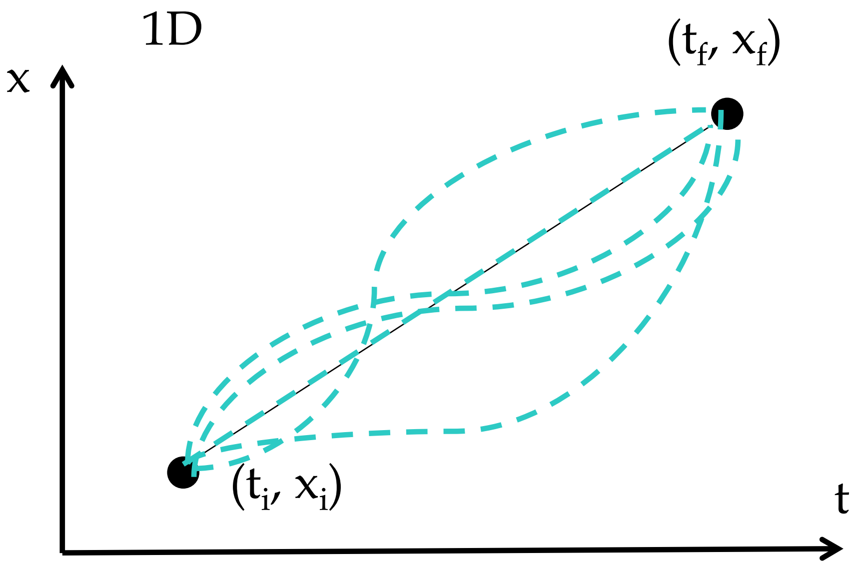
the classical path corresponds to the path with the minimum action

in quantum mechanics one sums over all possible paths

evolution of a quantum state
(1-D QM)

$$\langle x_f, t_f | x_i, t_i \rangle = \langle x_f | e^{-iH(t_f - t_i)} | x_i \rangle$$

The quantum propagation is expressed as a weighted sum over paths. The weight is a complex phase factor given by the exponential of i times the classical action *S*.



RULE

each path contributes
a phase given by the classical
action

$$A_i \propto \exp\left(i \int_i^f dt\, L(q(t)) \right)$$

PATH INTEGRAL
Feynman, 1948

$$A = \int D(q)\, \exp\left(i \int_i^f dt\, L(q(t)) \right)$$

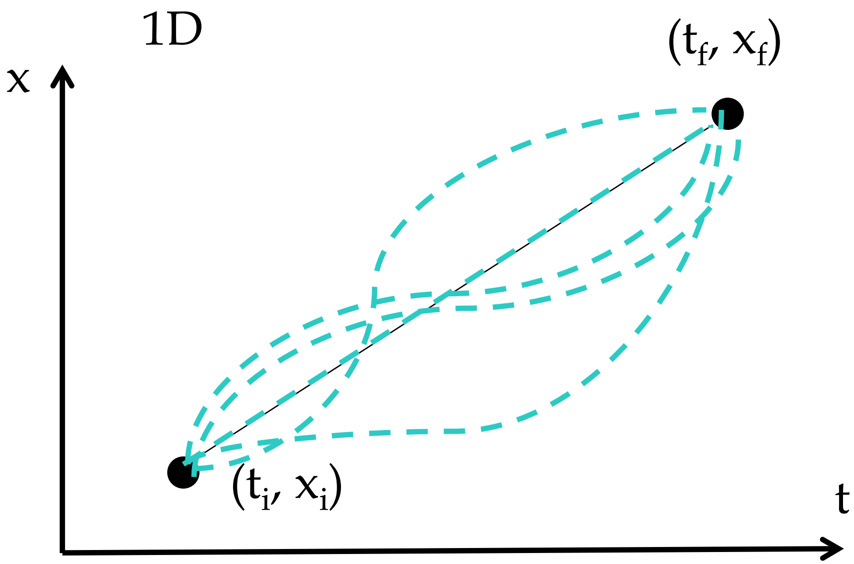
By rotating to Euclidean time:

$$x_0 \equiv t \rightarrow -ix_4 \equiv -i\tau$$

$$p_0 \equiv E \rightarrow ip_4$$

The propagation amplitude is re-expressed in terms of the Euclidean action, S_E

Brief introduction to the lattice formalism



RULE

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By rotating to Euclidean time:

$$x_0 \equiv t \rightarrow -ix_4 \equiv -i\tau$$

$$p_0 \equiv E \rightarrow ip_4$$

$$A = \int D(q)\ e^{\left(-\Delta\tau \sum_k \left[\frac{m}{2} \left(\frac{x_{k+1} - x_k}{\Delta t} \right)^2 + V(x_k) \right] \right)}$$

The propagation amplitude is re-expressed in terms of the Euclidean action, S_E

$$\int_{x(0)=x_i}^{x(t)=x_f} D\left[x_1,x_2,\ldots,x_{N-1}\right] e^{-\Delta\tau\sum_k\left[\frac{m}{2}\left(\frac{x_{k+1}-x_k}{\Delta t}\right)^2+V(x_k)\right]}$$

BASIS OF NUMERICAL SIMULATIONS

The weight of each path is a real positive quantity, looking like a ***Boltzmann factor***

Real
oscillating phase → decaying exponential

Analogy with the partition function of a classical statistical mechanics system

Expectation values:

$$\boxed{\langle O\rangle = \frac{1}{Z}\int DU\, Dq\, D\bar{q} O[q,\bar{q},A]\, e^{-S_E[\bar{q},q,A]}}$$

$$\langle G[\phi]\rangle_T = \frac{\sum_\phi e^{-\frac{E[\phi]}{kT}} G[\phi]}{\sum_\phi e^{-\frac{E[\phi]}{kT}}}$$

~Thermal average over configurations

are taken to satisfy a **Grassmann algebra**

$$\{\psi,\psi\} = \{\psi,\overline{\psi}\} = \{\overline{\psi},\overline{\psi}\} = 0$$

i.e. $\quad \psi\psi = \overline{\psi}\overline{\psi} = 0 \quad$ and $\quad \psi\overline{\psi} = -\overline{\psi}\psi$

QCD partition function in Euclidean space-time

$$Z = \int DA_\mu\, D\overline{\psi}\, D\psi\, \exp(-S_{QCD}) = \int DA_\mu\, D\overline{\psi}\, D\psi\, \exp\left(-\int d^4x \left(\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_f \overline{\psi}_f [D_\mu \gamma_\mu + m] \psi_f\right)\right)$$

matter fields

$$\psi^{iaf}: \quad \begin{array}{ll} i = 1,2,3,4 & \text{Dirac index} \\ a = 1,2,3 & \text{color index} \\ f = 1,2,\ldots n_f & \text{flavor index} \end{array}$$

are taken to satisfy a **Grassmann algebra**

$$\{\psi,\psi\} = \{\psi,\overline{\psi}\} = \{\overline{\psi},\overline{\psi}\} = 0$$

i.e. $\psi\psi = \overline{\psi}\overline{\psi} = 0$ and $\psi\overline{\psi} = -\overline{\psi}\psi$

$$\boxed{\langle O\rangle = \frac{1}{Z}\int DA_\mu\, D\psi\; D\overline{\psi} O[\psi,\overline{\psi},A]\, e^{-S_E[\overline{\psi},\psi,A]}}$$

expectation values

Integration over the quark fields

$$Z = \int DA_\mu\, D\overline{\psi}\, D\psi \exp(-S_{QCD}) = \int DA_\mu\, D\overline{\psi}\, D\psi \exp\left(-\int d^4x\left(\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a + \sum_f \overline{\psi}_f[D_\mu\gamma_\mu + m]\psi_f\right)\right)$$

$$Z = \int DA_\mu \exp(-S) = \int DA_\mu \det\left[M_f(A)\right]\exp(-S_{gluon})$$

$(S = S_{gluon} + S_f)$ very demanding

$S_f = \overline{\psi}M_f(A)\psi$

$$\det\left[M_f(A)\right] \equiv \det\left(\not{D}[A] + m\right) = \prod_{q=1}^{N_f}\det\left(\not{D}[A] + m_q\right)$$

fermion (quark) matrix nonlocal function of U

When computing expectation values of any given operator O, the quark fields in O are re-expressed in terms of quark propagators using Wick's Theorem: write all possible contractions for the fields (removing the dependence of quarks as dynamical fields)

$$\boxed{\langle O \rangle = \frac{1}{Z}\int DA\, D\psi\;\, D\overline{\psi}\, O[\psi,\overline{\psi},A]\, e^{-S_E[\overline{\psi},\psi,A]}}$$

expectation values

Integration over the quark fields

$$\langle \overline{\psi}(x_1)\psi(x_1)\overline{\psi}(x_0)\overline{\psi}(x_0)\rangle = \frac{1}{Z}\int D\psi\, D\overline{\psi}\, DA\, \overline{\psi}(x_1)\psi(x_1)\overline{\psi}(x_0)\psi(x_0)\, e^{-S[\psi,\overline{\psi},A]}$$

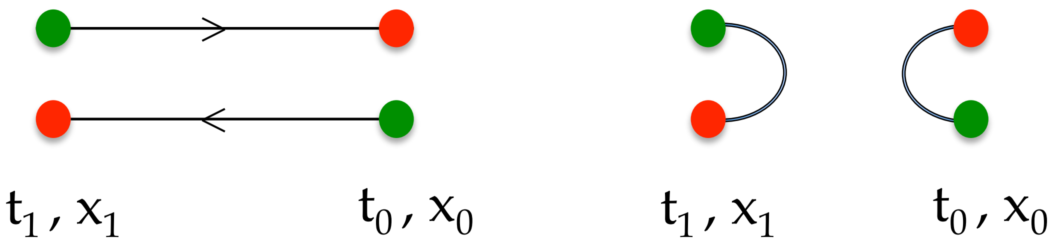
$$\downarrow$$

$$-\left[M^{-1}(A)\right]_{x_1x_0}\left[M^{-1}(A)\right]_{x_0x_1} + \left[M^{-1}(A)\right]_{x_1x_1}\left[M^{-1}(A)\right]_{x_0x_0}$$

more fields imply
more combinations

$\downarrow$

more involved



connected diagrams

disconnected diagrams

Correlation functions

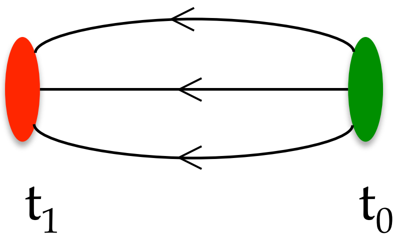
One-hadron correlation function

$$C(\Gamma^{\nu},\vec{p},t)=\boxed{\sum_{\vec{x}_1}} e^{-i\vec{p}\vec{x}_1}\,\Gamma^{\nu}\boxed{\left\langle J(\vec{x}_1,t_1)\bar{J}(\vec{x}_0,t_0)\right\rangle}$$

interpolating operators

spin tensor

momentum projection



for mesons:

scalar	$\bar{\Psi}_i\Psi_j$
pseudoscalar	$\bar{\Psi}_i\gamma_5\Psi_j$
vector	$\bar{\Psi}_i\gamma_k\Psi_j$

Correlation functions

NPLQCD

One-hadron correlation function

$$C(\Gamma^\nu,\vec{p},t)=\boxed{\sum_{\vec{x}_1}}\, e^{-i\vec{p}\vec{x}_1}\,\Gamma^\nu \boxed{\left\langle J(\vec{x}_1,t_1)\bar{J}(\vec{x}_0,t_0)\right\rangle}$$

interpolating operators

spin tensor

momentum projection

for baryons:

$$O_{(\alpha\beta)\gamma}=\left(\psi^T_{i,\alpha}C\gamma_5\psi_{j,\beta}\right)\psi_{k,\gamma}\,\varepsilon^{ijk}$$

color variables (ijk) are antisymmetrized

with $O_{(\beta\alpha)\gamma}=-O_{(\alpha\beta)\gamma}$ $\quad C=\gamma_2\gamma_0$

