Hyperon-Nucleon and Hyperon-Hyperon Interactions in Free Space and Nuclear Matter

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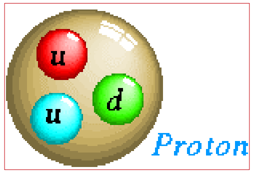
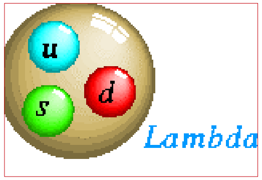
Outline

- Motivation
- Boson Exchange Interaction based on SU(3)
- Free Space Results
- In-Medium Effect via Pauli Projector
- Results

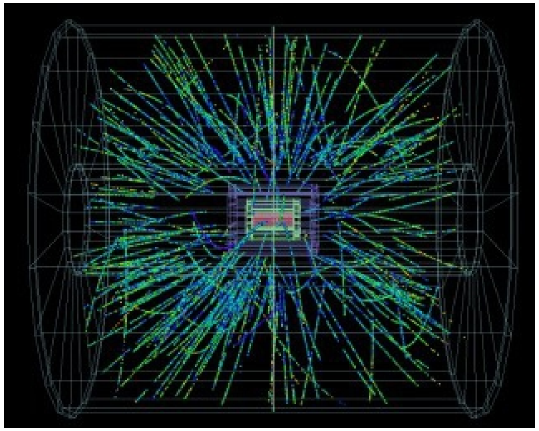
Motivation

Hyperon : Baryon containing strange quark

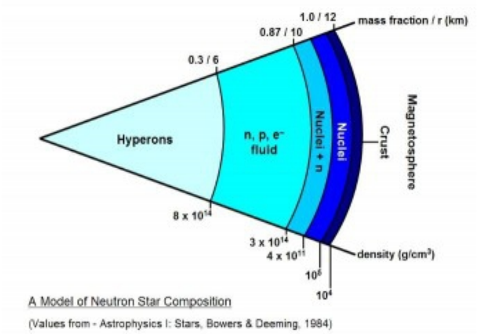




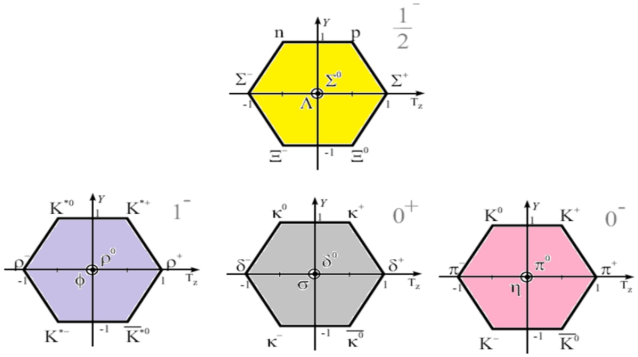
Heavy Ion Collisions



Astrophysics

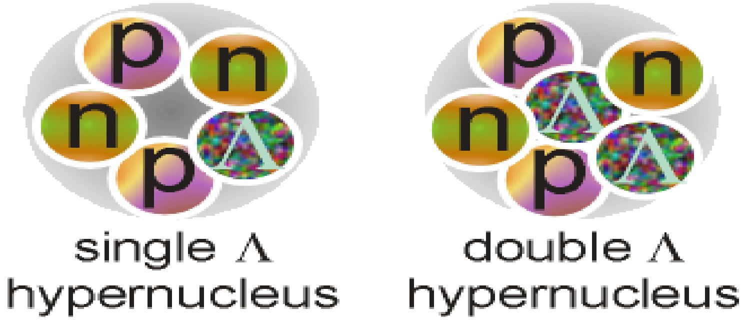


Octet interaction

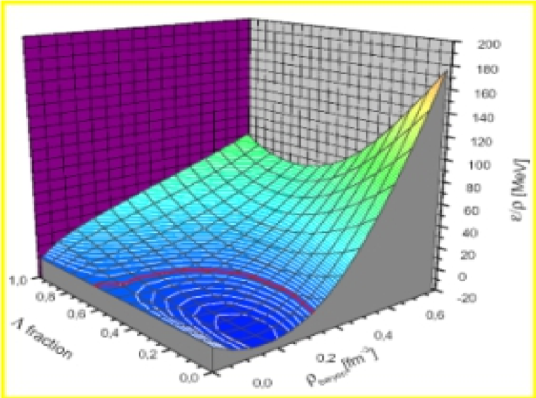


Hyper Nuclear Structure

(Talk by S. N. Nakamura, T. Nagae)



