

# **Electroproduction of medium-heavy $\Lambda$ -hypernuclei**

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EU SPHERE Network Meeting***

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# CONTENTS

- 1. Basic motivations** for medium-heavy systems
- 2. Go to sd-shell:** “Simplest” target ( $^{19}\text{F}$ ) and the  $(d_{5/2})^6$  model to demonstrate the present theoretical treatments.
- 3. Realistic predictions** for photo-production reaction with a typical target  $^{28}\text{Si}(\gamma, \text{K}^+)_{\Lambda}^{28}\text{Al}$
- 4. Extension** of the approach to produce heavier hypernuclei around A=40-52
5. Outlook

# 1. Basic motivation:

## *Why medium-heavy hypernuclei ?*

(1)

The great success of Hall A and Hall C experiments at Jlab:

- sub-MeV ( $\Gamma$ = approx. 0.5 MeV)
- predictions for p-shell confirmed

encourages

extension of high-resolution reaction spectroscopy to heavier hypernuclei:

$(K^-, \pi^-)$

$(\pi^+, K^+)$

played a great role of exciting high-spin series

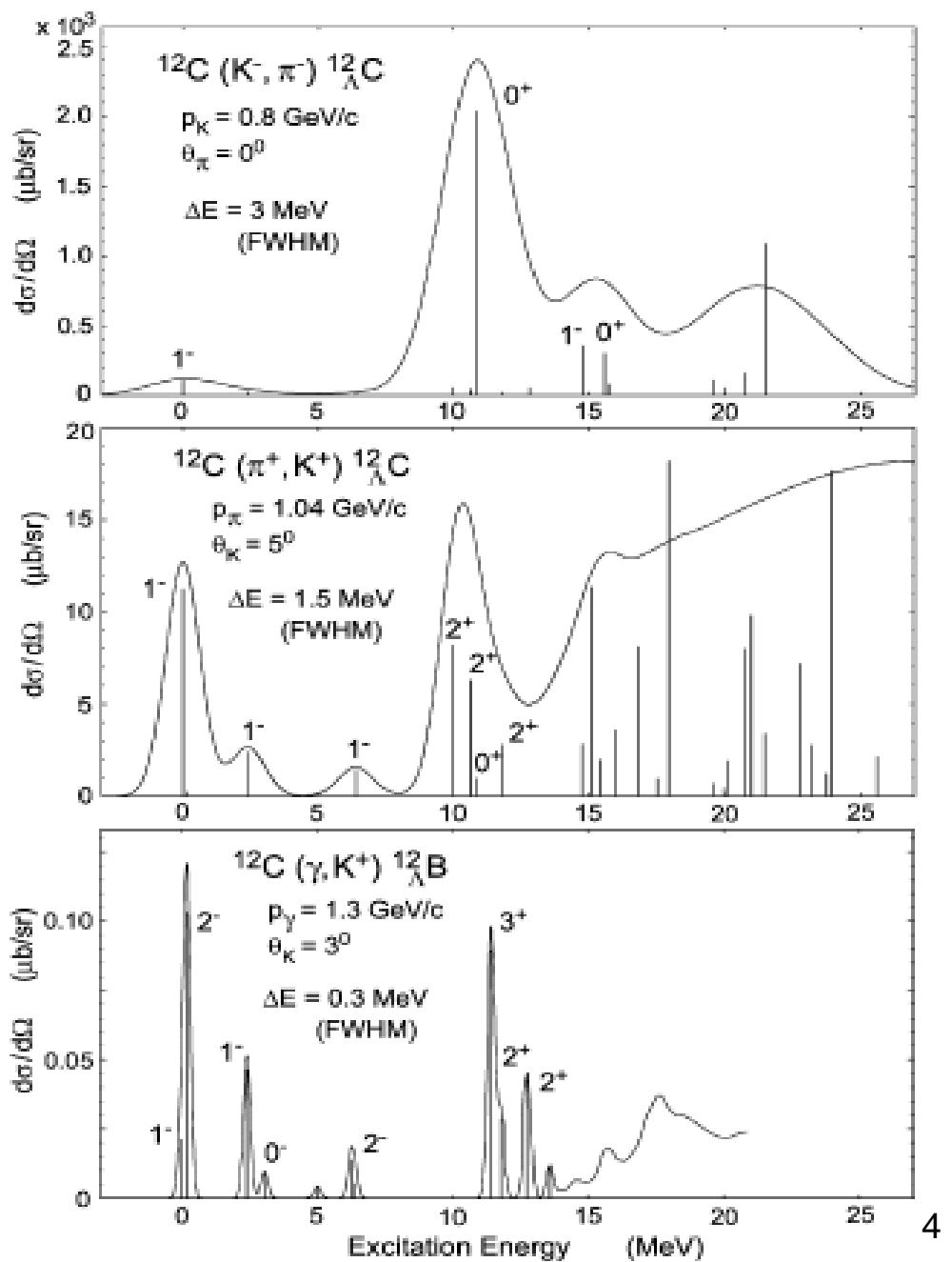
$\Gamma = 1.5 \text{ MeV (best)}$

$(e, e' K^+), (\gamma, K^+)$

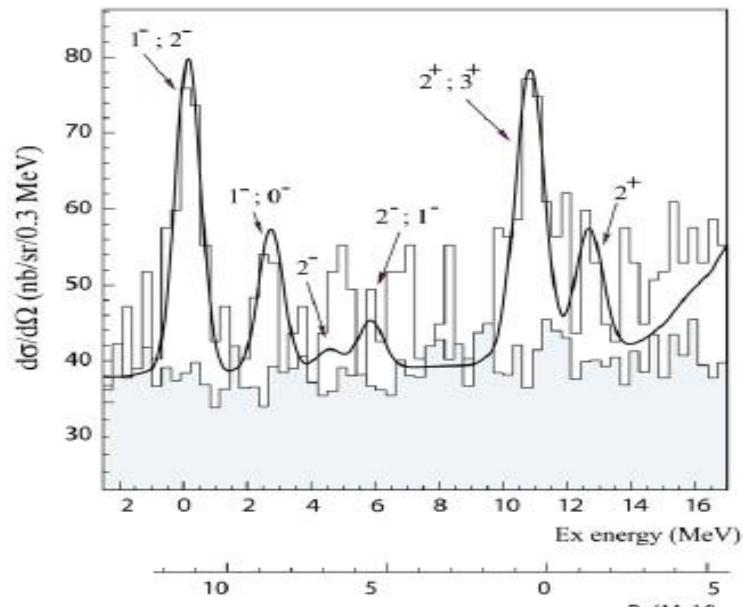
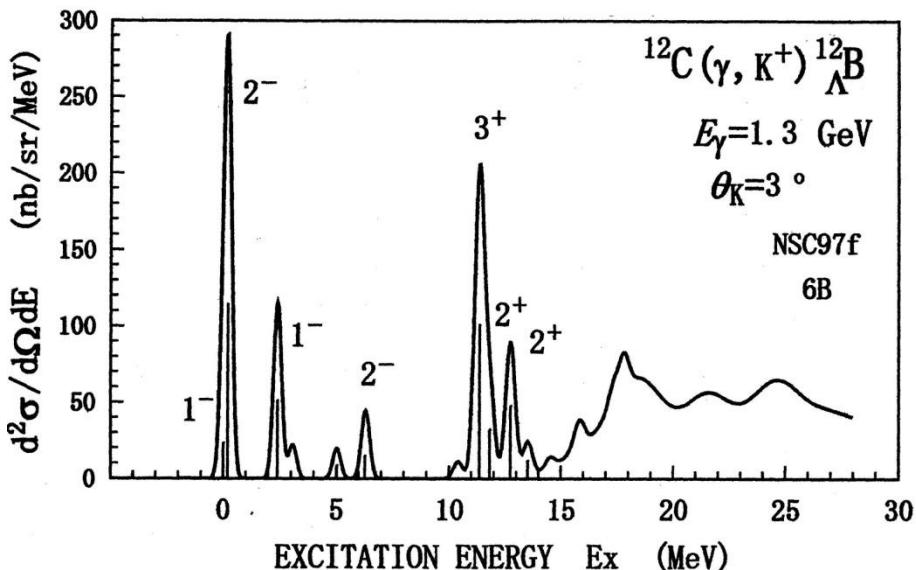
Motoba, Sotona, Itonaga,  
Prog.Theor.Phys.S.117(1994)

T.M. Mesons & Light Nuclei  
(2000) updated w/NSC97f.