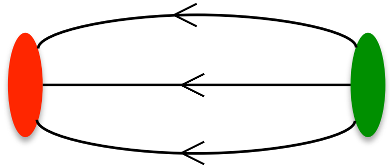
(flavor indices $\alpha\beta\gamma$)

$$J^\pi=\left(\frac{1}{2}\right)^+\quad \begin{array}{l} p_\alpha(\vec{x},t)=\varepsilon^{ijk}u^i_\alpha(\vec{x},t)\left(u^{j^T}_\alpha(\vec{x},t)C\gamma_5 d^k(\vec{x},t)\right) \\ \Lambda_\alpha(\vec{x},t)=\varepsilon^{ijk}s^i_\alpha(\vec{x},t)\left(u^{j^T}_\alpha(\vec{x},t)C\gamma_5 d^k(\vec{x},t)\right)\end{array}\quad \begin{array}{l} \Sigma^+_\alpha(\vec{x},t)=\varepsilon^{ijk}u^i_\alpha(\vec{x},t)\left(u^{j^T}_\alpha(\vec{x},t)C\gamma_5 s^k(\vec{x},t)\right) \\ \Xi^0_\alpha(\vec{x},t)=\varepsilon^{ijk}s^i_\alpha(\vec{x},t)\left(u^{j^T}_\alpha(\vec{x},t)C\gamma_5 s^k(\vec{x},t)\right)\end{array}$$

one can produce positive/negative (parity) energy states by using the projector:

$$\Gamma_\pm=\frac{1}{2}(1\pm\gamma_0)$$

Masses of (colourless) QCD bound states



t $\qquad$ t_0=0

(locate the source at t_0=0)

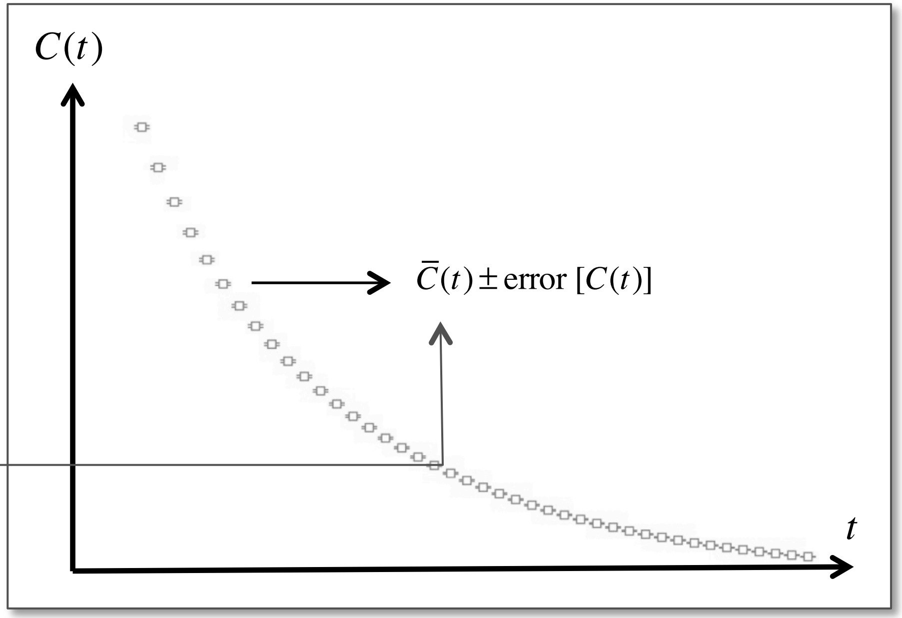
$$C(t) = \langle 0 | \phi(t) \phi^\dagger(0) | 0 \rangle \xrightarrow[\phi(t)=e^{Ht}\phi e^{-Ht}]{} \langle \phi | e^{-Ht} | \phi \rangle$$

Insert a complete set of energy eigenstates:

$$C(t) = \sum_n \langle \phi | e^{-Ht} | n \rangle \langle n | \phi \rangle = \sum \left| \langle \phi | n \rangle \right|^2 e^{-E_n t} \xrightarrow{t \to \infty} Z_0 \, e^{-E_0 t}$$

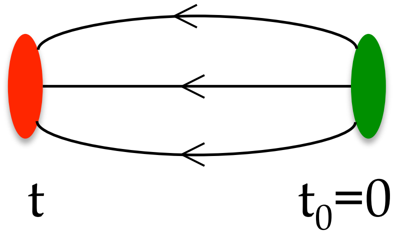
mass

i.e. one can obtain the energy of the state provided we see the large time exponential fall-off of the correlation function (Euclidean time evolution suppresses excited states)



average over N gauge-field configurations $\{U_{x\mu}\}^i$ with $i = 1,2,\ldots N$

Masses of (colourless) QCD bound states



$$C(t) = \langle 0|\phi(t)\phi^\dagger(0)|0\rangle \xrightarrow[\phi(t)=e^{Ht}\phi e^{-Ht}]{} \langle \phi|e^{-Ht}|\phi\rangle$$

Insert a complete set of energy eigenstates:

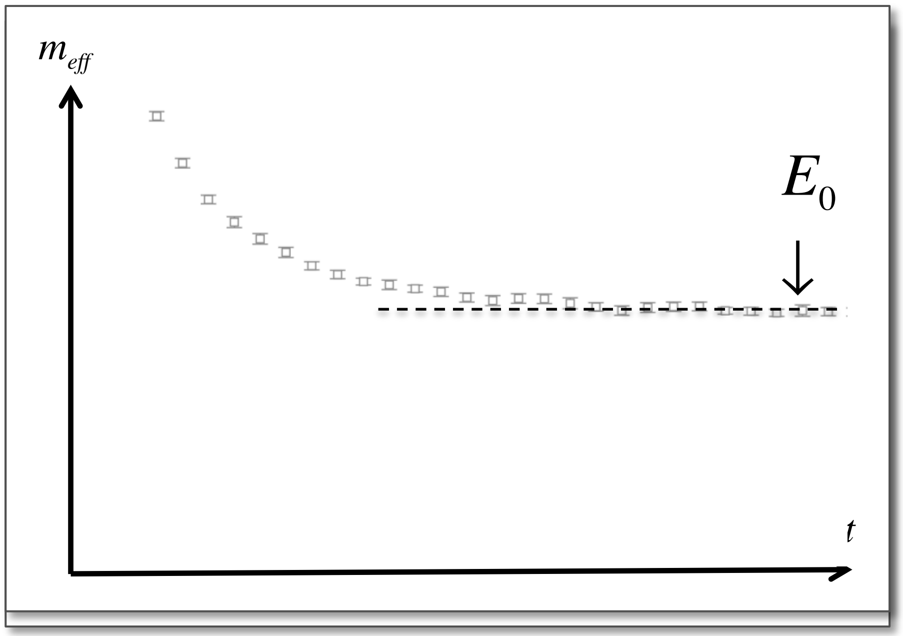
$$C(t) = \sum_n \langle \phi|e^{-Ht}|n\rangle\langle n|\phi\rangle = \sum |\langle \phi|n\rangle|^2 e^{-E_n t} \xrightarrow{t\to\infty} Z_0\, e^{-E_0 t}$$

mass

The g.s. energy can be more clearly seen by looking at the effective mass plots:

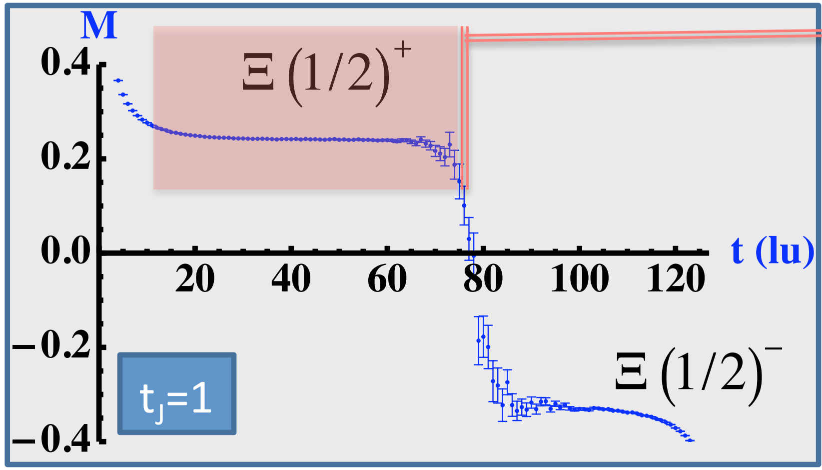
$$\begin{aligned} C(t) &\sim Z_0\, e^{-E_0 t} \\ C(t+\delta t) &\sim Z_0\, e^{-E_0 (t+\delta t)} \end{aligned} \rightarrow m_{eff} = \frac{1}{\delta t}\log\frac{C(t)}{C(t+\delta t)}$$

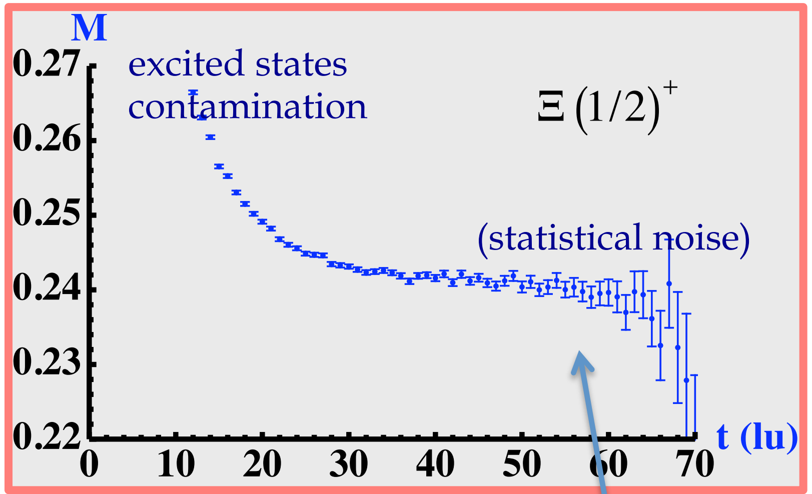
$$\sim -\frac{d}{dt}\log C(t)$$



Baryons, an example: Ξ mass (uss)

$$\Xi^0_\alpha(\vec{x},t) = \varepsilon^{ijk} s^i_\alpha(\vec{x},t)\left(u^{j^T}_\alpha(\vec{x},t)\, C\gamma_5\, s^k(\vec{x},t)\right)$$





(anti-periodic bc in time)

$$\frac{1}{t_J}\log\frac{C_A(t)}{C_A(t+t_J)} = m_A$$

Generalized effective plots

ground states dominate in this region

g.s. energy from plateau (mass)

for baryons, the noise grows exponentially with time
poor signal-to-noise ratio

Lepage, 1989

$$N\sigma^2(C) = \langle CC^\dagger \rangle - |\langle C \rangle|^2$$

$$\langle C \rangle \sim A_1\, e^{-m_1 t}$$

$$\langle CC^\dagger \rangle \sim A_2\, e^{-m_2 t}$$

$$\frac{\langle C \rangle}{\sigma} \sim \frac{\sqrt{N} A_1\, e^{-m_1 t}}{\sqrt{A_2\, e^{-m_2 t} - A_1^2\, e^{-2 m_1 t}}}$$

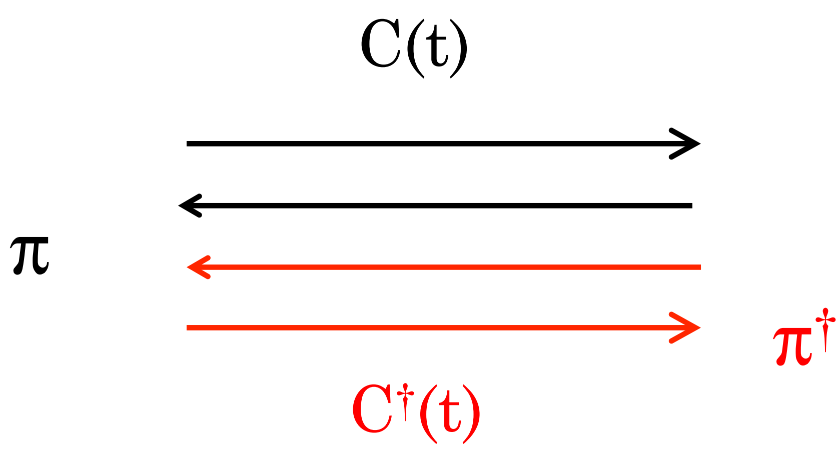
pion: $m_1 = m_\pi$ and $m_2 = 2m_\pi$

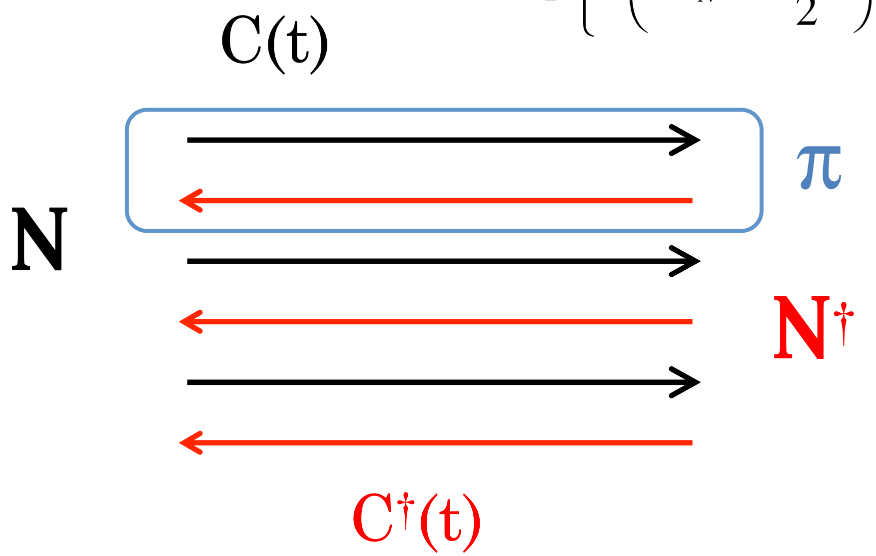
nucleon: $m_1 = m_N$ and $m_2 = 3m_\pi$

pions: $$\frac{\sqrt{N} A_1\, e^{-m_\pi t}}{\sqrt{A_2\, e^{-2m_\pi t} - A_1^2\, e^{-2m_\pi t}}} \rightarrow \frac{\sqrt{N} A_1}{\sqrt{A_2 - A_1^2}} \sim \sqrt{N}$$