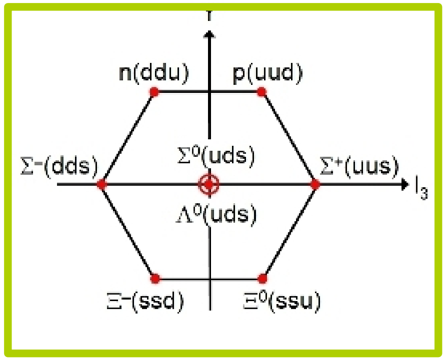
Hyper matter Equation of State



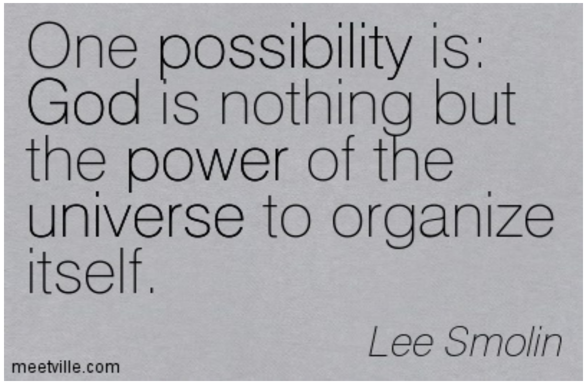
How to proceed??

- Proceed through octet model

  - {u,d,s} quark –> SU(3)$_{flavour}$ symmetry



(Jülich, Nijmegen)



- Nucleon Nucleon extension to hyperon-nucleon (YY) and hyperon-hyperon(YY)
- BB interactions from LQCD (NPLQCD, talk by A. Parreño and HALQCD from Japan)
- Chiral EFT as QCD-inspired SU(3) approach (e.g J. Haidenbauer (Jülich))
- Quark-cluster model (Fujiwara et al.)

Model Description

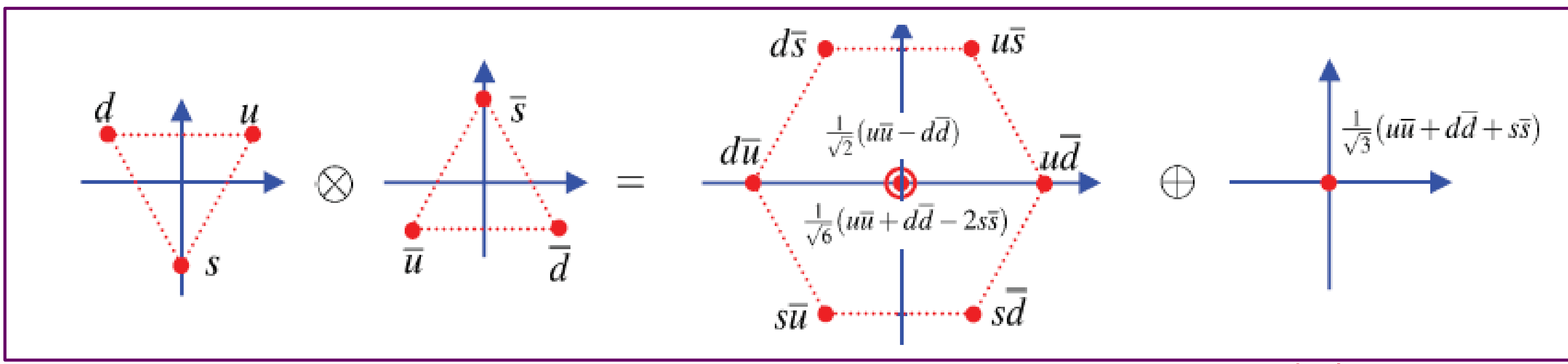
Group Theory Interpretation ::The Quark Model

-Strong interaction treats all quark flavours equally (u ≡ d ≡ s quark)

- Arrange particles according to I_3 and Y to form multiplets (Y=B+S)

- Gell-Mann and Zweig: Patterns of multiplets could be explained if all hadrons were made of quarks

