REALISTIC CALCULATIONS OF $\bar{K}NN$, $\bar{K}NNN$, AND $\bar{K}\bar{K}NN$ QUASIBOUND STATES

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LEANNIS Meeting - Prague
May 2012

האוניברסיטה העברית בירושלים
The Hebrew University of Jerusalem

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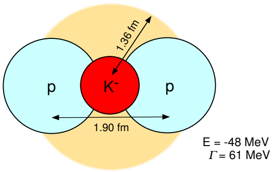
The Hebrew University of Jerusalem

In collaboration with
Avraham Gal and Evgeny Liverts

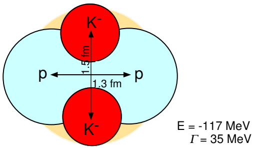
OUTLINE

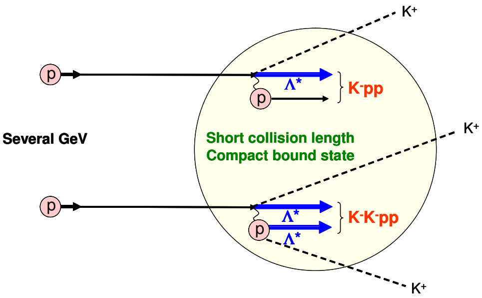
K^-pp and K^-K^-pp production [1] [2] [3]

K-pp



K-K-pp



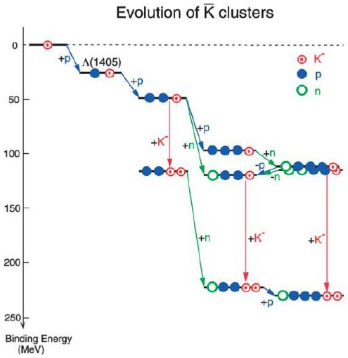


[1] T. Yamazaki, Y. Akaishi, M. Hassanvand, Proc. Jpn. Acad. Ser. B 87 (2011) 362
[2] M. Hassanvand, Y. Akaishi, T. Yamazaki, Phys. Rev. C 84 (2011) 015207
[3] T. Yamazaki, A. Doté, Y. Akaishi, Phys. Lett. B 587 (2004) 167

Prediction of the spectrum of light $\bar{K}$-nuclear systems

- $\bar{K}N$ potentials fitted to reproduce the Λ1405 as a bound $\bar{K}N$ state.
- Do not include $\bar{K}N - \pi\Sigma$ coupling.
- No energy dependence.
- B.E. $\bar{K}pp$ About 50MeV
- B.E. $\bar{K}ppp$ About 100MeV
- B.E. $\bar{K}\bar{K}pp$ About 120MeV

T. Yamazaki, A. Doté, Y. Akaishi, Phys. Lett. B 587 (2004) 167



CHIRAL $\bar{K}N$ INTERACTION

The Weinberg-Tomozawa chiral SU(3) potential[4]

$$V_{ij}(\sqrt{s}) = \frac{C_{ij}}{4f^2}(2\sqrt{s} - M_i - M_j)\sqrt{\frac{E_i + M_i}{M_i}}\sqrt{\frac{E_j + M_j}{M_j}}$$

$\sqrt{s}$ is the Mandelstam variable.
For $I = 0$ the channels are $(\bar{K}N, \pi\Sigma, \eta\Lambda, K\Xi)$

$$C_{ij}^{I=0} = \begin{pmatrix} 3 & -\sqrt{\frac{3}{2}} & \frac{3}{\sqrt{2}} & 0 \\ & 4 & 0 & \sqrt{\frac{3}{2}} \\ & & 0 & -\frac{3}{\sqrt{2}} \\ & & & 3 \end{pmatrix}$$

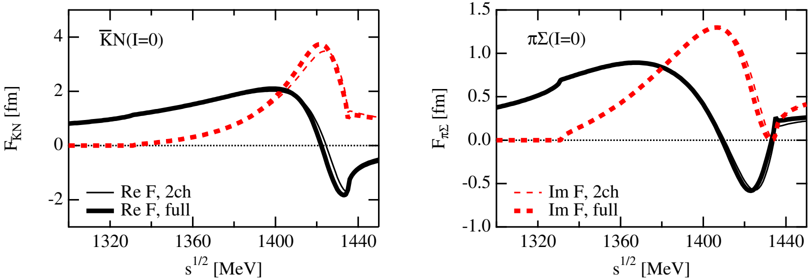
An effective $\bar{K}N$ potential is constructed demanding a "phase equivalent" potential, $T_{11}^{eff} = T_{11}$.