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**JLab Exp't :  $\Gamma = 0.5 \text{ MeV}$**



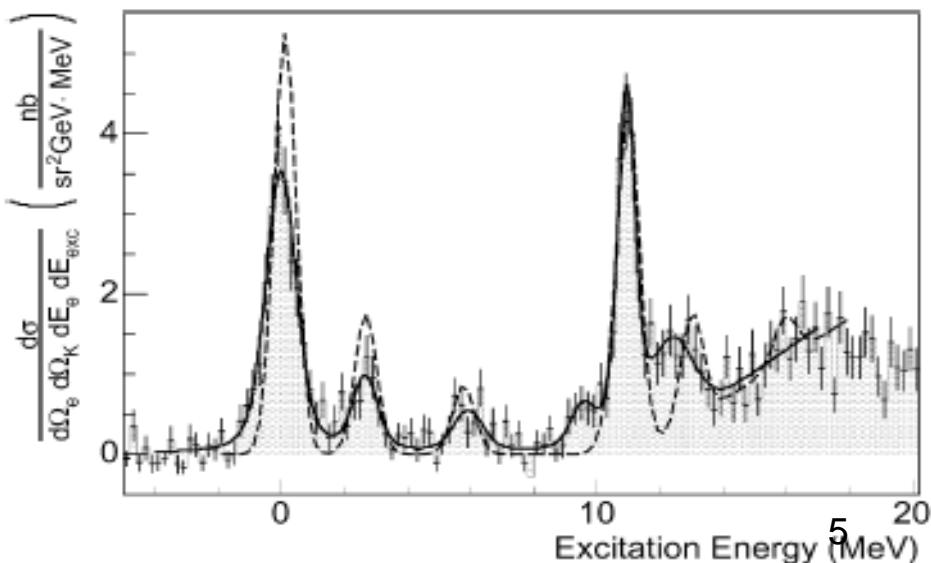
# Theor. prediction vs. ( $e, e' K^+$ ) experiments



Motoba, Sotona, Itonaga,  
*Prog. Theor. Phys. Sup.* **117** (1994)  
 T.M. Mesons & Light Nuclei (2000)  
 updated w/NSC97f.

----- Sotona's Calc. ----→

Hall C (up) T. Miyoshi et al.  
*P.R.L.* **90** (2003) 232502.  $\Gamma=0.75 \text{ MeV}$   
 Hall A (bottom), J.J. LeRose et al.  
*N.P.* **A804** (2008) 116.  $\Gamma=0.67 \text{ MeV}$



# *Why medium-heavy hypernuclei ?*

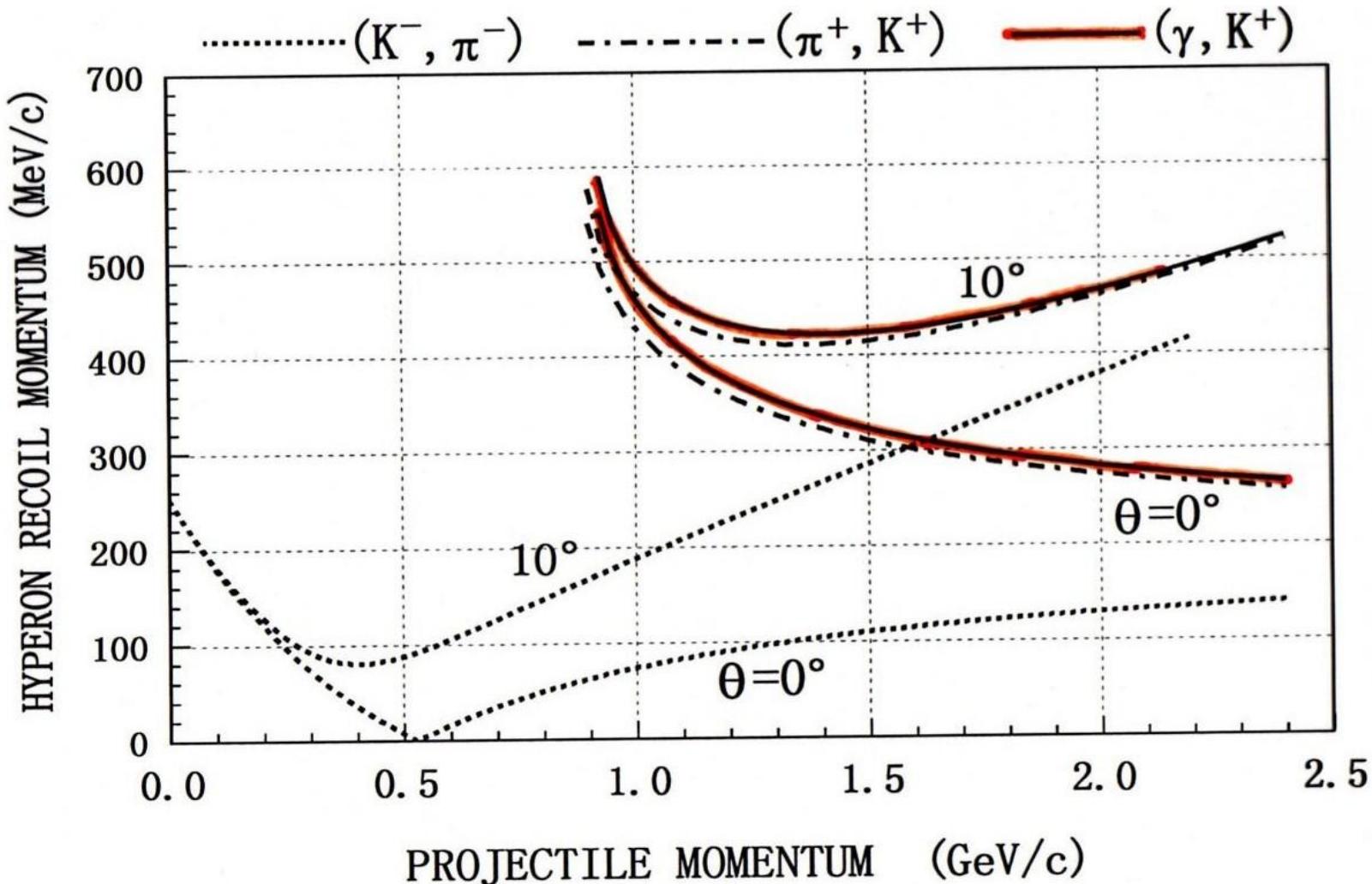
## **motivation (2)**

Unique characteristics of the  $(e, e' K^+)$ ,  $(\gamma, K^+)$  process are based on the **basic Properties** of elementary amplitudes for  $\gamma p \rightarrow \Lambda K^+$  :

- **sizable momentum transfer** to excite high-spin states, like  $(\pi^+, K^+)$
- **spin-flip dominance** of the operator, leading to unnatural parity states

# Comparison of the recoil momentum

$q_\Lambda = 350-420 \text{ MeV/c}$  at  $E\gamma = 1.3 \text{ GeV}$



# Elementary amplitudes (2CM): $\gamma p \rightarrow \Lambda K^+$

$$\mathcal{M} \equiv \frac{(2\pi)^2}{\sqrt{s}} \sqrt{e_r e_p e_\Lambda e_K} \langle q, -q | t | \kappa, -\kappa \rangle$$

$$= iF_1(\sigma \cdot \epsilon) + F_2(\sigma \cdot \hat{q})[\sigma \cdot (\hat{\kappa} \times \epsilon)] + iF_3(\sigma \cdot \hat{\kappa})(\hat{q} \cdot \epsilon) + iF_4(\sigma \cdot \hat{q})(\hat{q} \cdot \epsilon).$$

$$\begin{aligned} \left. \frac{d\sigma}{d\Omega} \right|_{\text{2CM}} &= \frac{q}{\kappa} \left\{ |F_1|^2 + |F_2|^2 - 2 \cos \theta_{\text{CM}} \operatorname{Re}(F_1^* F_2) \right. \\ &\quad \left. + \frac{1}{2} \sin^2 \theta_{\text{CM}} [|F_3|^2 + |F_4|^2 + \operatorname{Re}(F_1^* F_4 + F_2^* F_3 + F_3^* F_4 \cos \theta_{\text{CM}})] \right\}, \end{aligned}$$

$$\begin{aligned} \mathcal{P}_Y \left. \frac{d\sigma}{d\Omega} \right|_{\text{2CM}} &= \frac{q}{\kappa} \sin \theta_{\text{CM}} \operatorname{Im} \left\{ -2F_1^* F_2 - F_1^* F_3 + F_2^* F_4 + \sin^2 \theta_{\text{CM}} F_3^* F_4 \right. \\ &\quad \left. + \cos \theta_{\text{CM}} [F_2^* F_3 - F_1^* F_4] \right\}, \end{aligned}$$

# Lab $d\sigma/d\Omega$ for photoproduction (2Lab)

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{2Lab}} = \frac{(2\pi)^4 p^2 E_K E_\gamma E_A}{k\{p(E_A + E_K) - kE_K \cos\theta_L\}} \left| \langle \mathbf{k} - \mathbf{p}, \mathbf{p} | t | \mathbf{k}, 0 \rangle_L \right|^2, \quad (2.4)$$

$$\langle \mathbf{k} - \mathbf{p}, \mathbf{p} | t | \mathbf{k}, 0 \rangle_L = a_1(\boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}) + a_2(\boldsymbol{\sigma} \cdot \hat{\mathbf{k}})(\hat{\mathbf{p}} \cdot \boldsymbol{\epsilon}) + a_3(\boldsymbol{\sigma} \cdot \hat{\mathbf{p}})(\hat{\mathbf{p}} \cdot \boldsymbol{\epsilon}) + a_4\{(\hat{\mathbf{k}} \times \hat{\mathbf{p}}) \cdot \boldsymbol{\epsilon}\}. \quad (2.5)$$

$$\langle \mathbf{k} - \mathbf{p}, \mathbf{p} | t | \mathbf{k}, 0 \rangle_L = \epsilon_0(f_0 + g_0\sigma_0) + \epsilon_x(g_1\sigma_1 + g_{-1}\sigma_{-1}) \quad (2.11)$$

with definitions of the coefficients:

$$f_0 = a_4 \sin\theta_L,$$

$$g_0 = a_1,$$

$$g_{\pm 1} = \frac{1}{\sqrt{2}} \{ \mp(a_1 + a_3 \sin^2\theta_L) - i \sin\theta_L (a_2 + a_3 \cos\theta_L) \}. \quad (2.12)$$