nucleons: $$\frac{\sqrt{N} A_1\, e^{-m_N t}}{\sqrt{A_2\, e^{-3m_\pi t} - A_1^2\, e^{-2m_N t}}} \rightarrow \frac{\sqrt{N}}{\sqrt{\frac{A_2}{A_1^2}\, e^{2m_N t - 3m_\pi t} - 1}}$$

$$\sim \sqrt{N} \exp\left\{ -\left(M_N - \frac{3m_\pi}{2} \right) t \right\}$$





baryons: exponential degradation of the signal with time

$$N\sigma^2(C) = \langle CC^\dagger \rangle - |\langle C \rangle|^2$$

$$\langle C \rangle \sim A_1\, e^{-m_1 t}$$

$$\langle CC^\dagger \rangle \sim A_2\, e^{-m_2 t}$$

pion: $m_1 = m_\pi$ and $m_2 = 2m_\pi$

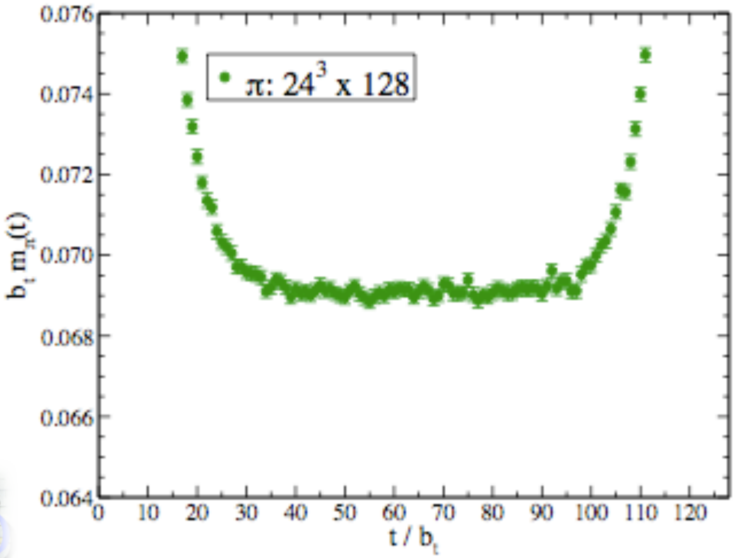
nucleon: $m_1 = m_N$ and $m_2 = 3m_\pi$

Lepage, 1989

$$\boxed{\text{A nucleons:} \quad \frac{\langle C \rangle}{\sigma} \sim \sqrt{N} \exp\left\{-A\left(M_N - \frac{3m_\pi}{2}\right)t\right\}}$$

signal-to-noise independent of time

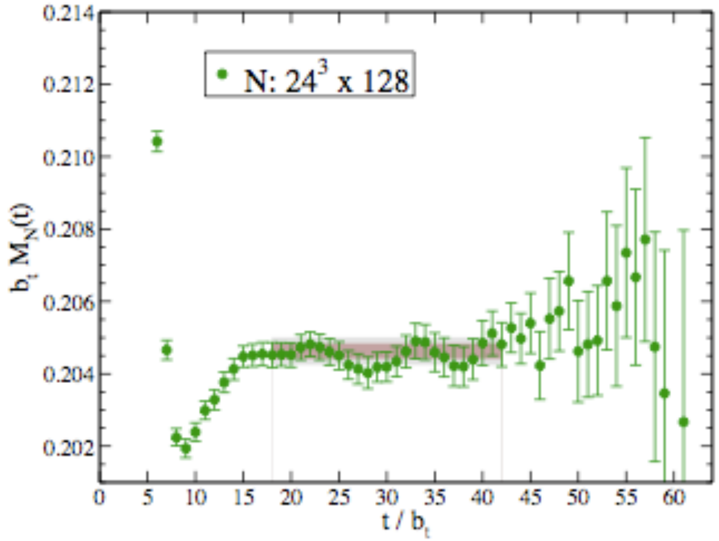
pion: $$\frac{\langle C(t) \rangle}{\sigma(t)} \sim \frac{\sqrt{N} A_0\, e^{-m_\pi t}}{\sqrt{\left(A_2 - A_0^2\right) e^{-m_\pi t}}} \sim \sqrt{N}$$



signal-to-noise degradation with time

nucleon: $$\frac{\langle C \rangle}{\sigma} \sim \sqrt{N} \exp\left\{-\left(M_N - \frac{3m_\pi}{2}\right)t\right\}$$

NPLQCD, Phys.Rev. D84 (2011) 014507



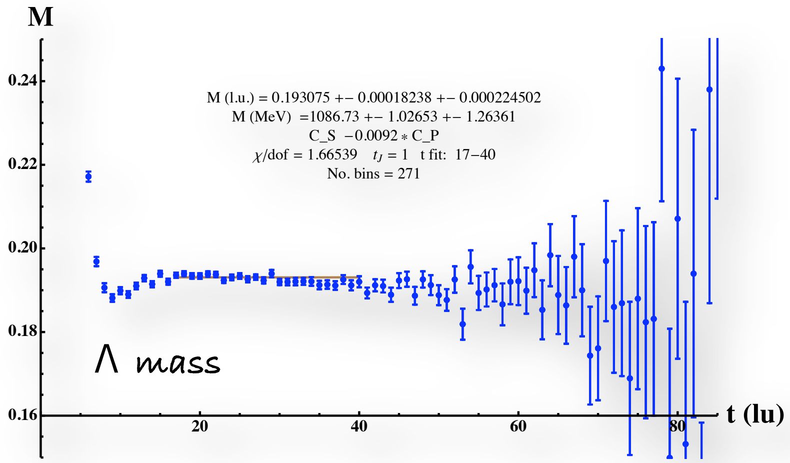
NPLQCD

Some current challenges in the extraction of hadron masses:

Optimize the intepolating fiels in order to maximize the weight of a given state

$$\varphi^{s}(\vec{x},t) \equiv \sum_{\vec{y}} f_{s}(\vec{x},\vec{y})\,\varphi(\vec{y},t)$$

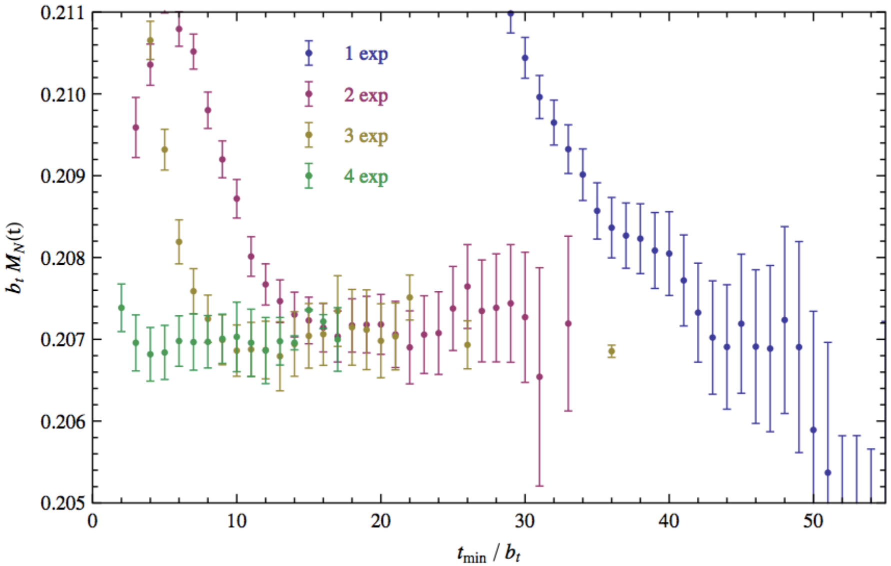
different choices for the smearing function:
The use of **gaussian smeared operators**
optimizes the overlap onto the ground-state hadrons



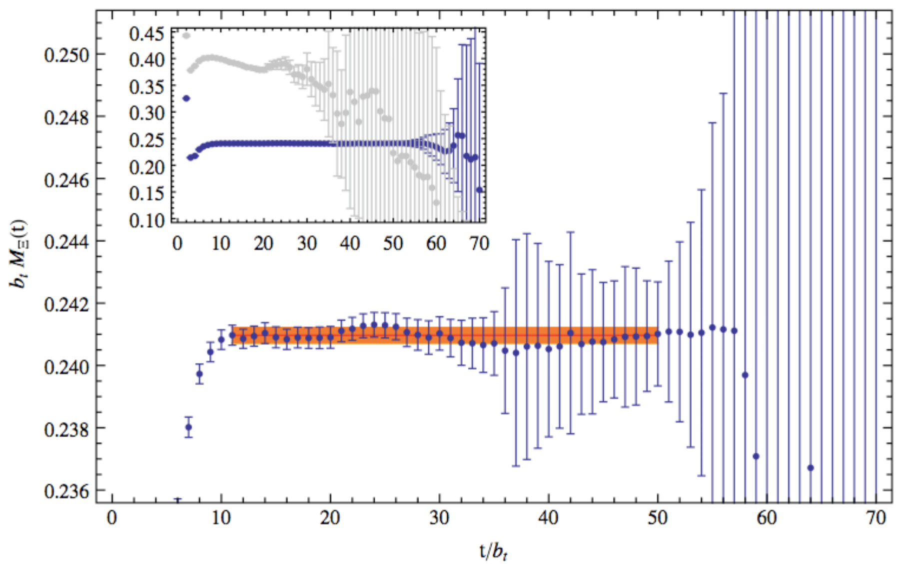
results obtained with a smeared source and working with point-like and smeared sink

Some current challenges in the extraction of hadron masses:

Develop analysis techniques to extract excited states from the two-point correlators: multiexponential fits, generalize the Effective Mass method to two or more exponential functions (Matrix-Prony), etc.