$$\begin{aligned} T_{11} &= V_{11} + \sum_k V_{1k} G_k T_{k1} \\ T_{11}^{eff} &= V_{11}^{eff} + V_{11}^{eff} G_1 T_{11}^{eff} \end{aligned}$$

[4] T. Hyodo, W. Weise, Phys. Rev. C 77 (2008) 035204

CHIRAL $\bar{K}N$ INTERACTION

The $I = 0$ coupled channel scattering amplitudes[5]



There are 2 poles in the scattering amplitude

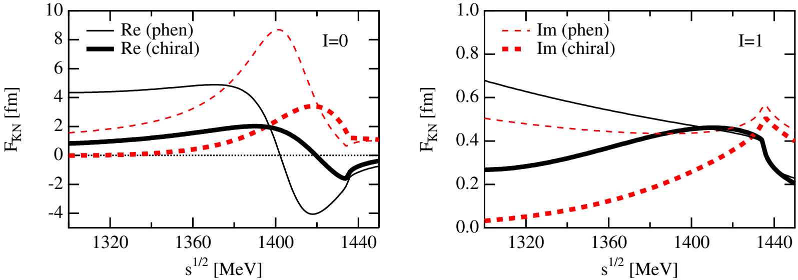
$$z_1 = 1428 - 17i \text{ MeV}$$

$$z_2 = 1400 - 76i \text{ MeV}$$

The first pole is dominated by $\bar{K}N$ QBS at $\sqrt{s} \approx 1420$ MeV.
The second pole is dominated by $\pi\Sigma$ QBS at $\sqrt{s} \approx 1405$ MeV.

[5] T. Hyodo, W. Weise, Phys. Rev. C 77 (2008) 035204

Chiral vs Phenomenological $\bar{K}N$ Interaction



Conclusions:

Binding Energies [MeV]

	Phenomenological	Chiral
$\bar{K}p$	27	10 – 13
$\bar{K}pp$	50 – 100	7 – 23
$\bar{K}ppp$	$\approx$ 100	?
$\bar{K}\bar{K}pp$	$\approx$ 120	?

Interactions

- *NN* - We used the Argonne AV4' potential [6] derived from the full AV18 potential.

$$V_{NN}(r) = V_0(r) + \sigma_1 \cdot \sigma_2 V_\sigma(r) + \tau_1 \cdot \tau_2 V_\tau(r) + (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) V_{\sigma\tau}(r)$$

- $\bar{K}N$ - Chiral SU(3) HNJH effective interaction[7],

$$V^{(I)}_{\bar{K}N}(r;\sqrt{s}) = V^{(I)}_{\bar{K}N}(\sqrt{s}) \exp(-r^2/b^2) \ ; \ \ b = 0.47 \text{ fm}$$

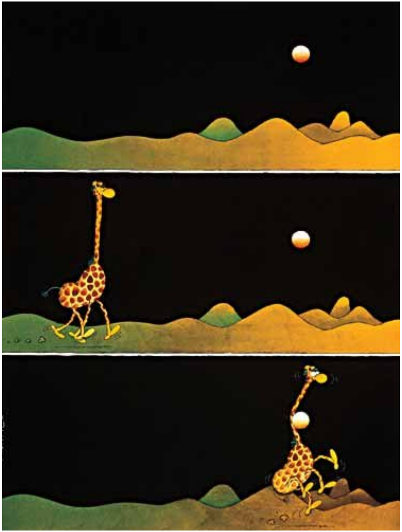
- $\bar{K}\bar{K}$[8] -

$$V^{(I)}_{\bar{K}\bar{K}}(r) = V^{(I)}_{\bar{K}\bar{K}} \exp(-r^2/b^2) \ ; \ \ b = 0.47 \text{ fm}$$

$V^{(I=0)}_{\bar{K}\bar{K}} = 0$ at low energies where *s* waves dominate.
$V^{(I=1)}_{\bar{K}\bar{K}} = 313 \text{ MeV}$ fitted to the *s*-wave scattering length.