*These characteristic merits of the  
 $\gamma p \rightarrow \Lambda K^+$  process (ability to excite  
high-spin unnatural-parity states)  
should be realized better in heavier  
systems with large  $j_p$  and large  $j_\Lambda$*

$$(e, e' K^+) \quad d^3\sigma / dE_e d\Omega_e d\Omega_K = \Gamma \times \frac{d\sigma}{d\Omega_K}$$

$\Gamma$  : virtual photon flux (kinematics)

Hereafter we discuss  $\frac{d\sigma}{d\Omega_K}$  for  ${}^A Z (\gamma, K+) {}^A Z'$

## **2. Go to *sd*-shell:**

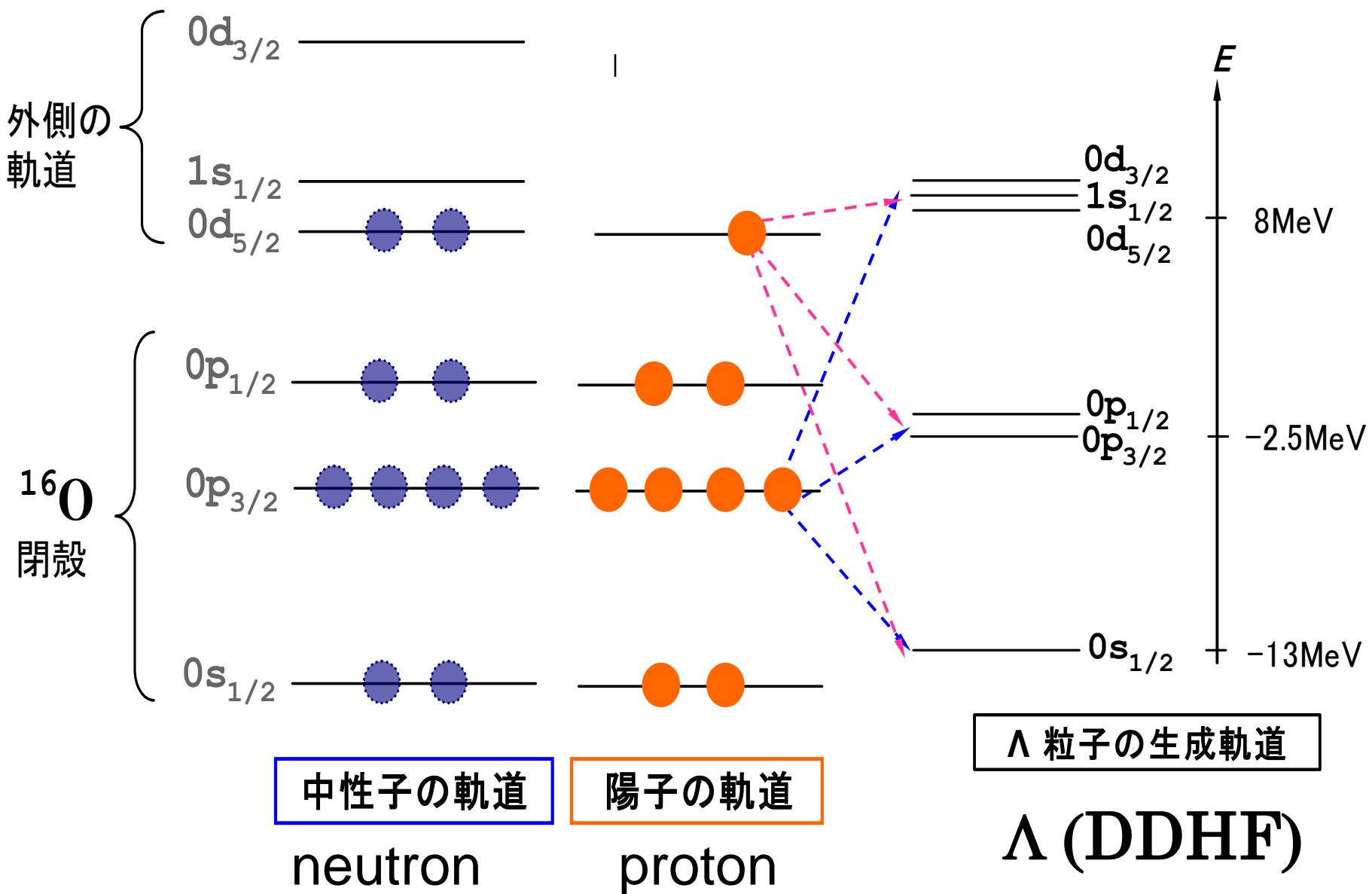
### **2-1. Simplest *sd*-shell target**

**Choose  $^{19}\text{F}(1/2^+)$  target**

for demonstration of the hypernuclear  
photoproduction

(asking the feasibility as a practical target )

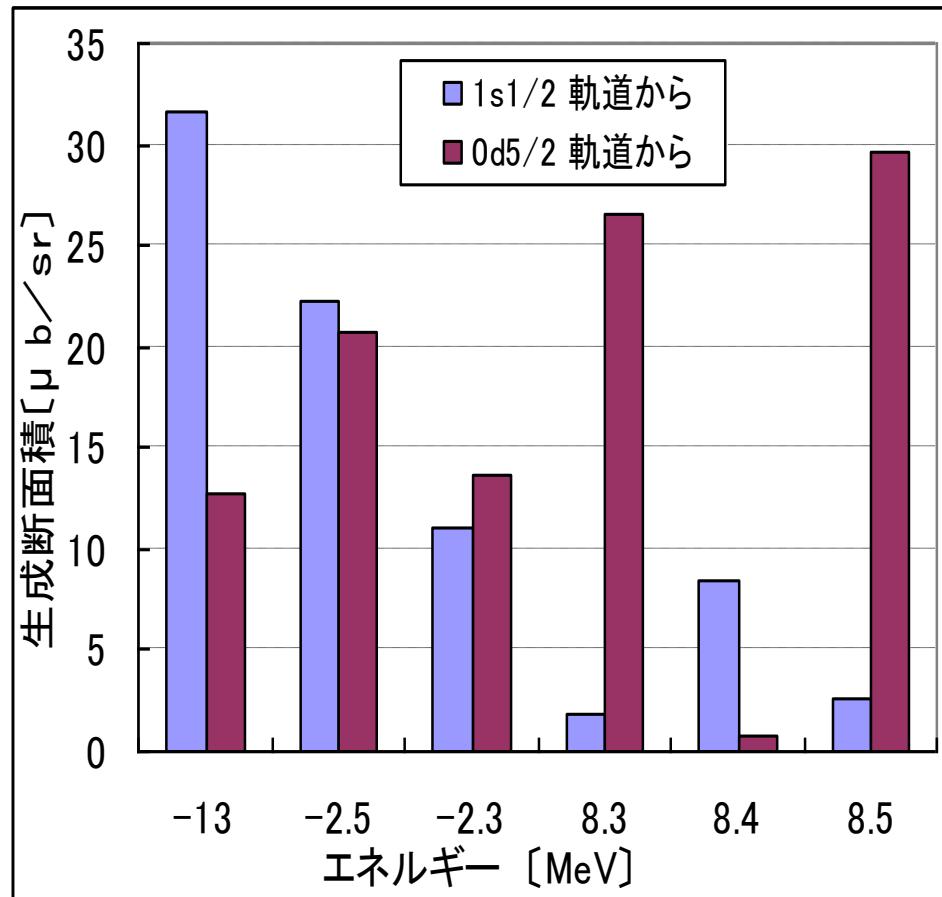
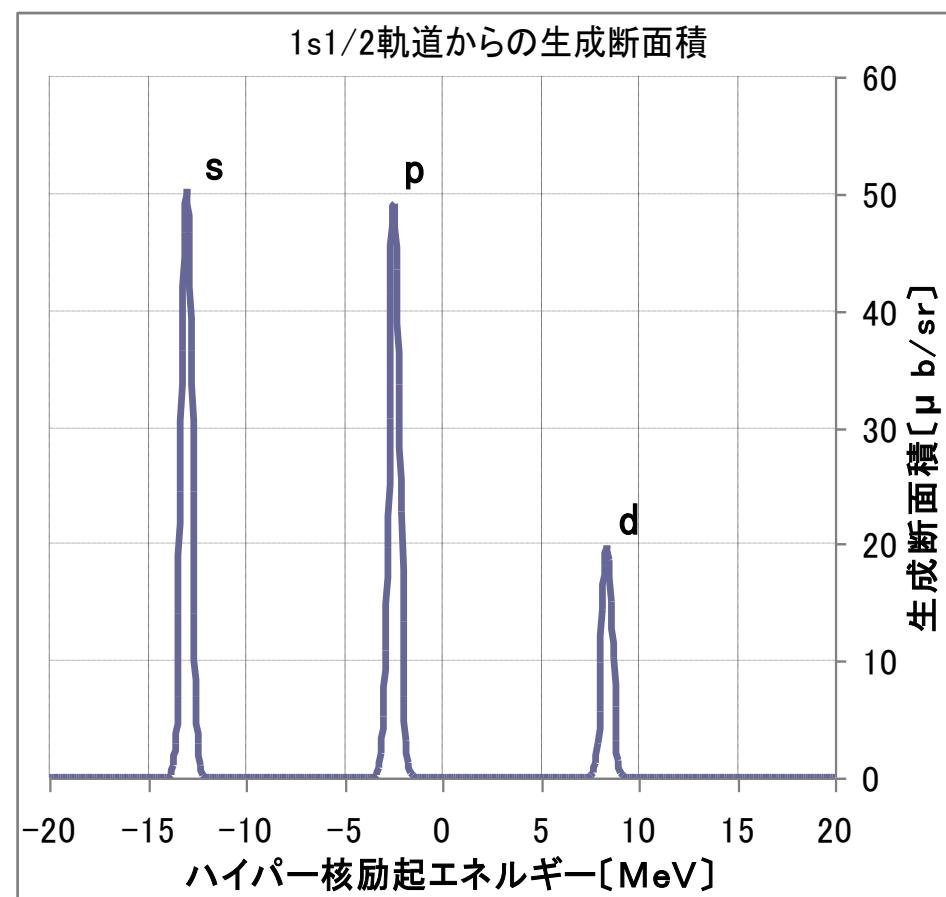
# Choose $^{19}\text{F}(1/2^+)$ target for demonstration