NPLQCD, PRD 79, 114502 (2009)



extraction of 2 states

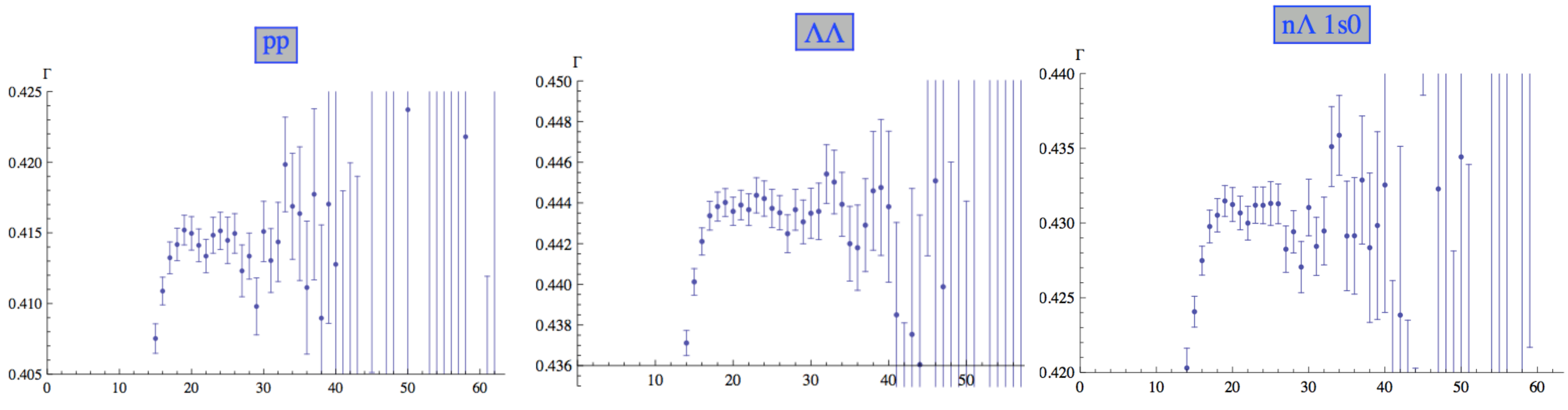
Increasing the number of particles

Two-particle correlators $\longrightarrow$ Energy of the interacting 2-particle system

$$C_{H_A H_B,\Gamma}(\vec{p}_1,\vec{p}_2,t) = \sum_{\vec{x}_1 \vec{x}_2} e^{i\vec{p}_1\vec{x}_1} e^{i\vec{p}_2\vec{x}_2} \underbrace{\Gamma^{\beta_1\beta_2}_{\alpha_1\alpha_2}}_{\text{spin tensor}} \left\langle \underbrace{J_{H_A,\alpha_1}(\vec{x}_1,t) J_{H_B,\alpha_2}(\vec{x}_2,t) \bar{J}_{H_A,\beta_1}(x_0,0) \bar{J}_{H_B,\beta_2}(x_0,0)}_{\text{interpolating operators}} \right\rangle$$

at large t $\quad C_{H_A H_B}(\vec{p},-\vec{p},t) \sim \sum_n Z^{(i)}_{n;AB}(\vec{p})\, Z^{(f)}_{n;AB}(\vec{p})\, e^{-E_n^{AB}(\vec{0})t}$

$m_\pi \sim 390$ MeV, $L_s \sim 2.5$ fm



LQCD calculations involving A > 2 particles

Greater complexity of multinucleon systems as compared to single meson and baryon calcs

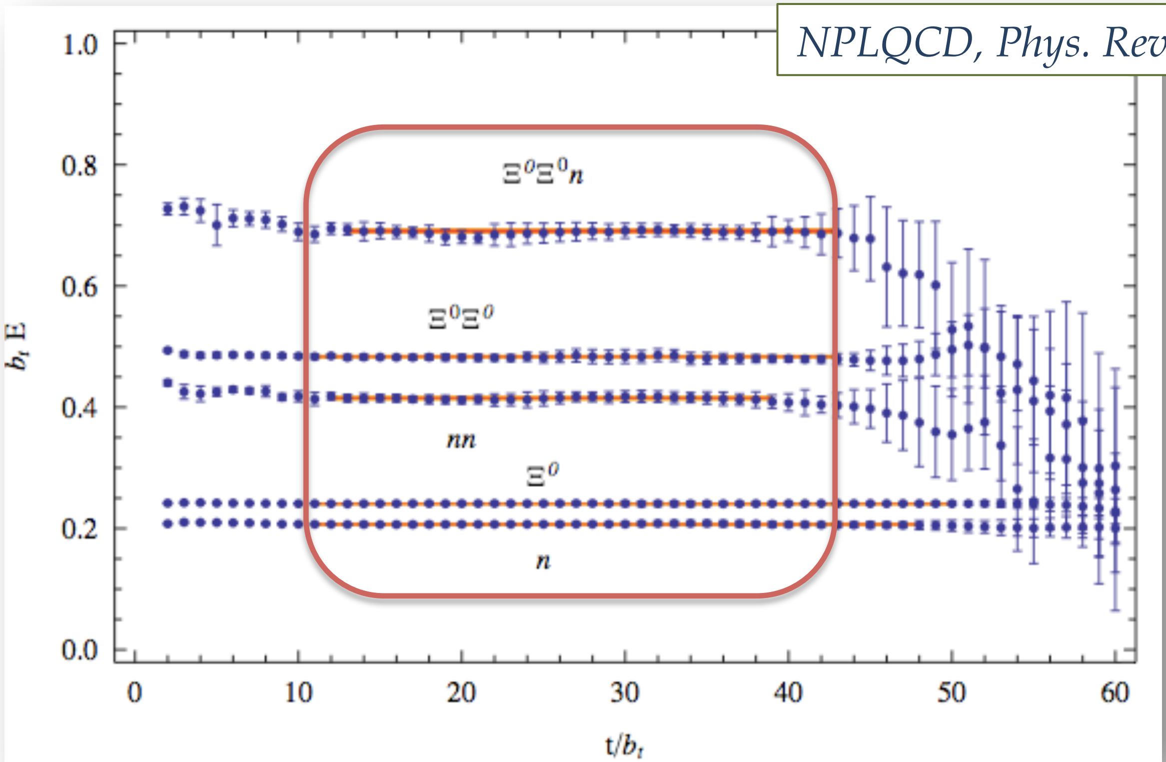
\# Wick contractions at que quark level, to form the correlation function is naively $N_u!\, N_d!\, N_s!$

$$\left((A+Z)!\,(2A-Z)!\right)$$

$$^{3}\mathrm{H} \rightarrow 2880 \qquad {}^{4}\mathrm{He} \rightarrow 518\,400$$

cheapest 3-baryon system: $\Xi^0\Xi^0 n$, with 3! 2! 4! = 288 Wick contractions

NPLQCD, Phys. Rev. D80 (2009) 074501



m_π ~390 MeV
L_s ~ 2.5 fm

How do we get the low-energy scattering parameters and binding energies?

The Maiani-Testa Theorem (*C. Michael, Nucl. Phys. B327(1989; L. Maiani and M. Testa, Phys. Lett. B245, 585 (1990)*) tells us that one cannot extract multi-hadron S-matrix elements from Euclidean space Green functions at infinite volume except for kinematical thresholds.

Scattering amplitudes are in general complex

$$\left|NN\right\rangle_{out} = S\left|NN\right\rangle_{in} \rightarrow \quad {}_{out}\left\langle NN \mid NN\right\rangle_{in} = {}_{in}\left\langle NN \mid S \mid NN\right\rangle_{in} = e^{i2\delta}$$

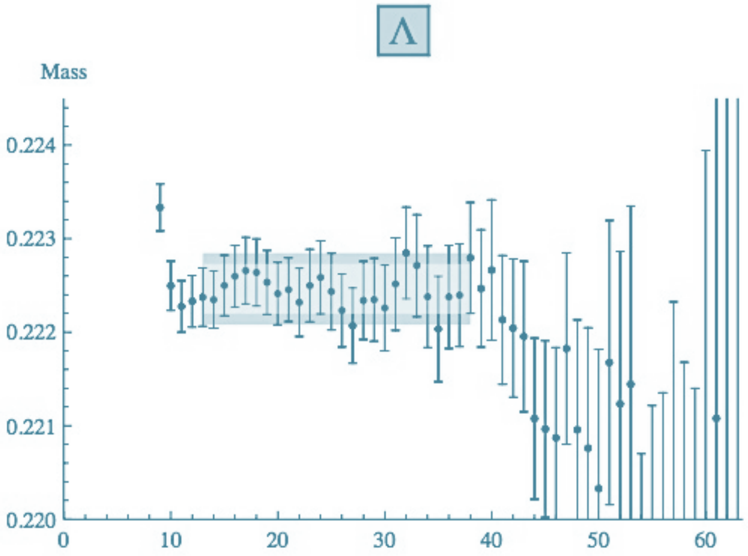
below inelastic thresholds

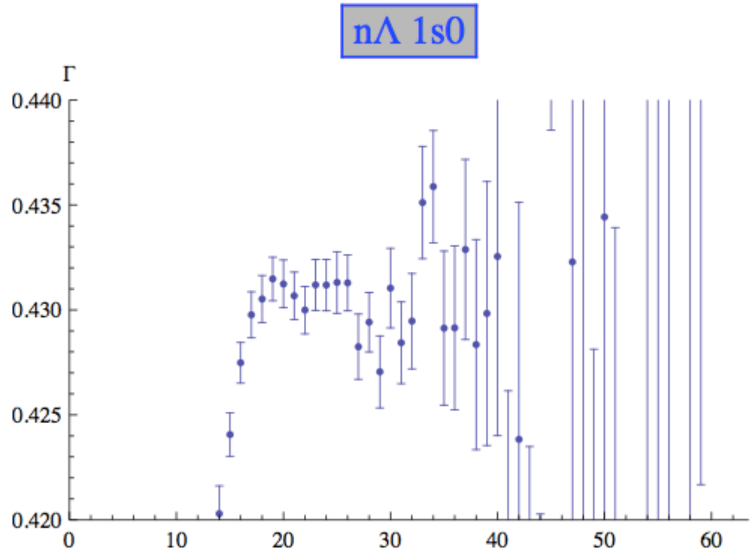
In Euclidean space-time, there is no distinction between in- and out-states. Therefore, the matrix elements one can extract are real numbers, and any signal of the phase due to the interaction is lost.

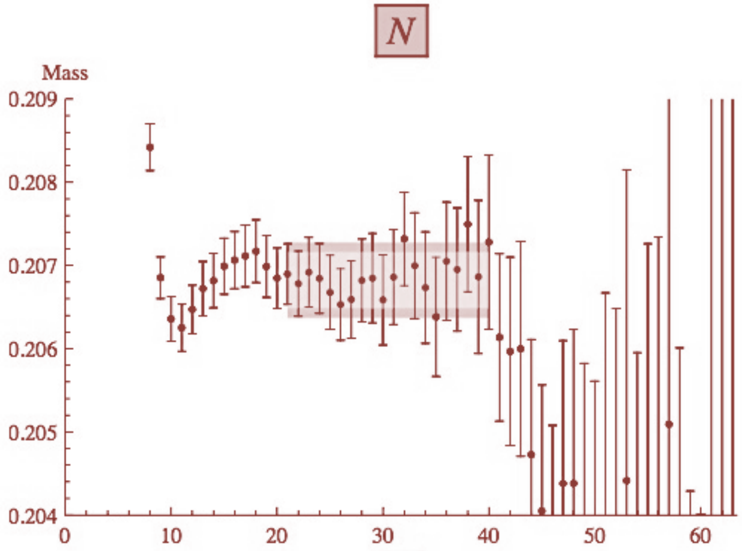
In non-relativistic Quantum Mechanics, it was known (*K. Huang and C.N. Yang, Phys. Rev. 105, 767 (1957)*) that placing the particles in a finite volume shifts their energies, and these shifts depend on their interactions.

M. Lüscher generalized this results to Quantum Field Theory (*M. Lüscher, Commun. Math. Phys. 105 153 (1986); Nucl. Phys. B354, 531 (1991)*): extract the scattering length from the volume dependence of two-particle energy levels at finite volume (up to inelastic thresholds)

First, extract the energy-shift due to the interaction:







$$G_{\Lambda N(^1S_0)}(t) = \frac{C_{\Lambda N(^1S_0)}(t)}{C_\Lambda(t)\, C_N(t)} \rightarrow A_0\, e^{-\Delta E_{\Lambda N(^1S_0)} t}$$

Effective Mass method

$$\frac{1}{t_J} \log \frac{G(t)}{G(t+t_J)} \rightarrow \text{extract} \quad \Delta E$$

How do we get scattering parameters or binding energies ?