[6] R.B. Wiringa, S.C. Pieper, Phys. Rev. Lett. 89 (2002) 182501
[7] T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Phys. Rev. C 68 (2003) 018201
[8] Y. Kanada-Eńyo, D. Jido, Phys. Rev. C 78 (2008) 025212

Solving the Schrödinger equation
The Hyperspherical Harmonics Expansion

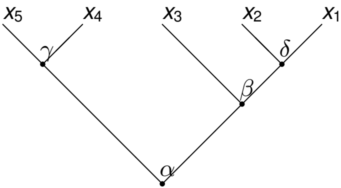


The Hyperspherical Harmonics

1. The HH were introduced in 1935 by Zernike and Brinkman.
2. They were reintroduced 25 years later by Delves and Smith.
3. In the 1970 Reynal and Revai derived the HH transformation coefficients.
4. and in 1972 Kil'dushov derives the HH recoupling coefficients.

$$\begin{aligned} x_1 &= \rho \cos(\alpha)\cos(\beta)\cos(\delta) \\ x_2 &= \rho \cos(\alpha)\cos(\beta)\sin(\delta) \\ x_3 &= \rho \cos(\alpha)\sin(\beta) \\ x_4 &= \rho \sin(\alpha)\cos(\gamma) \\ x_5 &= \rho \sin(\alpha)\sin(\gamma) \end{aligned}$$

The "Tree" diagram



THE HYPERSPHERICAL HARMONICS

1. Hyperspherical coordinates
   $x_1, x_2, x_3, \ldots x_D \longrightarrow \rho = \sqrt{\sum x_i^2}, \Omega$

$$
\begin{aligned}
x_1 &= \rho\cos(\alpha)\cos(\beta)\cos(\delta) \\
x_2 &= \rho\cos(\alpha)\cos(\beta)\sin(\delta) \\
x_3 &= \rho\cos(\alpha)\sin(\beta) \\
x_4 &= \rho\sin(\alpha)\cos(\gamma) \\
x_5 &= \rho\sin(\alpha)\sin(\gamma)
\end{aligned}
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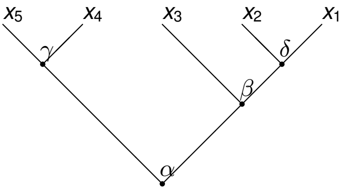
The "Tree" diagram



THE HYPERSPHERICAL HARMONICS

1. Hyperspherical coordinates
   $x_1, x_2, x_3, ...x_D \longrightarrow \rho = \sqrt{\sum x_i^2}, \Omega$
2. In hyperspherical coordinates

$$\Delta = \frac{\partial^2}{\partial \rho^2} + \frac{D-1}{\rho}\frac{\partial}{\partial \rho} - \frac{\hat{K}^2}{\rho^2}$$

$$\begin{aligned}
x_1 &= \rho \cos(\alpha)\cos(\beta)\cos(\delta) \\
x_2 &= \rho \cos(\alpha)\cos(\beta)\sin(\delta) \\
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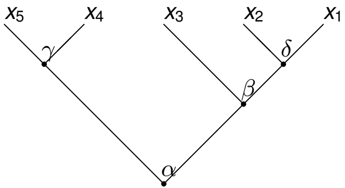

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   $x_1, x_2, x_3, \ldots x_D \longrightarrow \rho = \sqrt{\sum x_i^2}, \Omega$
2. In hyperspherical coordinates

$$\Delta = \frac{\partial^2}{\partial \rho^2} + \frac{D-1}{\rho}\frac{\partial}{\partial \rho} - \frac{\hat{K}^2}{\rho^2}$$

3. $\rho^K \mathcal{Y}_{[K]}(\Omega)$ is a Harmonic polynomial.

$$\begin{aligned} x_1 &= \rho \cos(\alpha)\cos(\beta)\cos(\delta) \\ x_2 &= \rho \cos(\alpha)\cos(\beta)\sin(\delta) \\ x_3 &= \rho \cos(\alpha)\sin(\beta) \\ x_4 &= \rho \sin(\alpha)\cos(\gamma) \\ x_5 &= \rho \sin(\alpha)\sin(\gamma) \end{aligned}$$

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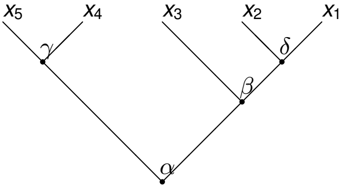

The Hyperspherical Harmonics

1. Hyperspherical coordinates
   $x_1, x_2, x_3, \ldots x_D \longrightarrow \rho = \sqrt{\sum x_i^2}, \Omega$
2. In hyperspherical coordinates