# Partial contributions

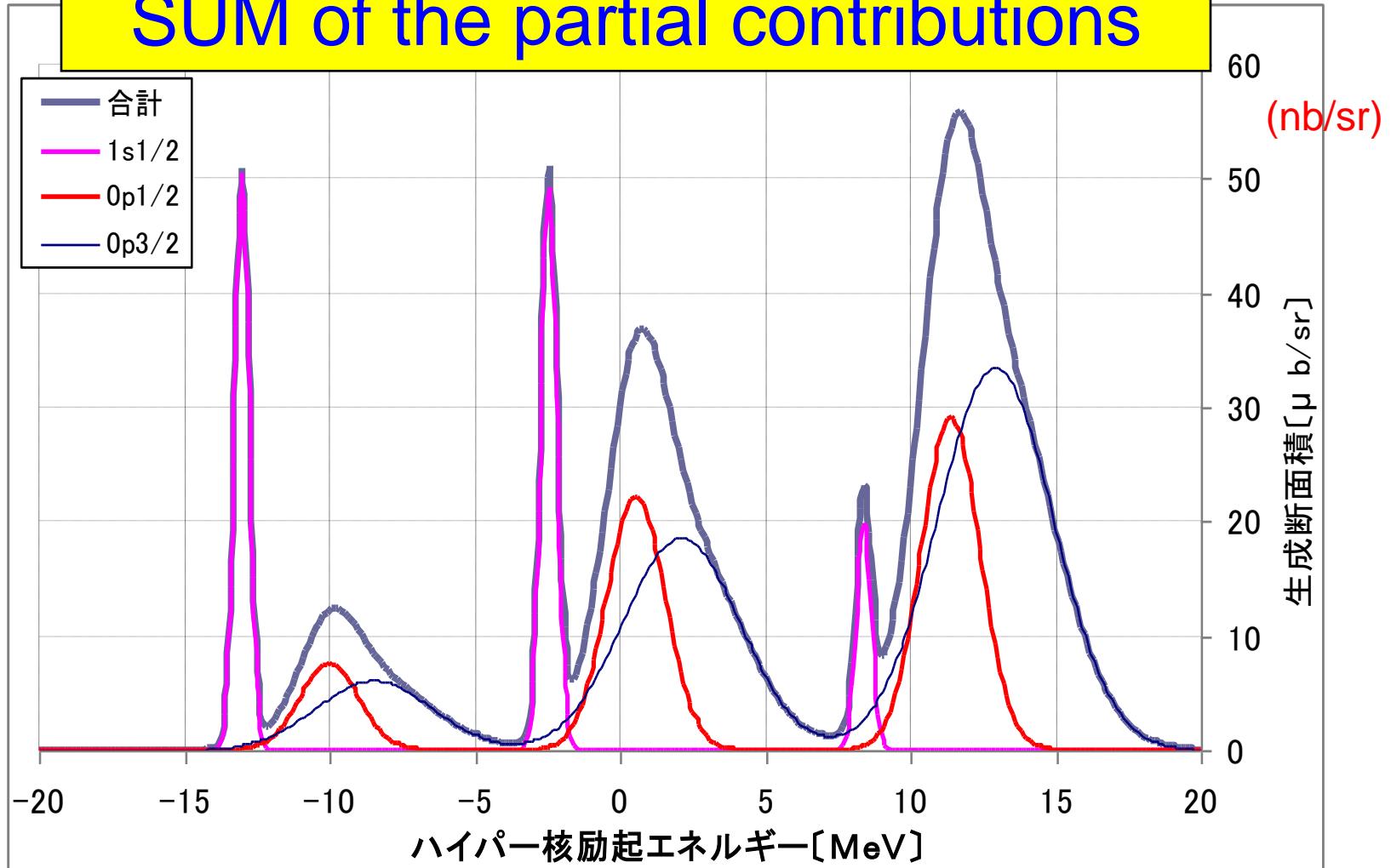
## Conversion of $1s_{1/2}$ -proton(nb/sr)

Cf. a trial calculation:  
If the last odd proton  
were in  $0d_{5/2}$ , then



XS in nb/sr ( correct units in the above figs.)

$^{19}\text{F}(\gamma, \text{K}^+) \Lambda \ ^{19}\text{O}$   
 SUM of the partial contributions



As a “closed core ( $^{18}\text{O}$ )”+  $\Lambda$ , cf. SO-splitting(0p)=152+-54 keV(C13)

# 2-1. Single- $j$ model for the $^{28}\text{Si}$ target

A typical example of medium-heavy  
target :  $^{28}\text{Si}$ :  $(d_{5/2})^6$

to show characteristics of  
the  $(\gamma, K^+)$  reaction with DDHF w.f.

( Spin-orbit splitting:  
consistent with  ${}_\Lambda^7\text{Li}$ ,  ${}^9\text{Be}$ ,  ${}^{13}\text{C}$ ,  ${}^{89}\text{Y}$  )

# Theor. x-section for $(d_{5/2})^6 (\gamma, K^+) [j_h - j_\Lambda] J$

DWIA		[ nb/sr ]					
Lambda=	s1/2L (-16.92)	p3/2L (-8.40)	p1/2L (-8.40)	1s1/2L (0.32)	d5/2L (0.69)	d3/2L (0.69)	
Proton hole							
d5/2 (-16.17)	2+ 29.2 3+ 63.8 (g.s.)	1- 5.4 2- 7.1 3- 4.2 4- 141.8	2- 19.4 3- 76.2	2+ 2.2 3+ 4.6	0+ 0.0 1+ 26.0 2+ 0.3 3+ 26.7 4+ 0.5 5+ 164.1	1+ 8.9 2+ 34.9 3+ 30.4 4+ 112.0	
p1/2 (-25.49)	0- 9.4 1- 30.5	0+ 0.0 1+ 2.0 2+ 66.9	0+ 0.0 1+ 28.3	0- 3.7 1- 12.2	1- 1.4 2- 10.7 3- 76.9	2- 1.4 2- 43.5	
p3/2 (-29.84)	1- 14.3 2- 59.1	0+ 0.0 1+ 8.9 2+ 0.4 3+ 109.1	1+ 1.8 2+ 62.5	1- 5.9 2- 24.8	0- 2.0 1- 3.2 2- 4.5 3- 4.5 4- 148.6	1+ 5.7 2+ 17.5 3+ 96.3	
s1/2 (-44.55)	0+ 0.1 1+ 19.2	0- 7.3 1- 12.1 2- 50.0	0+ 0.3 1+ 51.4	1+ 16.5 2+ 27.0 3+ 58.1	2+ 40.1		