- Fundamental representation of SU(3) : triplet of u, d,s quark



Pseudo scalar Meson octet

Singlet

$$3 \otimes \bar{3} = 8 \oplus 1$$ →Mesons

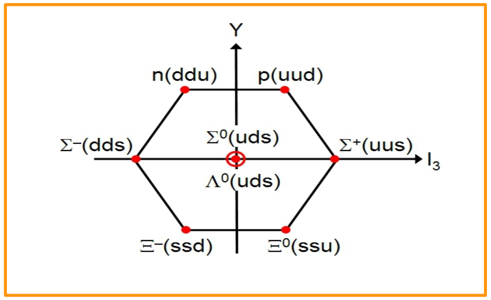
$$3 \otimes 3 \otimes 3 = 3 \otimes (6 \oplus \bar{3}) = 10_s \oplus 8_{M,S} \oplus 8_{M,A} \oplus 1$$ →Baryons

Flavour Symmetry

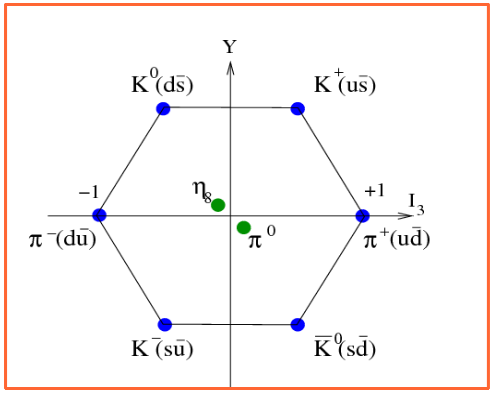
-SU(3) algebra is given by $\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = i f_{abc} \frac{\lambda_c}{2}$, λ: fundamental matrices of SU(3)

-**Baryons and Meson octets in matrix forms which are SU(3) invariant**

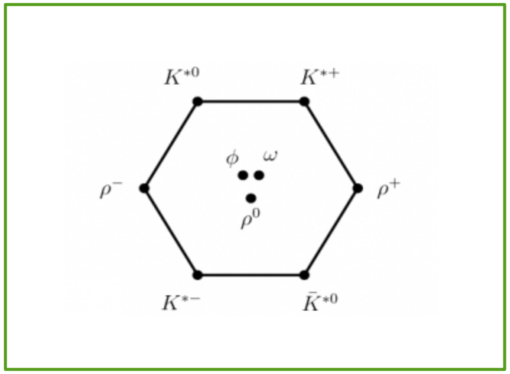
$$B = \frac{1}{\sqrt{2}} \sum_{a=1}^{8} \lambda^a B^a = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$



$$\phi_{ps} = \frac{1}{\sqrt{2}} \sum_{a=1}^{8} \lambda^a \phi^a_{ps} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}$$



$$\phi_v = \frac{1}{\sqrt{2}} \sum_{a=1}^{8} \lambda^a \phi^a_v = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\frac{2\omega}{\sqrt{6}} \end{pmatrix}$$



$$\phi_s = \frac{1}{\sqrt{2}} \sum_{a=1}^{8} \lambda^a \phi^a_s = \begin{pmatrix} \frac{a_0^0}{\sqrt{2}} + \frac{f_0}{\sqrt{6}} & a_0^+ & \kappa^+ \\ a_0^- & -\frac{a_0^0}{\sqrt{2}} + \frac{f_0}{\sqrt{6}} & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & -\frac{2f_0}{\sqrt{6}} \end{pmatrix}$$

Interaction model based on flavour symmetry

- Interaction Lagrangian with Yukawa type coupling (x: ps, scalar, vector)

$$\mathcal{L}^x_{int} = -g_x \alpha_x Tr([B,\bar{B}]\phi_x) + g_x(1-\alpha_x) Tr([\bar{B},B]\phi_x)$$

- SU(3)- invariant combinations :

$\{8_B\} \otimes \{8_B\} \otimes \{1_M\}$ and $\{8_B\} \otimes \{8_B\} \otimes \{8_M\}$

- Baryons have symmetric and antisymmetric representations » couple with different strength with $\{8_M\}$ (symmetric:: g_D, antisymmetric:: g_F)