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3. $\rho^K \mathcal{Y}_{[K]}(\Omega)$ is a Harmonic polynomial.
4. The HH are eigenstates of $\hat{K}^2$

$$\hat{K}^2 \mathcal{Y}(\Omega) = K(K+D-2)\mathcal{Y}(\Omega)$$

$$\begin{aligned}
x_1 &= \rho \cos(\alpha)\cos(\beta)\cos(\delta)\\
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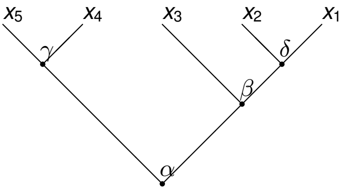
The "Tree" diagram



THE HYPERSPHERICAL HARMONICS

1. Hyperspherical coordinates
   $x_1, x_2, x_3, \ldots x_D \longrightarrow \rho = \sqrt{\sum x_i^2}, \Omega$
2. In hyperspherical coordinates

$$\Delta = \frac{\partial^2}{\partial \rho^2} + \frac{D-1}{\rho}\frac{\partial}{\partial \rho} - \frac{\hat{K}^2}{\rho^2}$$

3. $\rho^K \mathcal{Y}_{[K]}(\Omega)$ is a Harmonic polynomial.
4. The HH are eigenstates of $\hat{K}^2$

$$\hat{K}^2 \mathcal{Y}(\Omega) = K(K + D - 2)\mathcal{Y}(\Omega)$$

5. Using the tree structure one can easily construct HH starting from the leafs and uniting branches.

$$\begin{aligned} x_1 &= \rho \cos(\alpha)\cos(\beta)\cos(\delta) \\ x_2 &= \rho \cos(\alpha)\cos(\beta)\sin(\delta) \\ x_3 &= \rho \cos(\alpha)\sin(\beta) \\ x_4 &= \rho \sin(\alpha)\cos(\gamma) \\ x_5 &= \rho \sin(\alpha)\sin(\gamma) \end{aligned}$$

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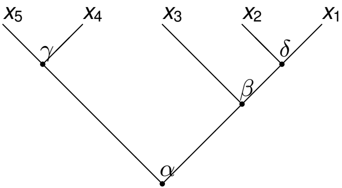

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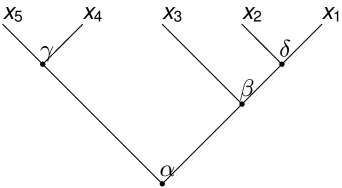
6. Each junction is associated with a quantum number.

$$\begin{aligned} x_1 &= \rho\cos(\alpha)\cos(\beta)\cos(\delta) \\ x_2 &= \rho\cos(\alpha)\cos(\beta)\sin(\delta) \\ x_3 &= \rho\cos(\alpha)\sin(\beta) \\ x_4 &= \rho\sin(\alpha)\cos(\gamma) \\ x_5 &= \rho\sin(\alpha)\sin(\gamma) \end{aligned}$$

The "Tree" diagram



The Hyperspherical Harmonics

1. Hyperspherical coordinates
   $x_1, x_2, x_3, \dots x_D \longrightarrow \rho = \sqrt{\sum x_i^2}, \Omega$
2. In hyperspherical coordinates

$$\Delta = \frac{\partial^2}{\partial \rho^2} + \frac{D-1}{\rho}\frac{\partial}{\partial \rho} - \frac{\hat{K}^2}{\rho^2}$$

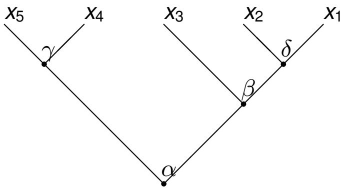
3. $\rho^K \mathcal{Y}_{[K]}(\Omega)$ is a Harmonic polynomial.
4. The HH are eigenstates of $\hat{K}^2$

$$\hat{K}^2 \mathcal{Y}(\Omega) = K(K+D-2)\mathcal{Y}(\Omega)$$

5. Using the tree structure one can easily construct HH starting from the leafs and uniting branches.
6. Each junction is associated with a quantum number.
7. Each junction adds a factor
   $N \cos^{K_R}(\theta) \sin^{K_L}(\theta) P^{(\alpha_R,\alpha_L)}_{(K-K_R-K_L)/2}(\cos(2\theta))$

$$
\begin{aligned}
x_1 &= \rho \cos(\alpha)\cos(\beta)\cos(\delta) \\
x_2 &= \rho \cos(\alpha)\cos(\beta)\sin(\delta) \\
x_3 &= \rho \cos(\alpha)\sin(\beta) \\
x_4 &= \rho \sin(\alpha)\cos(\gamma) \\
x_5 &= \rho \sin(\alpha)\sin(\gamma)
\end{aligned}
$$