In the finite volume, below the inelastic thresholds, the energies satisfy the eigenvalue equation:

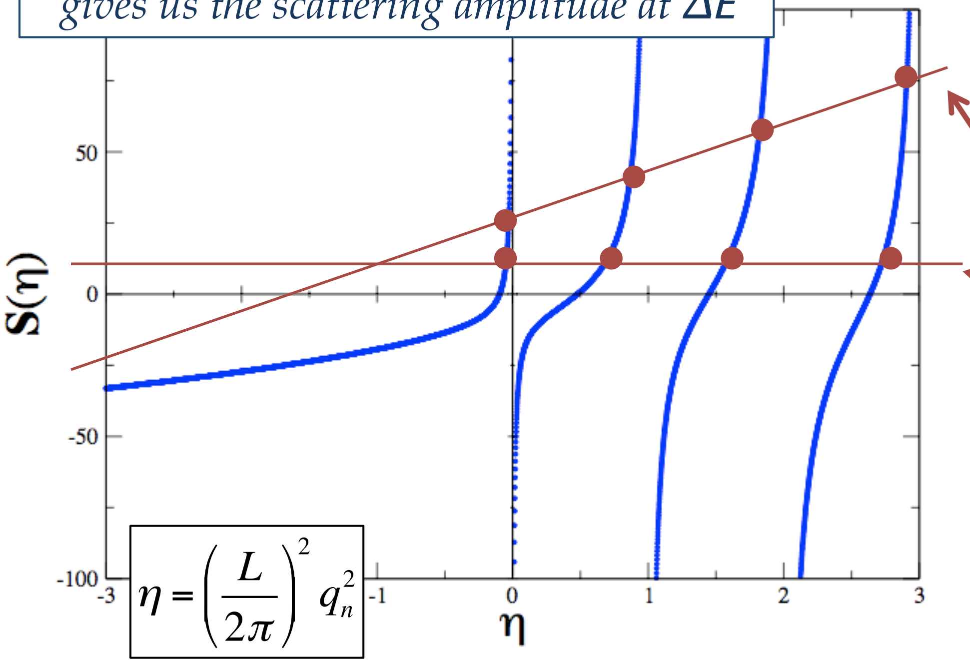
$$q_n \cot\delta(q_n) = \lim_{\Lambda_n \to \infty} \left\{ \frac{1}{\pi L} \sum_{\vec{n}}^{\Lambda_n} \frac{1}{|\vec{n}|^2 - \left(\frac{L q_n}{2\pi}\right)^2} - \frac{4\Lambda}{L} \right\} \equiv \frac{1}{\pi L} S\left(\frac{q_n^2 L^2}{4\pi^2}\right) \qquad + O(e^{-ML})$$

(3D zeta function)

$$\Delta E_n^{(AB)} \equiv \Delta E_n^{(AB)}(\vec{0}) \equiv E_n^{(AB)}(\vec{0}) - m_A - m_B$$
$$= \sqrt{q_n^{\ 2} + m_A^2} + \sqrt{q_n^{\ 2} + m_B^2} - m_A - m_B$$
$$= \frac{q_n^2}{2\mu_{AB}} + \dots$$

lattice extractions

gives us the scattering amplitude at ***ΔE***



$$\eta = \left(\frac{L}{2\pi}\right)^2 q_n^2$$

$$q_n \cot\delta(q_n) = -\frac{1}{a} + \frac{1}{2} r_0 q_n^2$$

● : location of the E eigenstates in the finite volume

Beane, Bedaque, Parreño, Savage, Phys. Lett. B585 (2004) 106-114

$$\Delta E_0 = \frac{p^2}{M} = \frac{4\pi a}{ML^3}\left[1 - c_1\frac{a}{L} + c_2\left(\frac{a}{L}\right)^2 + \ldots\right]$$

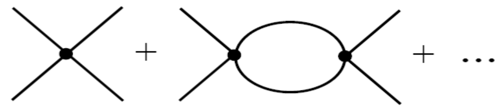
Ground state energy shift

Recovering M. Lüscher, Commun. Math. Phys. 105, 153 (1986) (L>>a)

$$\Delta E_1 = \frac{4\pi}{ML^2} - \frac{12\tan\delta_0}{ML^2}\left[1 + c'_1\tan\delta_0 + c'_2\tan^2\delta_0\right] + \ldots \qquad \text{with} \quad \delta_0 = \delta(p_{E_1})$$

First excited state energy shift

Bound states?

$$\mathcal{A} \sim$$ 

$$\mathcal{A} = \frac{4\pi}{M}\frac{1}{p\cot\delta(p) - ip}$$

infinite volume

b.s. $\quad p^2 = -\gamma^2$

$$\cot\delta(i\gamma) = i$$

finite volume:

$$\cot\delta(i\gamma)\Big|_{k=i\gamma} = i - i\sum_{\vec{m}\neq\vec{0}}\frac{e^{-|\vec{m}|\gamma L}}{|\vec{m}|\gamma L}$$

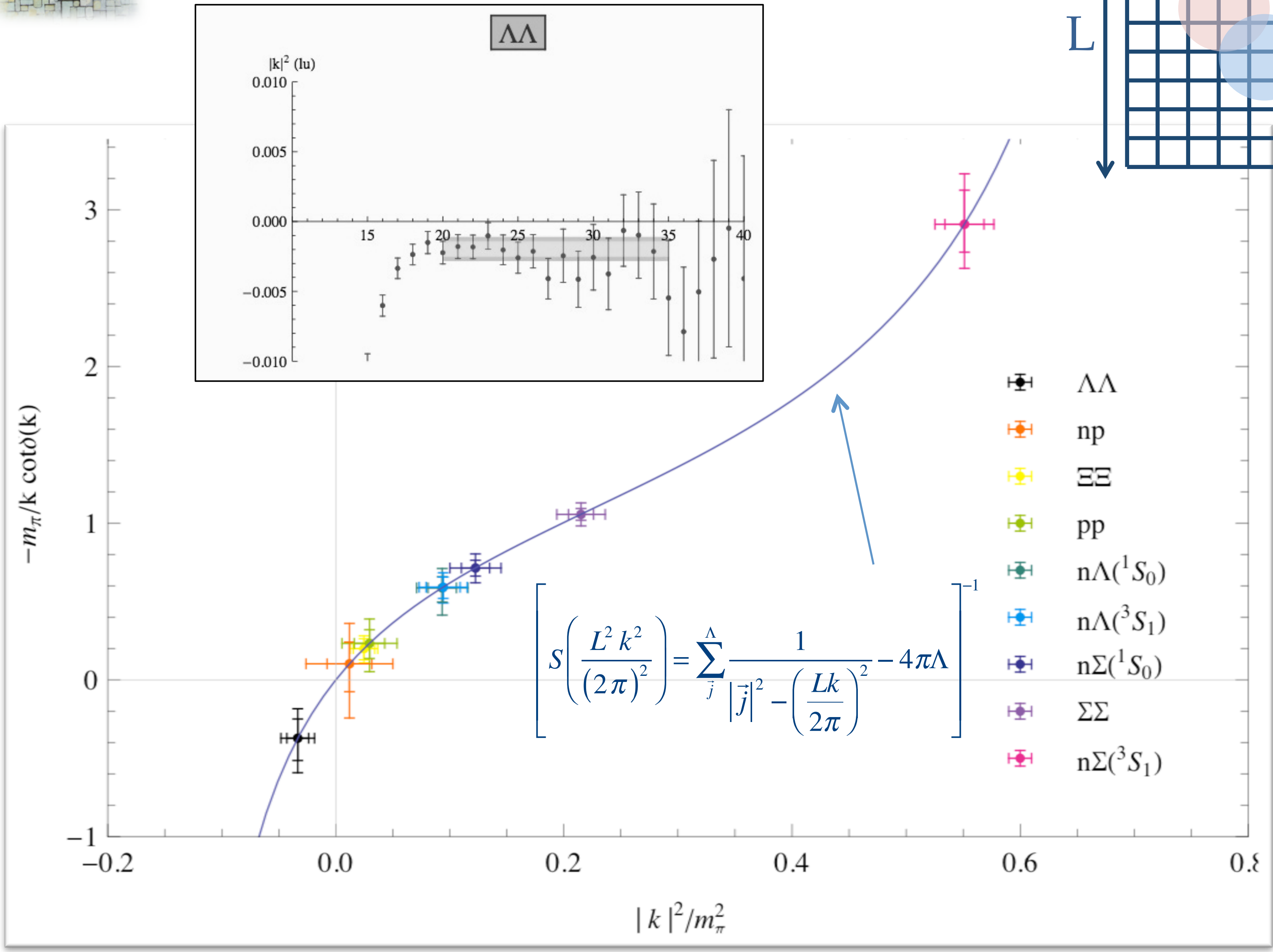
$$k^2 < 0, \quad k = i\kappa$$

$$\kappa = \gamma + \frac{g_1}{L}\left(e^{-\gamma L} + \sqrt{2}e^{-\sqrt{2}\gamma L} + \ldots\right)$$

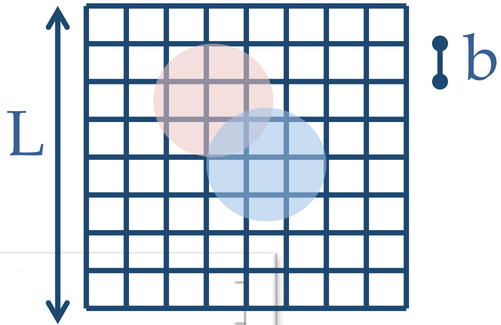
$\kappa \to \gamma$ for large L

$$B_\infty^H = \frac{\gamma^2}{M}$$

m_π ~390 MeV, L_s ~ 2.5 fm, b=0.123 fm



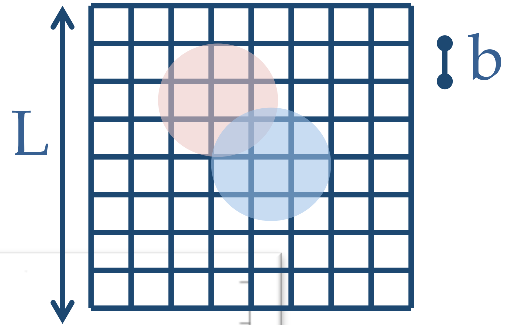
$$\left[S\left(\frac{L^2 k^2}{(2\pi)^2}\right) = \sum_{\vec{j}}^{\Lambda} \frac{1}{|\vec{j}|^2 - \left(\frac{Lk}{2\pi}\right)^2} - 4\pi\Lambda \right]^{-1}$$



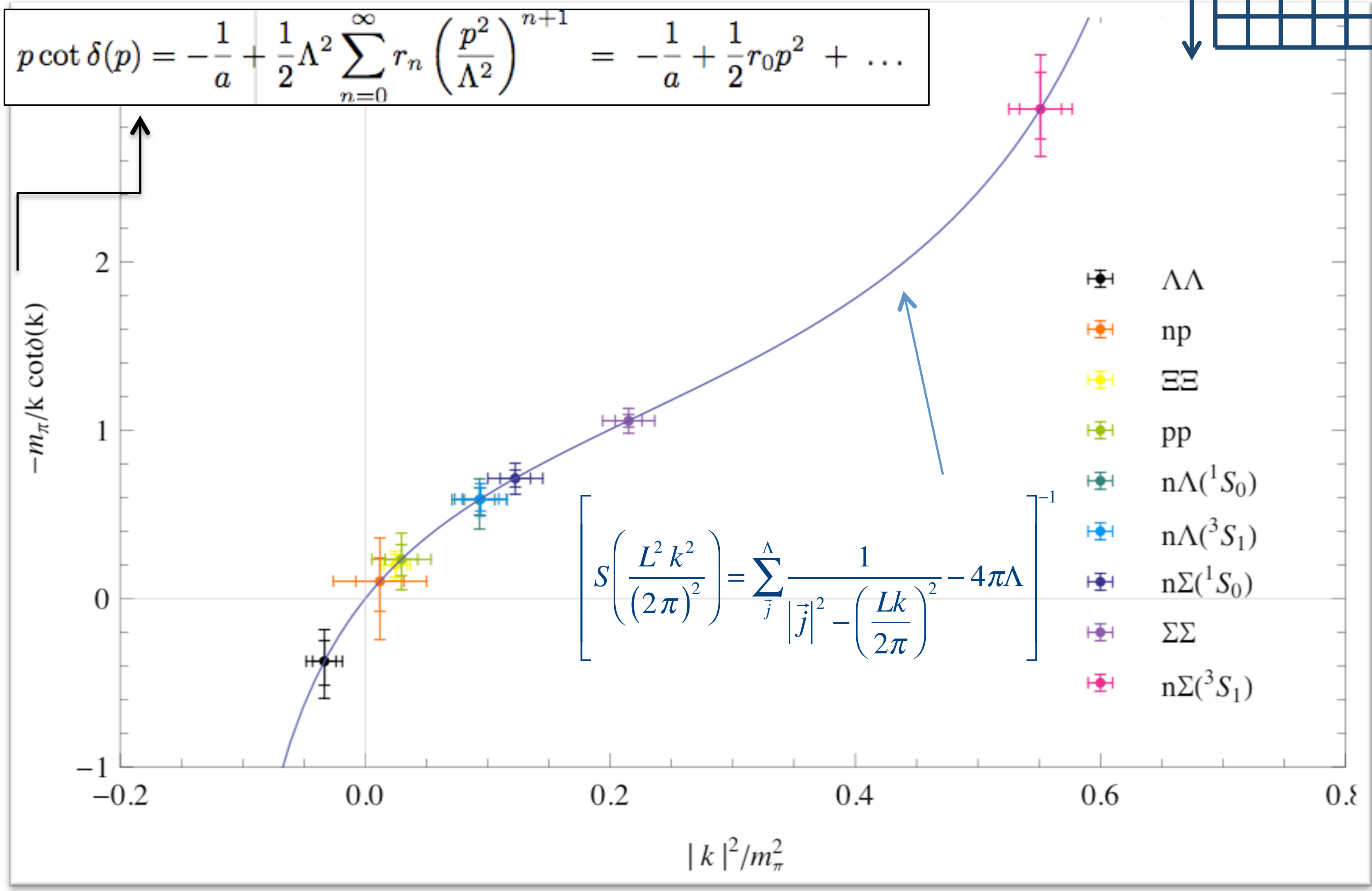
NPLQCD, Phys. Rev. D81 (2010) 054505

NPLQCD

$m_\pi \sim 390$ MeV, $L_s \sim 2.5$ fm , b=0.123 fm



$$p \cot \delta(p) = -\frac{1}{a} + \frac{1}{2}\Lambda^2 \sum_{n=0}^{\infty} r_n \left(\frac{p^2}{\Lambda^2}\right)^{n+1} = -\frac{1}{a} + \frac{1}{2} r_0 p^2 + \ldots$$



$$\left[S\left(\frac{L^2 k^2}{(2\pi)^2}\right) = \sum_{\vec{j}}^{\Lambda} \frac{1}{|\vec{j}|^2 - \left(\frac{Lk}{2\pi}\right)^2} - 4\pi\Lambda \right]^{-1}$$