$\chi S(J)$

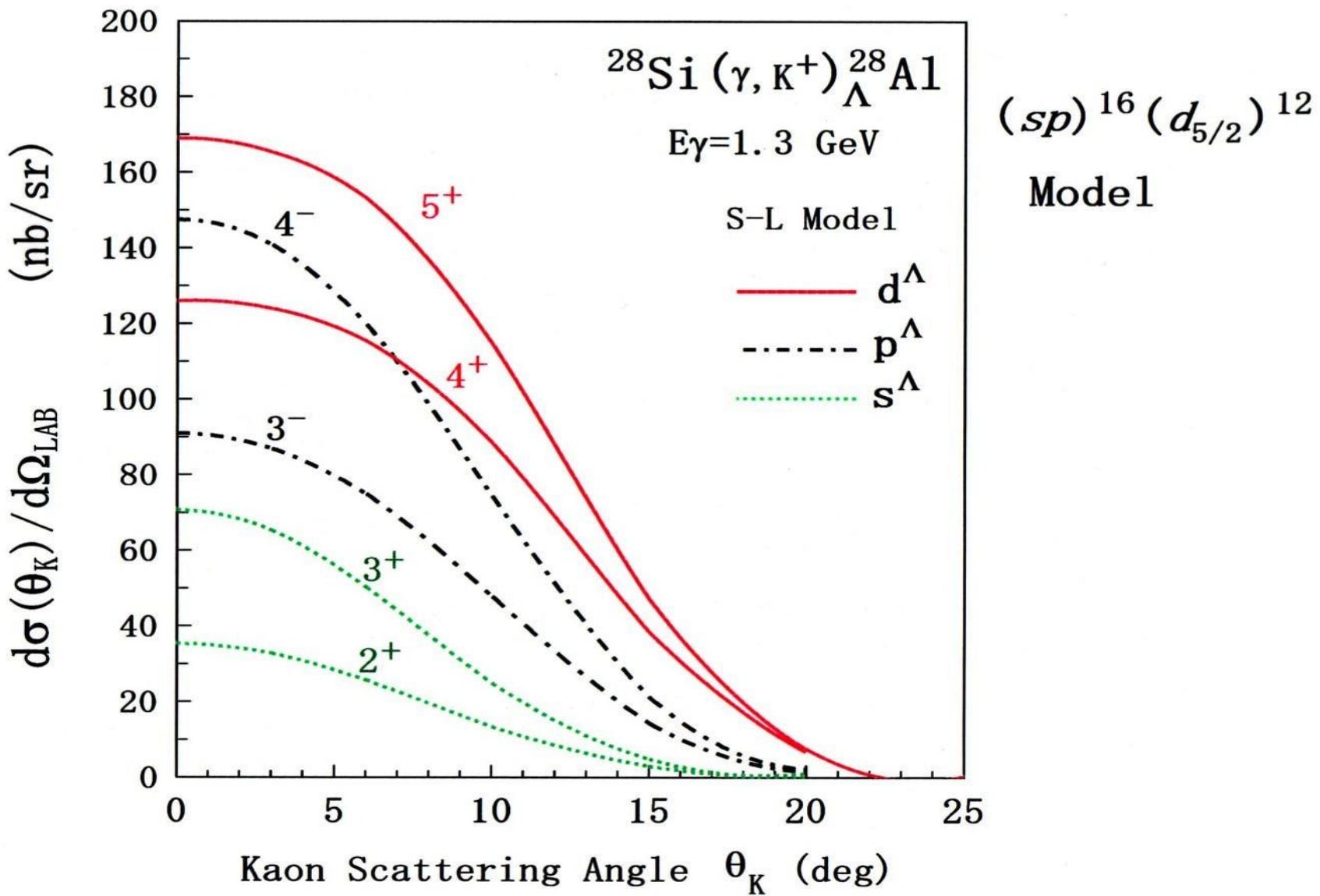
DWIA  
(65%)

VS.

PWIA

(a) DWIA		$0s_{1/2}^A$	$0p_{3/2}^A$	$0p_{1/2}^A$	$0d_{5/2}^A$	$0d_{3/2}^A$	$1s_{1/2}^A$	
		$(E_A)$	(-16.92)	(-8.40)	(-8.40)	(0.69)	(0.69)	(0.32)
(p-hole)	$J=0$	—	—	—	—	0.1	—	—
	$J=1$	—	5.3	—	27.6	7.2	—	—
(0d <sub>5/2</sub> <sup>-1</sup> )	$J=2$	32.9	2.9	15.7	0.2	33.6	1.6	—
	$J=3$	65.5	4.6	87.0	20.6	31.1	3.1	—
	$J=4$	—	141.0	—	0.3	124.1	—	—
	$J=5$	—	—	—	—	165.4	—	—
— — — — — — — — —								
(b) PWIA		$0s_{1/2}^A$	$0p_{3/2}^A$	$0p_{1/2}^A$	$0d_{5/2}^A$	$0d_{3/2}^A$	$1s_{1/2}^A$	
		$(E_A)$	(-16.92)	(-8.40)	(-8.40)	(0.69)	(0.69)	(0.32)
(p-hole)	$J=0$	—	—	—	—	0.2	—	—
	$J=1$	—	10.9	—	59.6	15.3	—	—
(0d <sub>5/2</sub> <sup>-1</sup> )	$J=2$	68.3	4.03	33.4	0.4	71.9	1.4	—
	$J=3$	135.6	8.3	155.3	46.5	58.9	2.7	—
	$J=4$	—	260.5	—	0.4	186.9	—	—
	$J=5$	—	—	—	—	248.1	—	—

# $d\sigma/d\Omega(\theta_K)$ : angular dependence



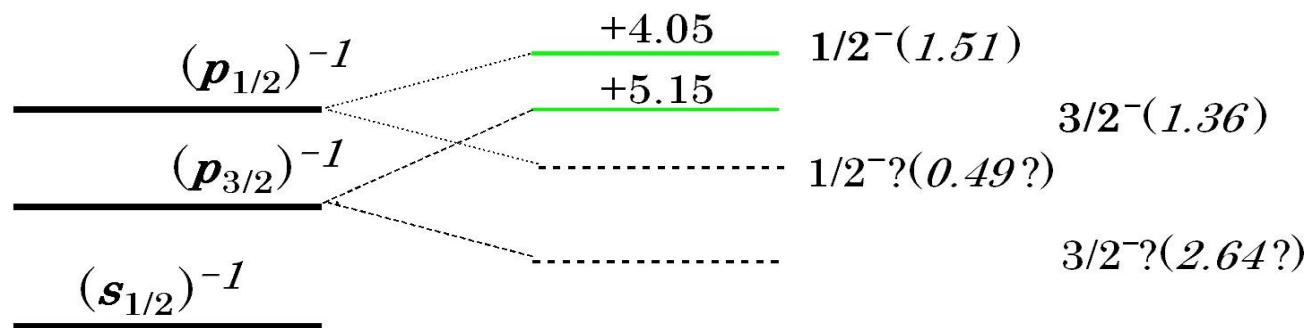
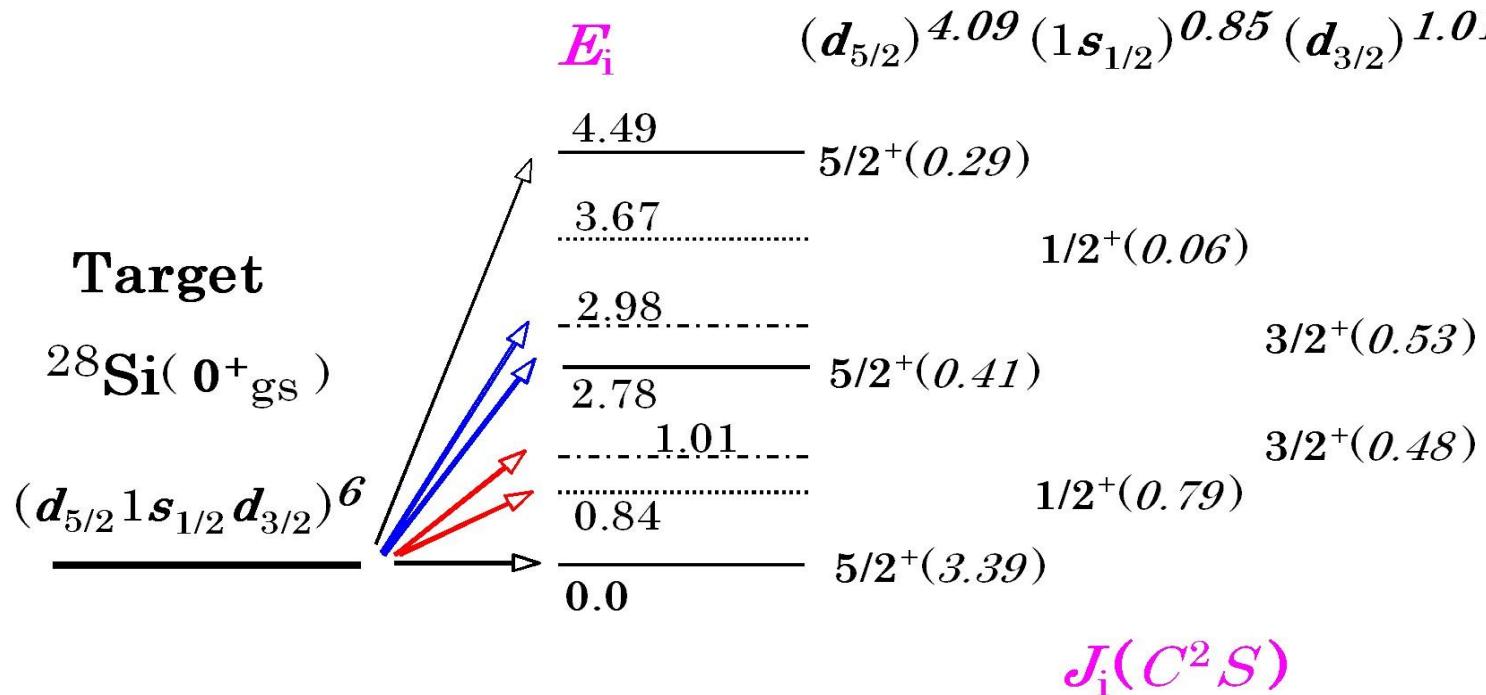
### 3. Realistic prediction for $^{28}\text{Si} (\gamma, \text{K}^+) \Lambda^{28}\text{Al}$