The “Tree” diagram



Removing the center of mass - The Jacobi coordinates

The 3-body case

$$\vec{\eta}_1 = \sqrt{\frac{M_1 M_2}{M_{12} m}}\,(\vec{r}_2 - \vec{r}_1)$$
$$\vec{\eta}_2 = \sqrt{\frac{M_{12} M_3}{M_{123} m}}\left(\vec{r}_3 - \frac{M_1\vec{r}_1 + M_2\vec{r}_2}{M_{12}}\right)$$

The 4-body case

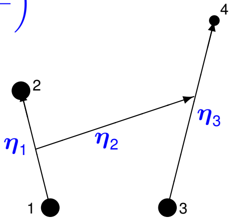
$$\vec{\eta}_1 = \sqrt{\frac{M_1 M_2}{M_{12} m}}\,(\vec{r}_2 - \vec{r}_1)$$
$$\vec{\eta}_2 = \sqrt{\frac{M_{12} M_{34}}{M_{1234} m}}\left(\frac{M_3\vec{r}_3 + M_4\vec{r}_4}{M_{34}} - \frac{M_1\vec{r}_1 + M_2\vec{r}_2}{M_{12}}\right)$$
$$\vec{\eta}_3 = \sqrt{\frac{M_3 M_4}{M_{34} m}}\,(\vec{r}_4 - \vec{r}_3)\,,$$

$$M_{ij} = M_i + M_j$$

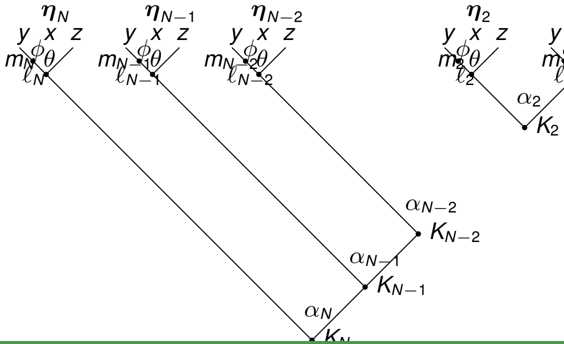
$$M_{123} = M_1 + M_2 + M_3$$

$$M_{1234} = M_1 + M_2 + M_3 + M_4$$

m is an arbitrary mass, we take $m = m_N$.



THE COMMON "TREE"



$$\mathcal{Y}_{[K]} = \left[\prod_{j=1}^{N} Y_{\ell_j,\, m_j}(\hat{\eta}_j)\right]$$

$$\times \left[\prod_{j=2}^{N} \mathcal{N}_{j,K_j}^{\ell_j,K_{j-1}} (\sin\alpha_j)^{\ell_j} (\cos\alpha_j)^{K_{j-1}} P_{\mu_j}^{(\ell_j+\frac{1}{2},K_{j-1}+\frac{3j-5}{2})}(\cos(2\alpha_j))\right]$$

THE MERITS OF THE HH EXPANSION

- A complete set of basis functions.

$$\sum_{[K]} \mathcal{Y}^*_{[K]}(\Omega')\mathcal{Y}_{[K]}(\Omega)\frac{\delta(\rho-\rho')}{\rho^{D-1}} = \prod_{i=1}^{N} \delta(\boldsymbol{\eta}_i - \boldsymbol{\eta}'_i)$$

- Easy transformation between configuration and momentum space

$$e^{i\sum \boldsymbol{\eta}_j \boldsymbol{q}_j} = \frac{(2\pi)^{D/2}}{(Q\rho)^{D/2-1}} \sum_{[K]} i^K \mathcal{Y}^*_{[K]}(\Omega_q)\mathcal{Y}_{[K]}(\Omega) J_{K+D/2-1}(Q\rho)$$

- Good asymptotics.
- With appropriate choice of Jacobi coordinates and states clusterization can be ”easily” treated.