NPLQCD, Phys. Rev. D81 (2010) 054505

infinite volume extrapolations with enough statistics for (hyper) nuclear systems

Heavier quark masses:

resources required to generate configurations and q-propagators are smaller

degradation in the signal-to-noise ratio in multinucleon correlation functions is reduced

calculations at the SU(3)-flavor symmetry point

L/b	T/b	β	$b\,m_q$	b (fm)	L (fm)	T (fm)	m_π (MeV)	$m_\pi L$	$m_\pi T$	$N_{\rm cfg}$	$N_{\rm src}$
24	48	6.1	−0.2450	0.145	3.4	6.7	806.5(0.3)(0)(8.9)	14.3	28.5	3822	96
32	48	6.1	−0.2450	0.145	4.5	6.7	806.9(0.3)(0.5)(8.9)	19.0	28.5	3050	72
48	64	6.1	−0.2450	0.145	6.7	9.0	806.7(0.3)(0)(8.9)	28.5	38.0	1905	54

NPLQCD, PRD 87, 034506 (2013); PRC 88, 024003 (2013)

no physical light-quark masses yet

only one lattice spacing

Anisotropic lattices: N_t >> N_s

(N_f=2+1 clover-improved Wilson fermion actions)

higher resolution in the time direction:

better study of noisy states

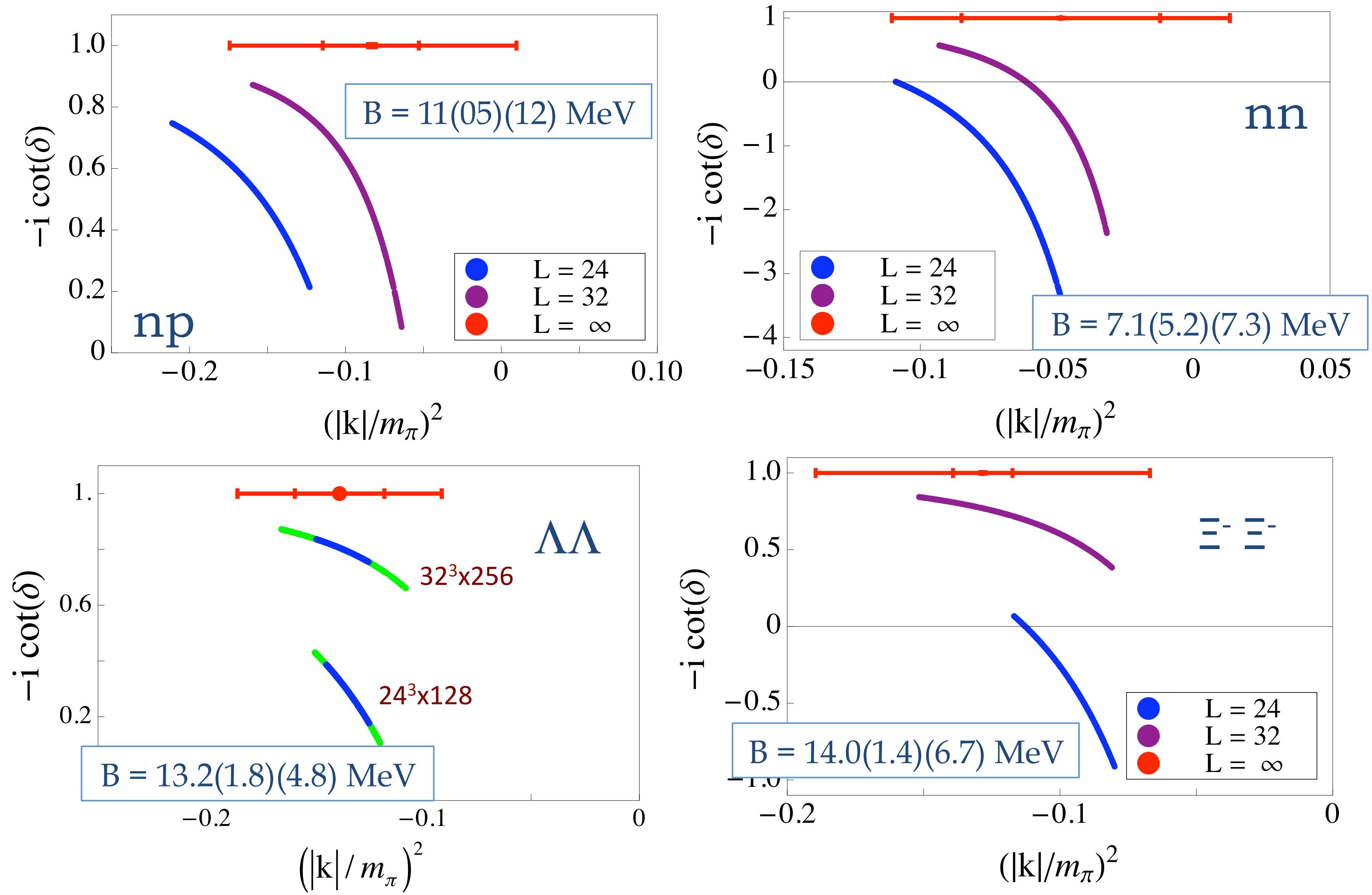
better extraction of excited states

reduce the systematic due to fitting (confident plateaus)

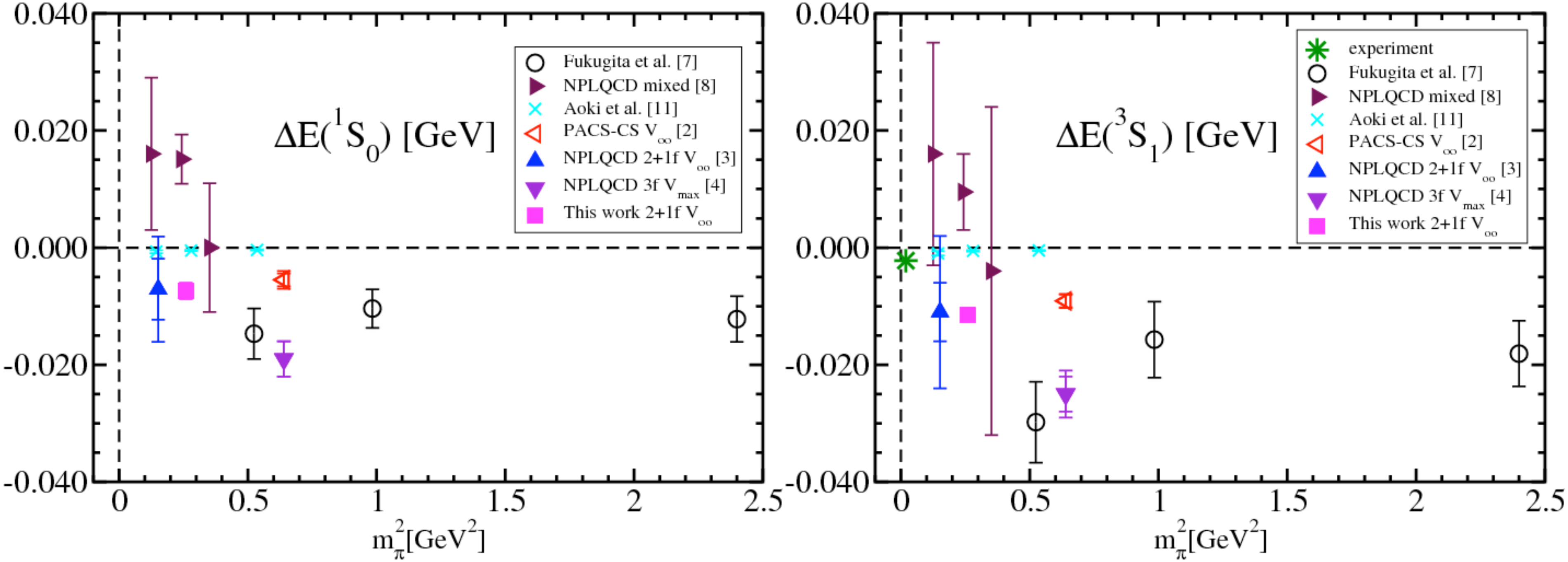
NPLQCD

m_π ~390 MeV, L_s ~ 2, 2.5, 3, 4 fm, b=0.123 fm

finite volume: $\cot\delta(i\gamma)\big|_{k=i\gamma} = i - i\sum_{\vec{m}\neq\vec{0}} \frac{e^{-|\vec{m}|\gamma L}}{|\vec{m}|\gamma L}$



Nucleon-Nucleon

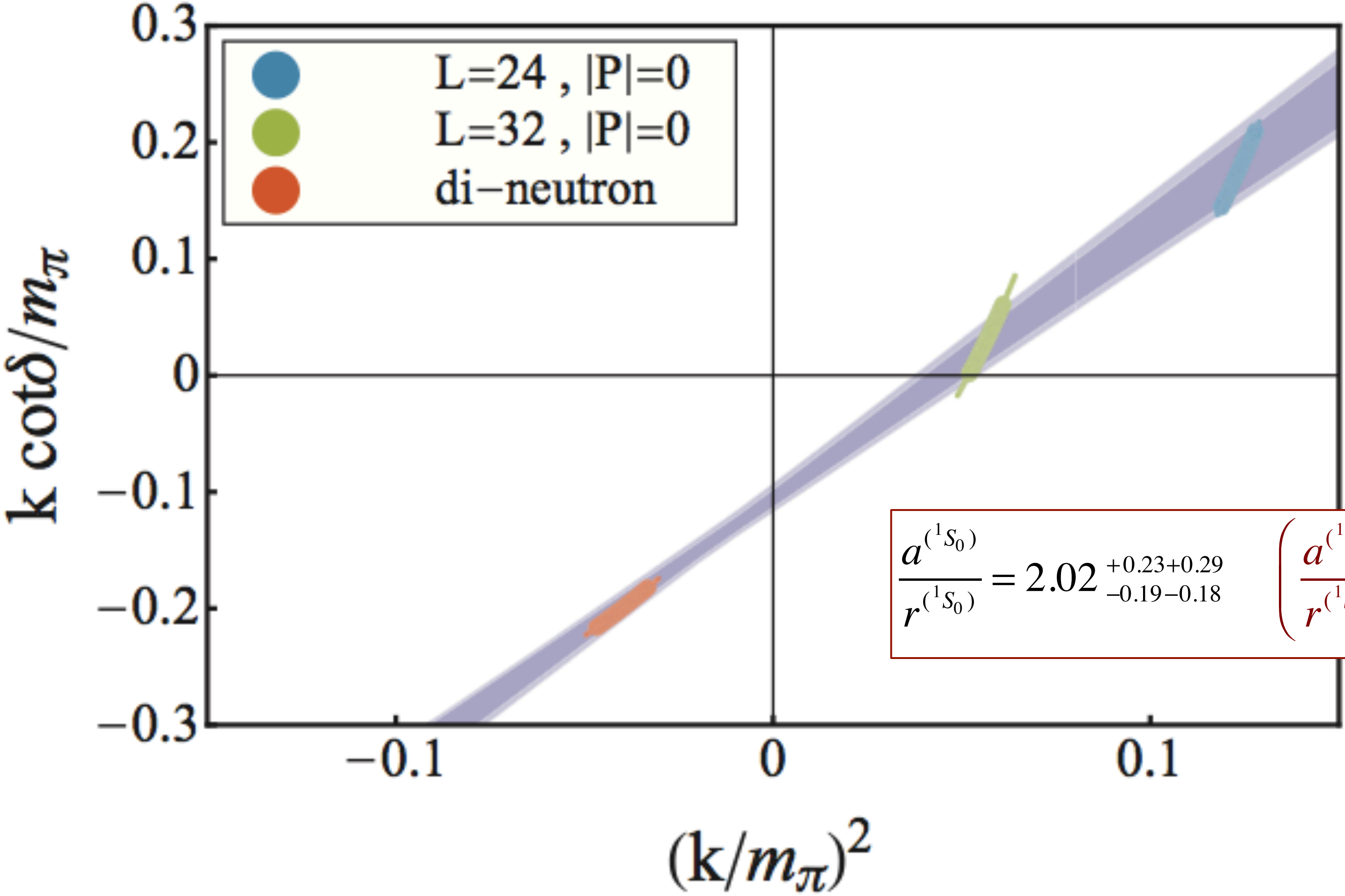


from T. Yamazaki, K. Ishikawa, Y. Kuramashi, A. Ukawa,
Phys. Rev. D86 (2012) 074514; arXiv:1207.4277