By fully taking account of

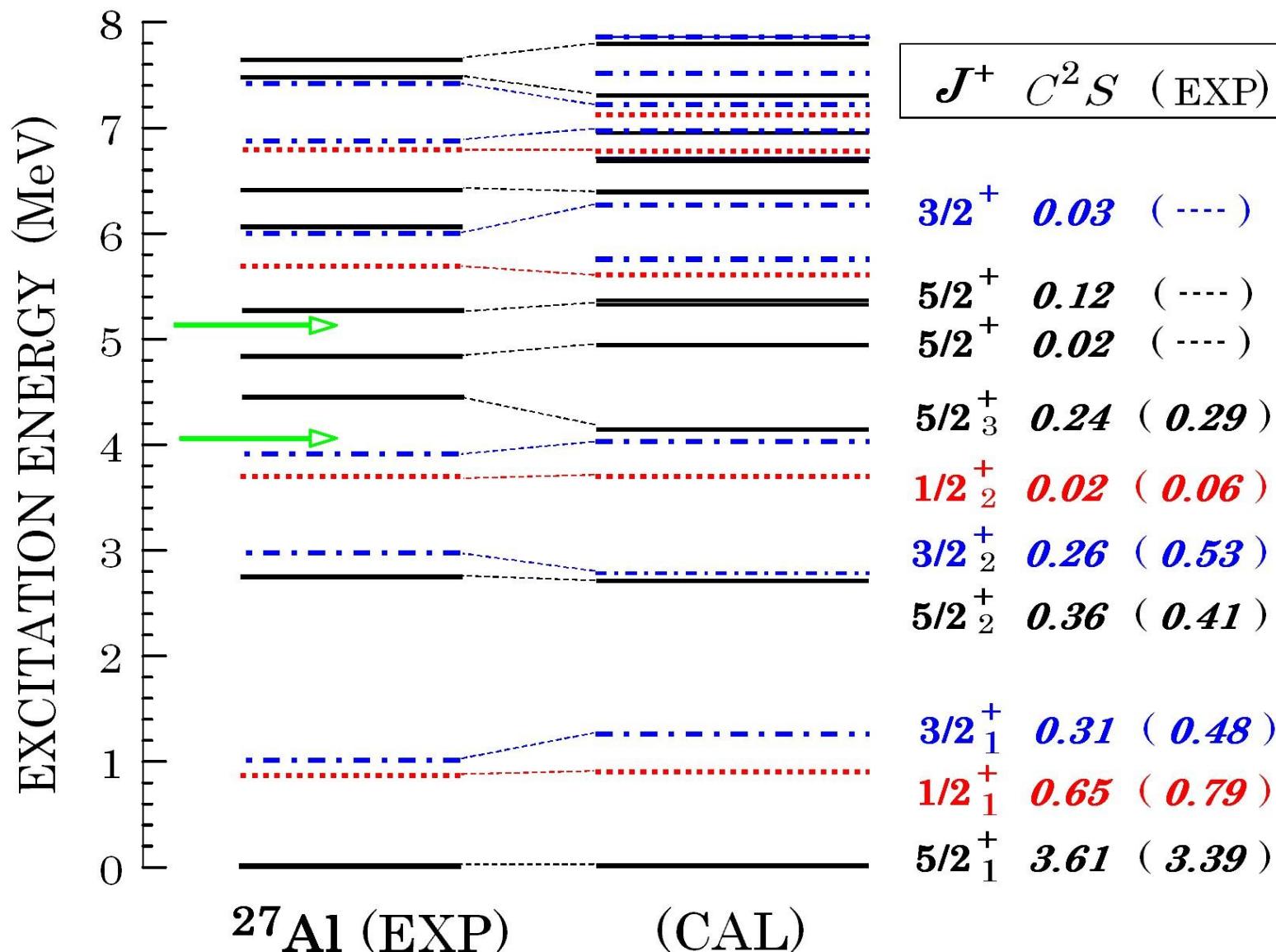
- full  $p(\text{sd})^6 \cdot n(\text{sd})^6$  configurations,
- fragmentations when a proton is converted,
- $^{27}\text{Al}$  core nuclear excitation
- $\text{K}^+$  wave distortion effects

→ Comparison with the  $^{28}\text{Si} (\text{e}, \text{e}'\text{K}^+)$  exp.

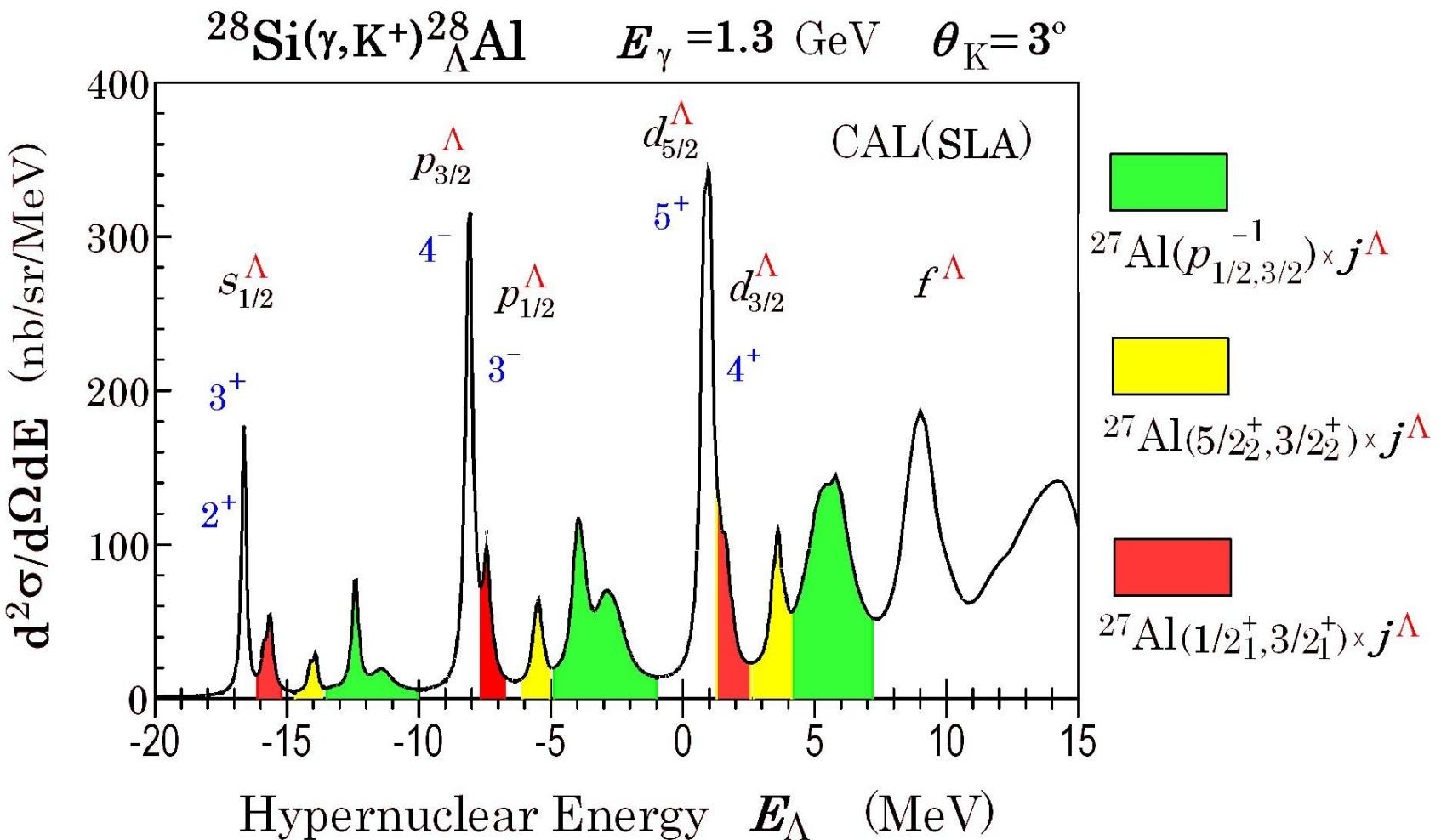
# proton-state fragmentations should be taken into account *to be realistic*