The HH expansion in 4 steps

1. Remove the center of mass

$$\vec{r}_1, \vec{r}_2, \ldots \vec{r}_A \longrightarrow \vec{R}_{c.m.}, \vec{\eta}_1 \vec{\eta}_2 \ldots \vec{\eta}_{A-1}$$

2. Introduce hyperspherical coordinates

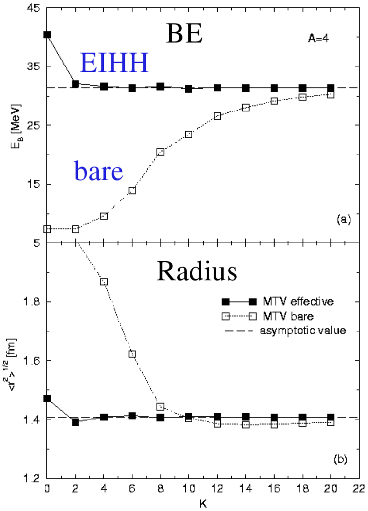
$$\vec{\eta}_1 \vec{\eta}_2 \ldots \vec{\eta}_{A-1} \longrightarrow \rho = \sqrt{\eta_1^2 + \eta_2^2 + \ldots + \eta_{A-1}^2}, \Omega$$

3. Expand the wave function using hyperspherical harmonics

$$\Psi(\rho, \Omega) = \sum_{K \leq K_{max}} R_{[K]}(\rho) \mathcal{Y}_{[K]}(\Omega)$$

4. Solve the Schrödinger equation $H\Psi = E\Psi$,

$$H = -\frac{1}{2} \left(\frac{\partial^2}{\partial \rho^2} + \frac{3A-4}{\rho} \frac{\partial}{\partial \rho} - \frac{\hat{K}^2}{\rho^2} \right) + \sum_{i<j} V_{ij} + \left[\sum_{i<j<k} V_{ijk} \right]$$



BE
A=4
EIHH
bare
(a)
Radius
MTV effective
MTV bare
asymptotic value
(b)
E_B [MeV]
<r²>^1/2 [fm]
K
45
35
25
15
5
1.8
1.6
1.4
1.2
0
2
4
6
8
10
12
14
16
18
20
22

PHYSICAL REVIEW C, VOLUME 64, 044001

Benchmark test calculation of a four-nucleon bound state

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(Received 20 April 2001; published 27 August 2001)

BENCHMARK FOR ^{4}HE GROUND STATE WITH AV8' POTENTIAL

H. KAMADA *et al.*, PRC 64 044001 (2001)

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	-128.33(10)	-25.94(5)	1.485(3)
CRCGV	102.25	-128.13	-25.90	1.482
SVM	102.35	-128.27	-25.92	1.486
HH	102.44	-128.34	-25.90(1)	1.483
GFMC	102.3(1.0)	-128.25(1.0)	-25.93(2)	1.490(5)
NCSM	103.35	-129.45	-25.80(20)	1.485
EIHH	100.8(9)	-126.7(9)	-25.944(10)	1.486

For a single $\bar{K}$ meson bound together with A define $\sqrt{s}_{\rm av}$ by

$$A\sqrt{s}_{\rm av} = \sum_{i=1}^{A} \sqrt{(E_K + E_i)^2 - (\vec{p}_K + \vec{p}_i)^2} \ ,$$

approximating it near threshold, $\sqrt{s_{\rm th}} \equiv m_N + m_K$ MeV, by

$$A\sqrt{s}_{\rm av} \approx A\sqrt{s_{\rm th}} - B - (A-1)B_K - \sum_{i=1}^{A} (\vec{p}_K + \vec{p}_i)^2 / 2E_{\rm th} \ ,$$

where B is the total binding energy of the system and $B_K = -E_K$.
Note that $\sqrt{s}_{\rm av} \approx \sqrt{s_{\rm th}} + \delta\sqrt{s}$ with $\delta\sqrt{s} < 0$.

$$\langle \delta\sqrt{s} \rangle = -\frac{B}{A} - \frac{A-1}{A} B_K - \xi_N \frac{A-1}{A} \langle T_{N:N} \rangle - \xi_K \left(\frac{A-1}{A} \right)^2 \langle T_K \rangle \ ,$$

$$\xi_N \equiv m_N/(m_N + m_K) \qquad \xi_K \equiv m_K/(m_N + m_K)$$

T_K - the $\bar{K}$ K.E., $T_{N:N}$ - the pairwise NN K.E.