- Convention : g_D,g_F replaced by octet coupling g_8 and α = F/(F+D)

$$g_D = \frac{40}{\sqrt{30}} g_8 (1-\alpha)$$

$$g_F = 4\sqrt{6}\, g_8 \alpha$$

Boson Exchange Interaction(Free Space)

- **Bonn potential** extended to include hyperons (Juelich model)
  (R. Machleidt, K. Holinde, and Ch. Elster, Physics Reports 149 (1987), 1 – 89)

$$\mathcal{L}_s = +g_s \bar{\psi}\psi\Phi^{(s)}$$

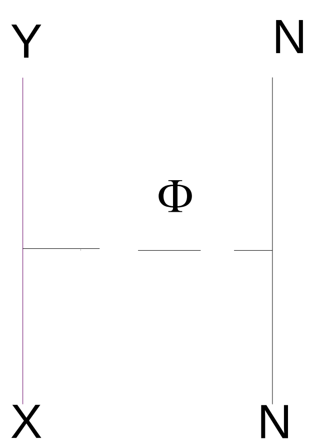
$$\mathcal{L}_{ps} = -g_{ps}\bar{\psi}i\gamma^5\psi\Phi^{(ps)}$$

$$\mathcal{L}_v = -g_v\bar{\psi}\gamma^\mu\psi\Phi^{(v)}_\mu - \frac{f_v}{2(M_{\bar{B}} + M_B)}\bar{\psi}\sigma^{\mu\nu}\psi(\partial_\mu\Phi^{(v)}_\nu - \partial_\nu\Phi^{(v)}_\mu)$$

- Overall strength by fit to data ⟶ not sufficient data for hyperons !!!

- **$SU(6)_{SF} \longrightarrow SU(3)_{Flavour}$ X $SU(2)_{Spin}$** or **SU(3)** coupling constants (g's)

(B. Holzenkamp, K. Holinde and J. Speth, NPA500 (1989) 485-528)



$$g_{XN\leftrightarrow YN} = g_{\Phi NN}\, g_{\Phi XY}$$

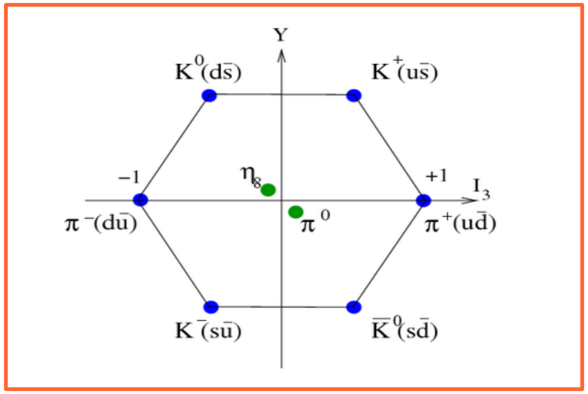
One Boson Exchange Diagram

Parameters of the Model

- Four Parameters for each type of meson octet:

  i) octet coupling (g_8) ii) mixing angle (θ)

  iii) $\alpha = F/(F+D)$ iv) singlet coupling (g_1)



- Example : Pseudoscalar mesons

$$g_{NN\pi} = g_8 \qquad g_{\Sigma\Sigma\eta_8} = \frac{2}{\sqrt{3}} g_8 (1-\alpha_{ps}) \qquad g_{\Lambda N K} = \frac{-1}{\sqrt{3}} g_8 (1+2\alpha_{ps})$$

$$g_{\Lambda\Sigma\pi} = \frac{2}{\sqrt{3}} g_8 (1-\alpha_{ps}) \qquad g_{NN\eta_8} = \frac{1}{\sqrt{3}} g_8 (4\alpha_{ps}-1) \qquad g_{\Sigma N K} = g_8 (1-2\alpha_{ps})$$

$$g_{\Sigma\Sigma\pi} = 2 g_8 \alpha_{ps} \qquad g_{\Lambda\Lambda\eta_8} = \frac{-2}{\sqrt{3}} g_8 (1-\alpha_{ps}) \qquad g_{\Xi\Lambda K} = \frac{1}{\sqrt{3}} g_8 (4\alpha_{ps}-1)$$