# Proton pickup from $^{28}\text{Si}(0^+)$ : $(sd)^6 = (d_{5/2})^4 (1S_{1/2})^{\color{red}0.9} (d_{3/2})^{\color{blue}1.0}$

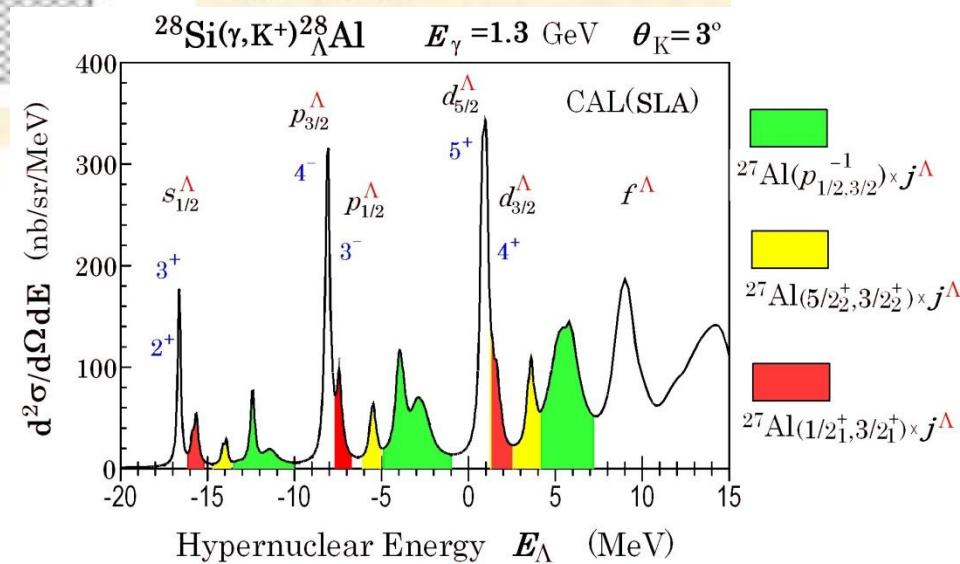
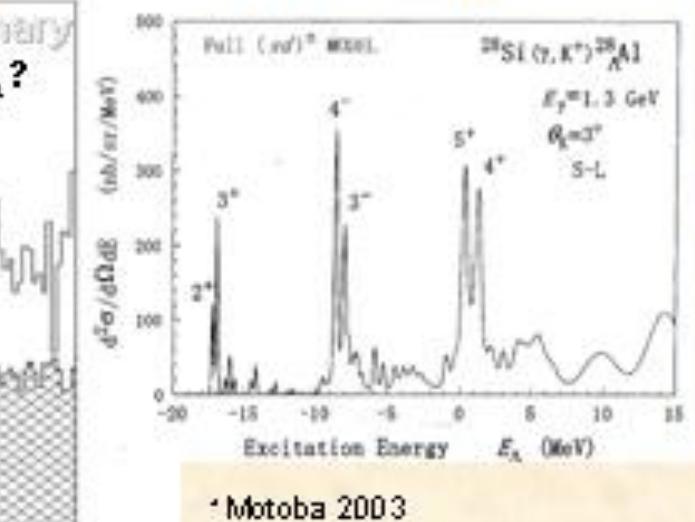
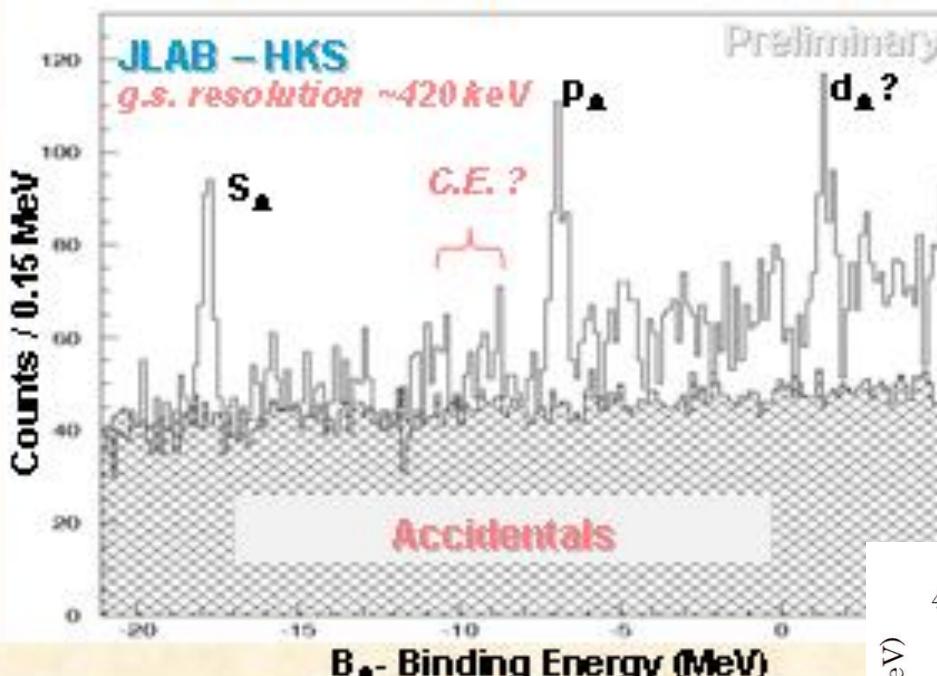


# Peaks can be classified by the characters



*Major peak series :*  $[{}^{27}\text{Al}(5/2_1^+, 3/2_2^+) \times j^\Lambda]_J$  with  $j^\Lambda = s, p, d, \dots$

# $^{28}\text{Si}(\text{e},\text{e}'\text{K}^+)^{28}_{\Lambda}\text{Al}$ – First Spectroscopy of $^{28}_{\Lambda}\text{Al}$



Major peak series :  $[^{27}\text{Al}(5/2_1^+) \times j^{\Lambda}]_J$  with  $j^{\Lambda} = s, p, d, \dots$

# Peak energies: $^{28}_{\Lambda}\text{Si}$ vs. $^{28}_{\Lambda}\text{Al}$

H.Hotchi et al, PRC 64(2001) vs. O.Hashimoto et al, NP A804(2008)

$j_{\Lambda}$	$^{28}\text{Si}(p^+, K^+)^{28}_{\Lambda}\text{Si}$ $E_{\Lambda} = -B_{\Lambda}$ (Ex )	$^{28}\text{Si}(e, e' K^+)^{28}_{\Lambda}\text{Al}$ (as read on the Sendai08 poster)	$(\gamma, K^+)$ CAL
s	-16.6+-0.2 (GS)	-17.85 ?? (GS)	-16.6 (GS)
		-15.7 ? -13.0 ? -10.8 ?	-11.9+-0.4 ( $E_x=4.7$ )
p	-7.0+-0.2 ( $E_x=9.6$ )	- 6.88 +- ?? ( $E_x=11$ )	-8.1 ( $E_x=8.5$ )
	-4.3+-0.2 ( $E_x=12.4$ )		-5.6, -4.0
d	+1.0+-0.8 $E_x=17.6$ )	+ 1.35 +/- ( $E_x=19.2$ )	+0.9 ( $E_x=17.5$ )

# 4. Extend to heavier nuclear Targets

$^{52}\text{Cr}$ :  $(f_{7/2})^4$  assumed

$^{40}\text{Ca}$ : (sd-shell LS-closed)

Well-separated series of peaks  
due to large  $q$  and spin-flip dominance:

$$j_>=l+1/2, \quad j_<=l-1/2$$

$[(nlj)_p^{-1} (nlj)^\Lambda]_J$  a series of pronounced peaks

**$jj$ -closed target** : (  $^{28}\text{Si}$ ,  $^{52}\text{Cr}$  )

$[j_>^{-1} j_>^\Lambda]_J \quad J=j_>+j_>^\Lambda = I_p + I_\Lambda + 1 = L_{\max} + 1$  (unnatural parity)