(N_f=2+1, b = 0.09 fm, m_p = 0.51 GeV, L:2.9 fm to 5.8 fm)

From calculations of the two-nucleons at rest and moving in the lattice volume

Two-parameters fit: $a^{(^1S_0)} = 2.33^{+0.19+0.27}_{-0.17-0.20}$ fm $\quad r^{(^1S_0)} = 1.130^{+0.071+0.059}_{-0.077-0.063}$ fm



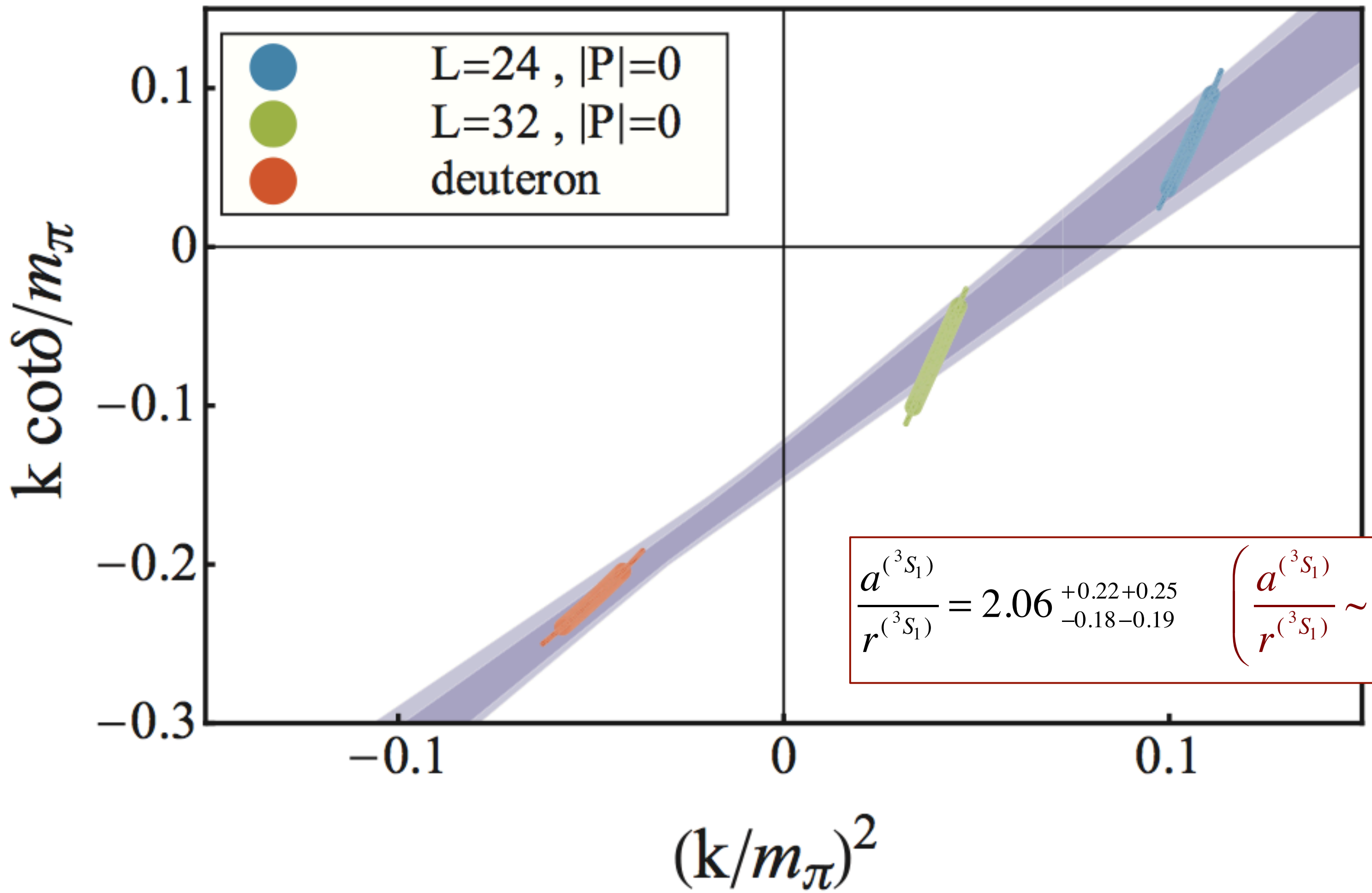
$$\frac{a^{(^1S_0)}}{r^{(^1S_0)}} = 2.02^{+0.23+0.29}_{-0.19-0.18} \qquad \left(\frac{a^{(^1S_0)}}{r^{(^1S_0)}} \sim -8.7\right)_{phys}$$

NPLQCD, PRC 88, 024003 (2013)

L ~ 3.4 fm, 4.5 fm, and 6.7 fm, b ~ 0.145 fm and @ m_π~800 MeV

From calculations of the two-nucleons at rest and moving in the lattice volume

Two-parameters fit: $a^{(^3S_1)} = 1.82\,^{+0.14+0.17}_{-0.13-0.12}$ fm $\quad r^{(^3S_1)} = 0.906\,^{+0.068+0.068}_{-0.075-0.084}$ fm



$$\frac{a^{(^3S_1)}}{r^{(^3S_1)}} = 2.06\,^{+0.22+0.25}_{-0.18-0.19} \qquad \left(\frac{a^{(^3S_1)}}{r^{(^3S_1)}} \sim +3.1\right)_{phys}$$

NPLQCD, PRC 88, 024003 (2013)

L ~ 3.4 fm, 4.5 fm, and 6.7 fm, b ~ 0.145 fm and @ m_π~800 MeV

Increasing complexity of performing contractions with the number of hadrons

To study the interaction of multi-meson/baryon states, a large number of contractions are required; in fact the number grows factorially with the number of particles involved.

Wick contractions to form the correlation function is naively $N_u!\ N_d!\ N_s!$

The triton (nnp) involves 2880 (N_u=4 and N_d=5)

However some careful counting, identifying redundant contractions, can reduce this number to a smaller # of distinct contractions.

Recursive algorithms would be needed to study even the light nuclei.
(Detmold & Savage, PRD82 (2010) 014511)

Increasing complexity of performing contractions with the number of hadrons

To study the interaction of multi-meson/baryon states, a large number of contractions are required; in fact the number grows factorially with the number of particles involved.

Wick contractions to form the correlation function is naively $N_u!\ N_d!\ N_s!$

The triton (nnp) involves 2880 (N_u=4 and N_d=5)

NPLQCD Phys. Rev. D87 (2013), 034506

flavor-SU(3) symmetry
physical strange quark mass
(no electromagnetic interactions)

$A \leq 4$ & $|s| \leq 2$:

(Quark-hadron contraction method: Detmold & Orginos, PRD87 (2013) 114512)

Isotropic clover

Label	L/b	T/b	β	$b\ m_q$	b [fm]	L [fm]	T [fm]	m_π [MeV]	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
A	24	48	6.1	-0.2450	0.145	3.4	6.7	806.5(0.3)(0)(8.9)	14.3	28.5	3822	48
B	32	48	6.1	-0.2450	0.145	4.5	6.7	806.9(0.3)(0.5)(8.9)	19.0	28.5	3050	24
C	48	64	6.1	-0.2450	0.145	6.7	9.0	806.7(0.3)(0)(8.9)	28.5	38.0	1212	32

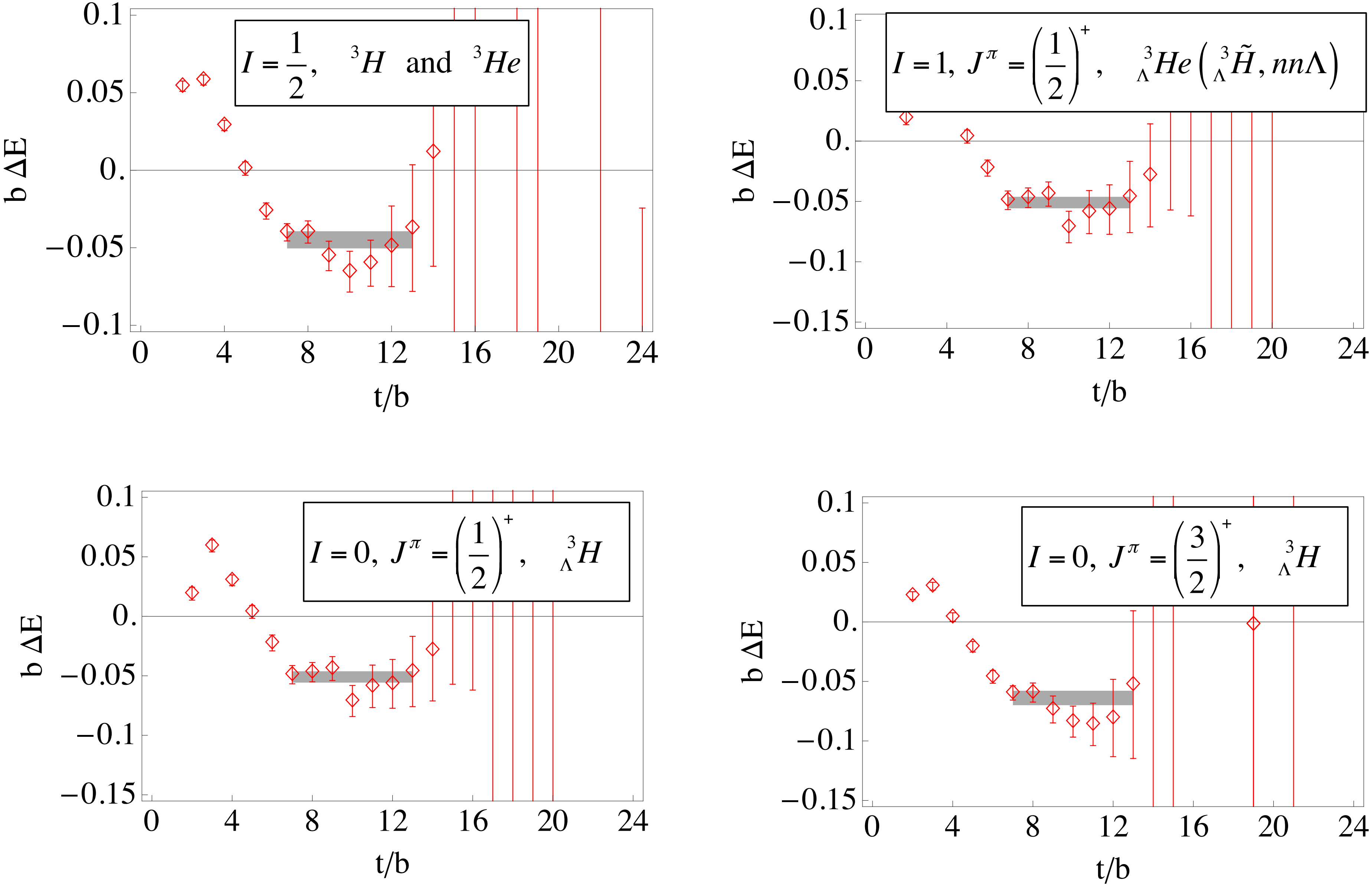
$(\pi, J^2, J_z, s, A, I^2, I_z)$

NPLQCD Phys.Rev. D87 (2013), 034506

Label	A	s	I	J^π	Local SU(3) irreps	This work
N	1	0	1/2	$1/2^+$	$\mathbf{8}$	$\mathbf{8}$
Λ	1	-1	0	$1/2^+$	$\mathbf{8}$	$\mathbf{8}$
Σ	1	-1	1	$1/2^+$	$\mathbf{8}$	$\mathbf{8}$
Ξ	1	-2	1/2	$1/2^+$	$\mathbf{8}$	$\mathbf{8}$
d	2	0	0	1^+	$\overline{\mathbf{10}}$	$\overline{\mathbf{10}}$
nn	2	0	1	0^+	$\mathbf{27}$	$\mathbf{27}$
$n\Lambda$	2	-1	1/2	0^+	$\mathbf{27}$	$\mathbf{27}$
$n\Lambda$	2	-1	1/2	1^+	$\mathbf{8}_A$, $\overline{\mathbf{10}}$	—
$n\Sigma$	2	-1	3/2	0^+	$\mathbf{27}$	$\mathbf{27}$
$n\Sigma$	2	-1	3/2	1^+	$\mathbf{10}$	$\mathbf{10}$
$n\Xi$	2	-2	0	1^+	$\mathbf{8}_A$	$\mathbf{8}_A$
$n\Xi$	2	-2	1	1^+	$\mathbf{8}_A$, $\mathbf{10}$, $\overline{\mathbf{10}}$	—
H	2	-2	0	0^+	$\mathbf{1}$, $\mathbf{27}$	$\mathbf{1}$, $\mathbf{27}$
^{3}H, ^{3}He	3	0	1/2	$1/2^+$	$\overline{\mathbf{35}}$	$\overline{\mathbf{35}}$
$^3_\Lambda$H$(1/2^+)$	3	-1	0	$1/2^+$	$\overline{\mathbf{35}}$	—
$^3_\Lambda$H$(3/2^+)$	3	-1	0	$3/2^+$	$\overline{\mathbf{10}}$	$\overline{\mathbf{10}}$
$^3_\Lambda$He, $^3_\Lambda\tilde{\text{H}}$, $nn\Lambda$	3	-1	1	$1/2^+$	$\mathbf{27}$, $\overline{\mathbf{35}}$	$\mathbf{27}$, $\overline{\mathbf{35}}$
$^3_\Sigma$He	3	-1	1	$3/2^+$	$\mathbf{27}$	$\mathbf{27}$
^{4}He	4	0	0	0^+	$\overline{\mathbf{28}}$	$\overline{\mathbf{28}}$
$^4_\Lambda$He, $^4_\Lambda$H	4	-1	1/2	0^+	$\overline{\mathbf{28}}$	—
$^4_{\Lambda\Lambda}$He	4	-2	0	0^+	$\mathbf{27}$, $\overline{\mathbf{28}}$	$\mathbf{27}$, $\overline{\mathbf{28}}$