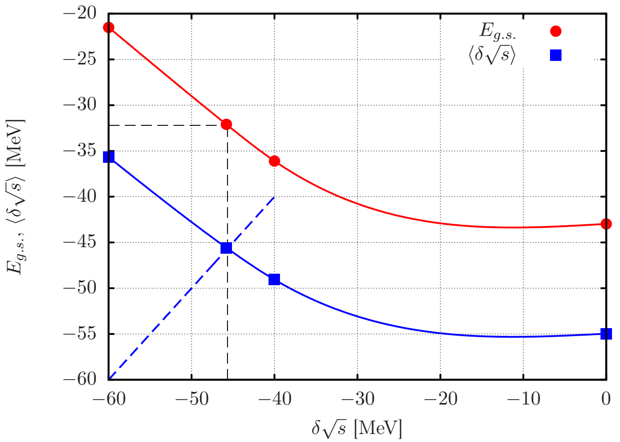
For $\bar{K}\bar{K}NN$

$$\langle \delta\sqrt{s} \rangle = -\frac{1}{2}\left(B + \xi_N \langle T_{N:N} \rangle + \xi_K \langle T_{K:K} \rangle\right)$$

$T_{K:K}$ is the pairwise $\bar{K}\bar{K}$ kinetic energy in c.m. frame.

In the limit $A \gg 1$, the K-nuclei $\sqrt{s}$ expansion coincides with the nuclear-matter formula[9].

[9] A. Cieplý, E. Friedman, A. Gal, D. Gazda, J. Mareš, Phys. Lett. B 702 (2011) 402, Phys. Rev. C 84 (2011) 045206

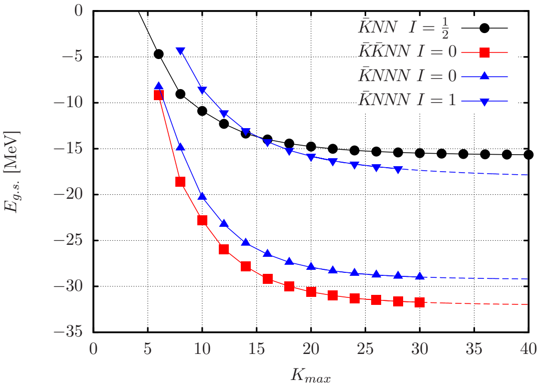


Self-consistency construction in $(\bar{K}\bar{K}NN)_{I=0,J^\pi=0^+}$ B.E. calculations.

Ground state energies

Convergence

Ground-state energies of $\bar{K}$ nuclear clusters, calculated self consistently, as a function of $K_{\rm max}$.



Asymptotic values of $E_{\rm g.s.}$ are found by fitting the formula

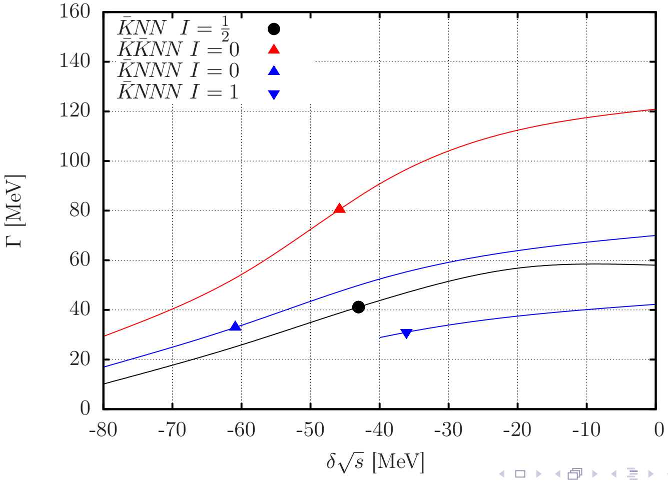
$$E(K_{\rm max}) = E_{\rm g.s.} + \frac{C}{K_{\rm max}^{\gamma}}$$

Conversion widths

Conversion widths Γ of $\bar{K}$ nuclear clusters calculated from

$$\Gamma = -2 \langle \Psi_{\rm g.s.} | {\rm Im}\, \mathcal{V}_{\bar{K}N} | \Psi_{\rm g.s.} \rangle ,$$

as a function of $\delta\sqrt{s}$.