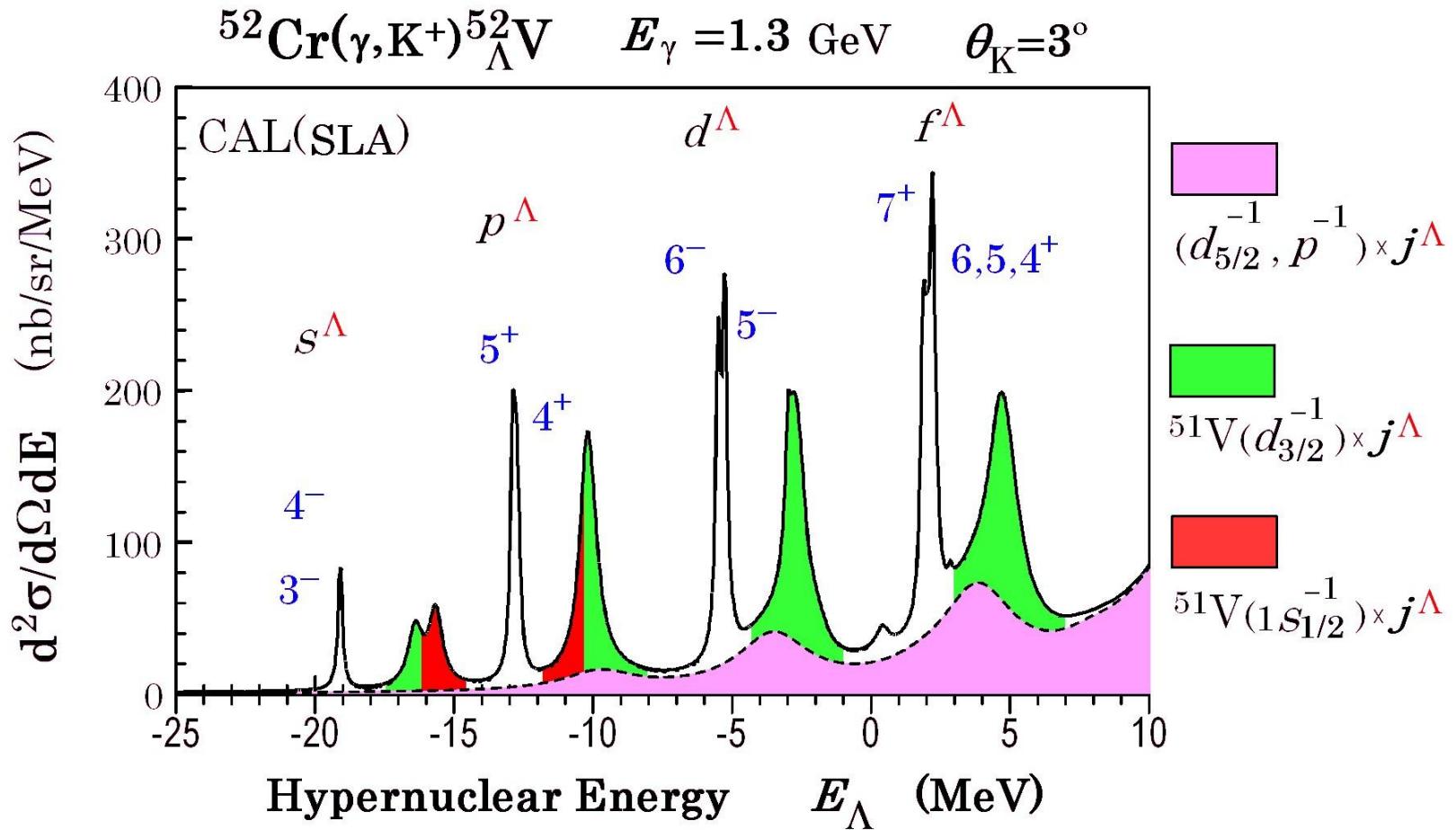
$[j_>^{-1} j_<^\Lambda]_J \quad J=j_>+j_<^\Lambda = I_p + I_\Lambda = L_{\max}$  (natural parity)

**$LS$ -closed target** : (  $^{40}\text{Ca}$  )

$[j_<^{-1} j_>^\Lambda]_J \quad J=j_<+j_>^\Lambda = I_p + I_\Lambda = L_{\max}$  (natural parity)

# $^{52}\text{Cr}$ ( $j_\sigma$ dominant target case)

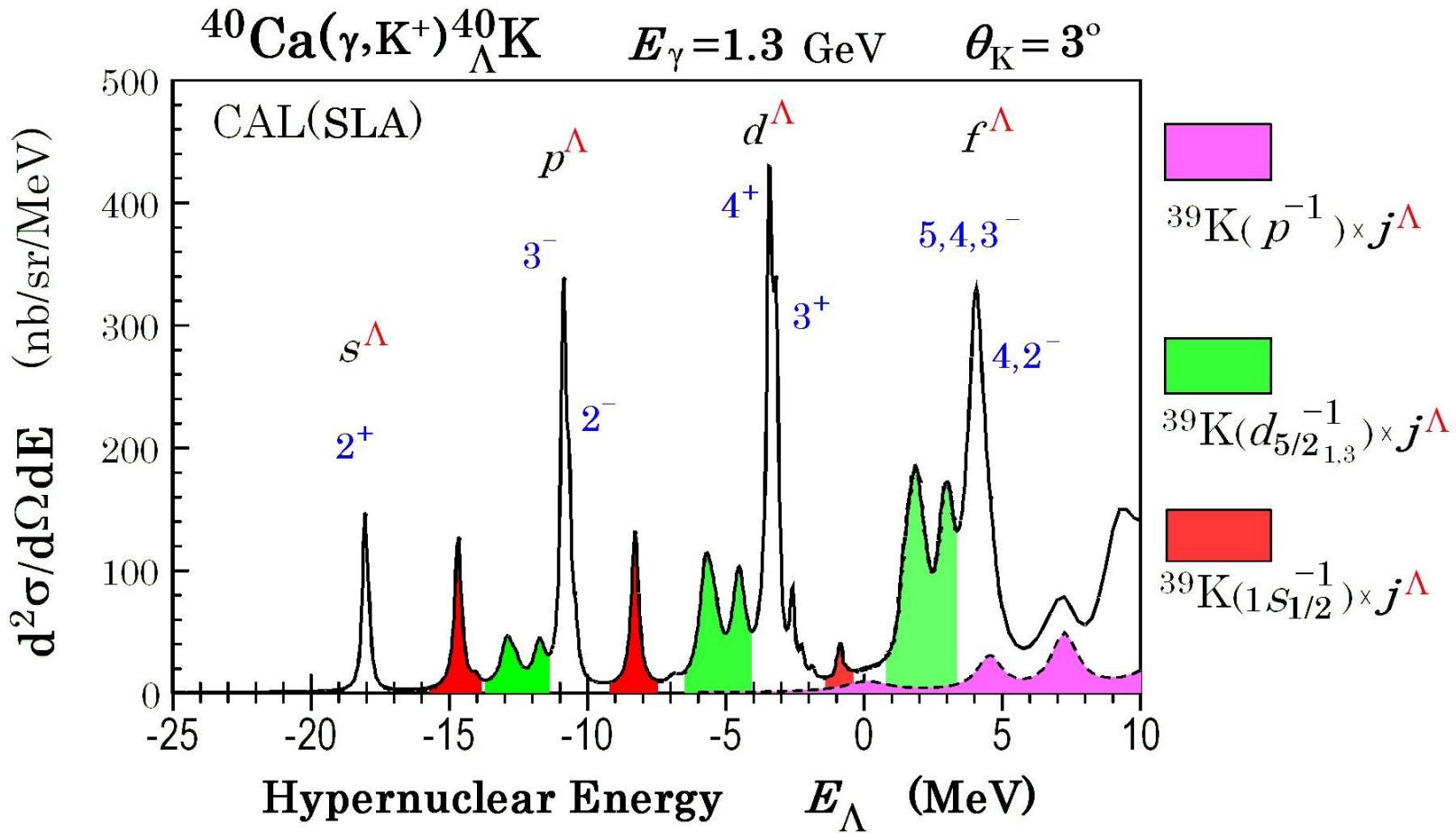
typical unnatural-parity high-spin states



Major peak series :  $[{}^{51}\text{V}(7/2^-; \text{gs}) \times j^\Lambda]_J$  with  $j^\Lambda = s, p, d, f, \dots$

# $^{40}\text{Ca}$ ( LS-closed shell case):

## high-spin states with natural-parity ( $2^+, 3^-, 4^+$ )



*Major peak series :*  $[{}^{39}\text{K}(d_{3/2}^{-1}; \text{gs}) \times j^\Lambda]_J$  with  $j^\Lambda = s, p, d, f, \dots$

## 6. SUMMARY

1) Basic properties of the  $\gamma p \rightarrow \Lambda K^+$  process are discussed with typical models for the amplitudes:

----- spin-flip dominance, large  $q$ ,

----- appreciable difference among theor. models (P).

2) Novel characteristics of photo/electroproduction of hypernuclei are clarified with the  $^{28}\text{Si}$  target:  $(d_{5/2})^{12}$  Model

$[\bar{j}_>^1 j_>^\Lambda]: J_{\max} = j_> + j_>^\Lambda = l_N + l_\Lambda + 1 = L_{\max} + 1$  unnatural parity

$[\bar{j}_>^1 j_<^\Lambda], [\bar{j}_<^1 j_>^\Lambda]: J'_{\max} = l_N + l_\Lambda = L_{\max}$  orbitally stretched

Selective excitation of high-spin states, providing a series of  $\Lambda$  orbits.

- 3) Realistic prediction for the  $^{28}\text{Si}$  target was made. The calculation is in good agreement with the recent JLab exp. The predictions are made also for  $^{40}\text{Ca}$  and  $^{52}\text{Cr}$ .
- 4) Almost all predictions for the p-shell targets have been confirmed by the recent Jlab exp.(Hall A, C).
- 5) **Medium-mass hypernuclear production** by  $(e,e'K^+)$  provide us with good opportunities in understanding the *details* of the hyperon motion in nuclear matter. ( $\Lambda$ -s.p.e. to establish “textbook”, Rotation/Vib.- $\Lambda$  coupling, Auger effect,  $\mu_\Lambda$ ,  $e_{\text{eff}}(\Lambda)$ , etc )