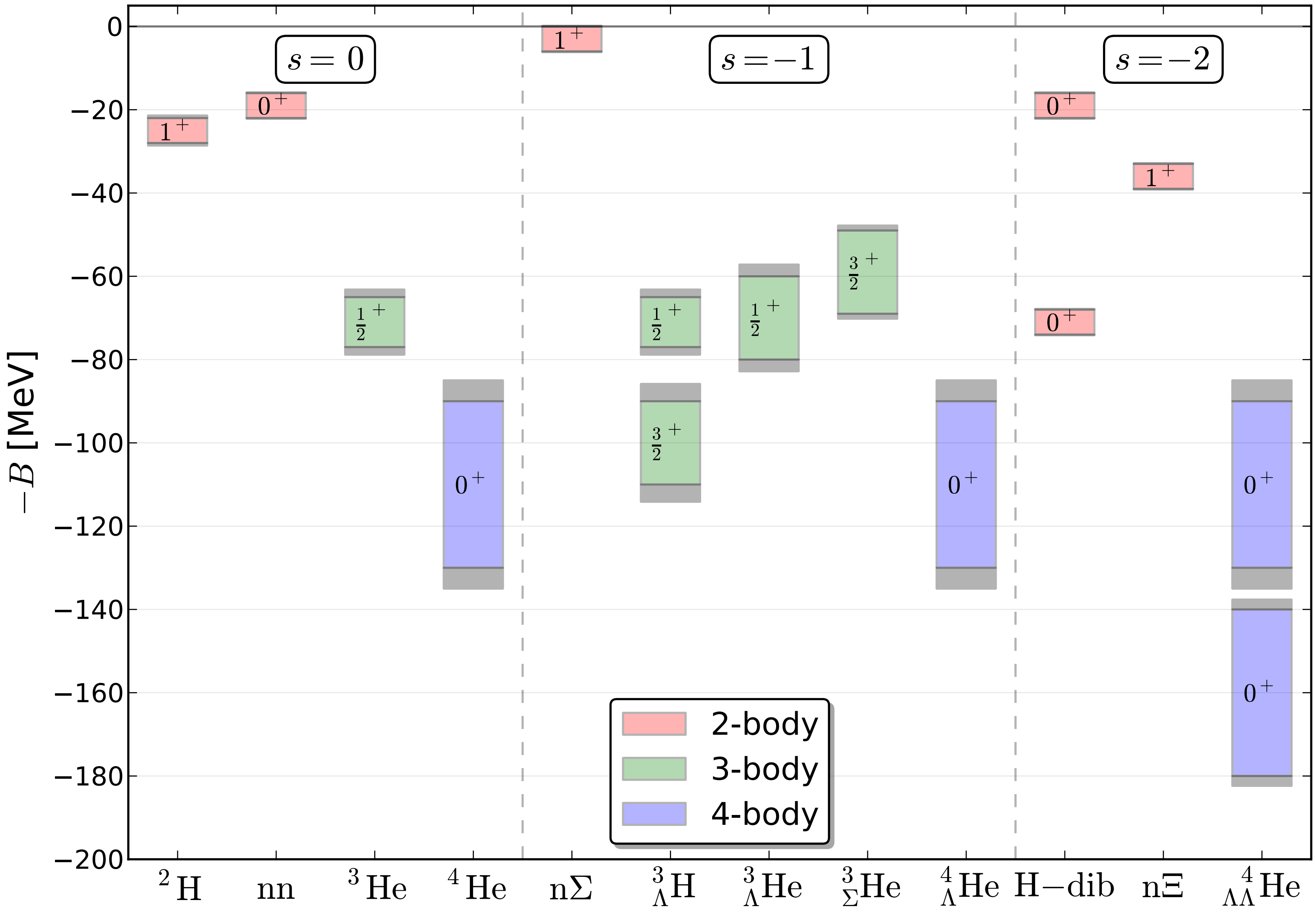
NPLQCD Phys.Rev. D87 (2013), 034506

For example, for the A=3 system: $(48^3 \times 64)$



NPLQCD Phys. Rev. D87 (2013) 3, 034506

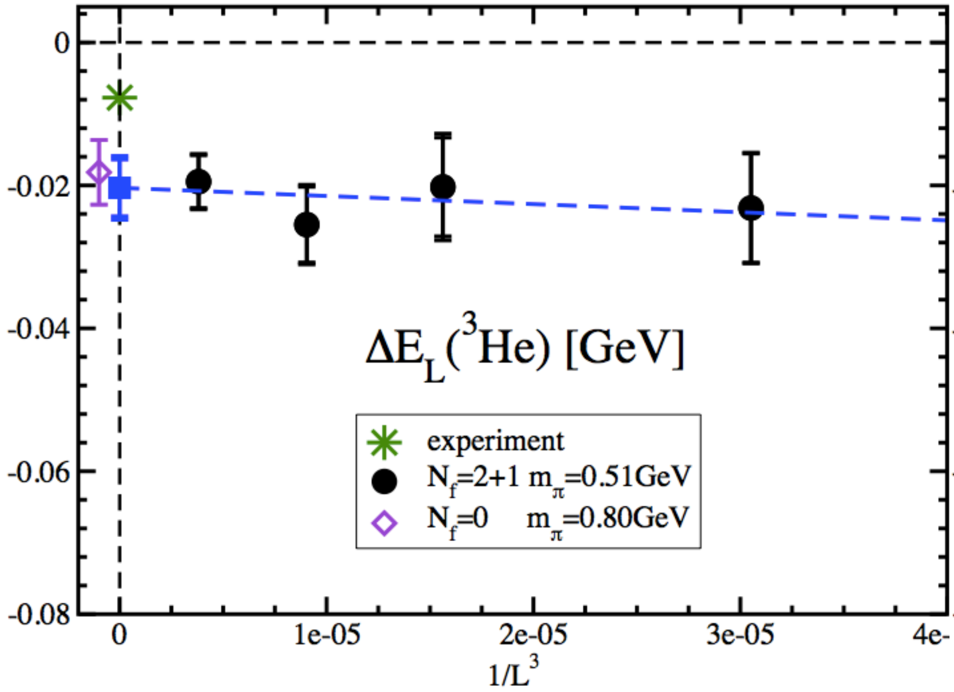
$SU(3)_f$

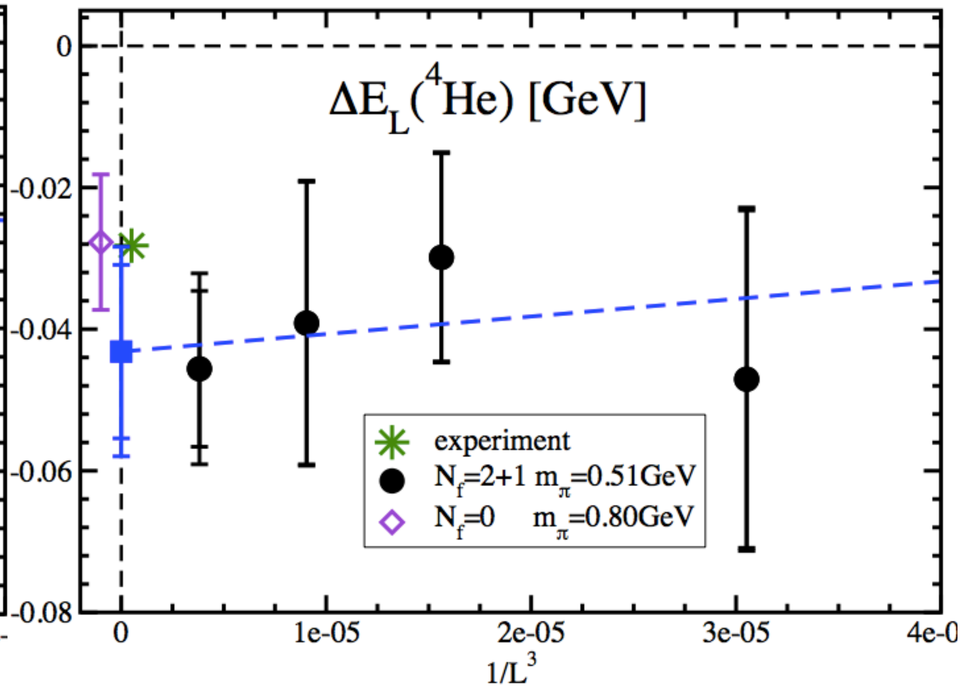


Other works

Remarkable progress up to A=4 systems

PACS-CS Yamazaki et al PRD86 (2012) 074514





$$-\Delta E_\infty = \begin{cases} 43(12)(8) & \text{MeV for } {}^4\text{He}, \\ 20.3(4.0)(2.0) & \text{MeV for } {}^3\text{He}, \\ 11.5(1.1)(0.6) & \text{MeV for } {}^3\text{S}_1, \\ 7.4(1.3)(0.6) & \text{MeV for } {}^1\text{S}_0. \end{cases}$$

Progress

Formalism developed that I did not cover:

✧ Including partial-wave mixing

1. M. Lüscher, Commun. Math. Phys. 105 (1986)
2. M. Lüscher, Nucl. Phys. B 354 (1991)
3. K. Rummukainen and S.A. Gotlieb, Nucl.Phys.B 450 (1995)
4. C.H. Kim, C.T.Sachrajda and S.R.Sharpe, Nucl.Phys.B 727 (2005)
5. T.Luu and M.J.Savage, PRD 83 (2011)
6. M.T.Hansen and S.R.Sharpe, PRD 86 (2012)
7. R.Briceño and Z.Davoudi, Phys.Rev. D88 (2013) 9, 094507
8. R.Briceño, Z.Davoudi and T.Luu, Phys.Rev. D88 (2013) 3, 034502
9. R.Briceño, Z.Davoudi, T.Luu and M.J.Savage, Phys.Rev. D88 (2013) 11, 114507

✧ Including the electromagnetic interaction

1. S. Drury et al., PoS LATTICE **2013** (2013) 268 [arXiv:1312.0477 [hep-lat]]
2. S.R. Beane and M.J. Savage, arXiv:1407.4846 [hep-lat]
3. K.U. Can et al., JHEP **1405** (2014) 125, [arXiv:1310.5915 [hep-lat]].

Summary

High statistics Lattice QCD determinations allow us to extract the infinite volume binding energies of light nuclear and hypernuclear sytems.

We have found evidence of bound NN ($^1S_0$ and $^3S_1$), ΛΛ and ΞΞ ($^1S_0$) systems at unphysical values of the quarks masses.

LQCD predictions for hypernuclear physics are possible:

Scattering phase-shifts for the $^1S_0$ and $^3S_1$ n Σ^- channels

We require enough computational resources in order to undertake simulations at lighter quark masses and at multiple lattice spacings.

These calculations will allow us to perform reliable extrapolations to the physical mass point and to the continuum.

We have evolved very fast to the point where first principles calculations of light nuclei are possible. We need to secure computational resources in powerful supercomputers to continue with this program and provide some useful input to hypernuclear physics.

Thank you for your attention