$\bar{K}N$ and $\bar{K}NN$

Comparison of $\bar{K}N$ and $\bar{K}NN$ QBS calculations.

QBS	I, J^π	Ref.	$\langle\delta\sqrt{s}\rangle$ [MeV]	B [MeV]	Γ [MeV]	B_K [MeV]	r_{NN} [fm]	r_{KN} [fm]
$\bar{K}N$	$0, \frac{1}{2}^-$	BGL	-11.4	11.4	43.6	11.4	–	1.87
		DHW	-11.5	11.5	$43.8^\dagger$	11.5	–	1.86
$\bar{K}NN$	$\frac{1}{2}, 0^-$	BGL	-43	15.7	41.2	35.5	2.41	2.15
		DHW	-39	16.9	47.0	38.9	2.21	1.97
$\bar{K}NN$	$\frac{1}{2}, 0^-$	BGL	-35	11.0	38.8	27.9	2.33	2.21
$I_N = 1$		DHW	-31	12.0	44.8	31.0	2.13	2.01

DHW[10], BGL[11]

[10] A. Doté, T. Hyodo, W. Weise, Nucl. Phys. A 804 (2008) 197, Phys. Rev. C 79 (2009) 014003

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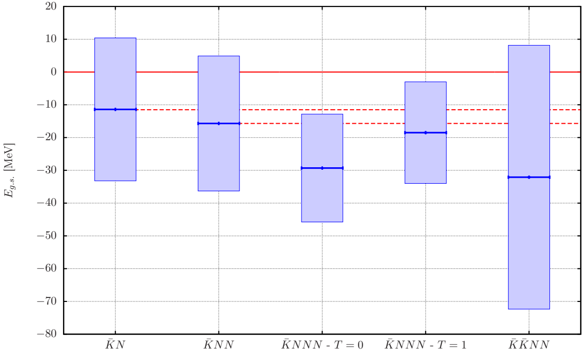
$\bar{K}NNN$ and $\bar{K}\bar{K}NN$

Results of $\bar{K}NNN$ and $\bar{K}\bar{K}NN$ QBS calculations.

QBS	I, J^π	$\langle\delta\sqrt{s}\rangle$ [MeV]	B [MeV]	Γ [MeV]	B_K [MeV]	r_{NN} [fm]	r_{NK} [fm]	r_{KK} [fm]
$\bar{K}NNN$	$0, \frac{1}{2}^+$	-61	29.3	32.9	36.6	2.07	2.05	–
	$1, \frac{1}{2}^+$	-36	18.5	31.0	21.0	2.33	2.55	–
$\bar{K}\bar{K}NN$	$0, 0^+$	-46	32.1	80.5	33.6	1.84	1.88	2.31
$V_{\bar{K}\bar{K}} = 0$		-52	36.1	83.2	37.9	1.71	1.70	2.01

Results of $\bar{K}NNN$ and $\bar{K}\bar{K}NN$ QBS calculations.

QBS	I, J^π	$\langle\delta\sqrt{s}\rangle$ [MeV]	B [MeV]	Γ [MeV]	B_K [MeV]	r_{NN} [fm]	r_{NK} [fm]	r_{KK} [fm]
$\bar{K}NNN$	$0, \frac{1}{2}^+$	-61	29.3	32.9	36.6	2.07	2.05	–
	$1, \frac{1}{2}^+$	-36	18.5	31.0	21.0	2.33	2.55	–
$\bar{K}\bar{K}NN$	$0, 0^+$	-46	32.1	80.5	33.6	1.84	1.88	2.31
$V_{\bar{K}\bar{K}} = 0$		-52	36.1	83.2	37.9	1.71	1.70	2.01



Conclusions

1. We have performed calculations of three-body $\bar{K}NN$ and four-body $\bar{K}NNN$ and $\bar{K}\bar{K}NN$ QBS systems.
2. For $K^- pp$ we confirmed the results of Doté et al. [12].
3. For $\bar{K}NNN$ and $\bar{K}\bar{K}NN$ we found $B \approx 30$ MeV in both case.
4. The widths are $\Gamma_{\bar{K}NNN} \approx 30$ MeV and $\Gamma_{\bar{K}\bar{K}NN} \approx 80$ MeV, without 3-body absorption.
5. These systems, are not as compact as suggested by Yamazaki et al. [13].
6. The energy dependence of the subthreshold $\bar{K}N$ potential[14] is restraining the binding of the 4-body systems.

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