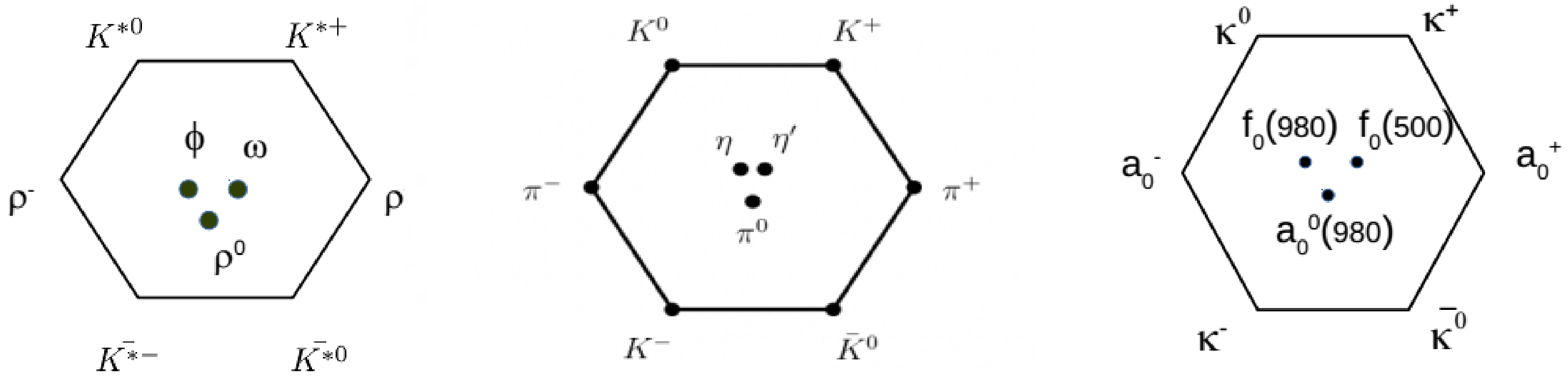
$$g_{\Xi\Xi\pi} = -g_8 (1-2\alpha_{ps}) \qquad g_{\Xi\Xi\eta_8} = \frac{-1}{\sqrt{3}} g_8 (1+2\alpha_{ps}) \qquad g_{\Xi\Sigma K} = -g_8$$

For singlet meson η_1 :: $g_{NN\eta_1} = g_{\Lambda\Lambda\eta_1} = g_{\Sigma\Sigma\eta_1} \equiv g_1$

Physical η coupling :: $g_{NN\eta} = \cos(\theta_{ps})\, g_{NN\eta_8} - \sin(\theta_{ps})\, g_{NN\eta_1}$

Parameters

- Similar relations for Vector meson and Scalar mesons



Form Factor: -used to regularize the large-momentum behaviour of amplitudes

-multiplied to each BBM vertex

$$\left(\frac{\Lambda_c^2 - m^2}{\Lambda_c^2 + k^2}\right)^{2n}, \quad n = 1,2$$

$$k = q' - q$$

Total parameters : -octet couplings(3)
-singlet couplings(Max. 3)
-mixing angles(Max. 3)
- alpha's(4)
-cut-off (pseudoscalar, scalar, vector)

Channel Coupling

Particle basis	Q= -2	Q=-1	Q=0	Q=1	Q=2
S=0			nn	np	pp
S= -1		$\Sigma^- n$	$\Lambda n, \Sigma^0 n, \Sigma^- p$	$\Lambda p, \Sigma^+ n, \Sigma^0 p$	
S= -2	$\Sigma^-\Sigma^-$	$\Xi^- n, \Sigma^-\Lambda, \Sigma^-\Sigma^0$	$\Lambda\Lambda, \Xi^0 n, \Xi^- p, \Sigma^0\Lambda, \Sigma^0\Sigma^0, \Sigma^-\Sigma^+$		$\Sigma^+\Sigma^+$
S= -3	$\Xi^-\Sigma^-$	$\Xi^-\Lambda, \Xi^0\Sigma^-, \Xi^-\Sigma^0$	$\Xi^0\Lambda, \Xi^0\Sigma^0, \Xi^-\Sigma^+$	$\Xi^0\Sigma^+$	
S= -4	$\Xi^-\Xi^-$	$\Xi^-\Xi^0$	$\Xi^0\Xi^0$		

NN (S=0)

YN (S= -1, S= -2)

YY (S= -3, S= -4)

Caveats

- Very few experimental data for hyperon sector
- Breaking of SU(3) symmetry ($m_s >> m_u$, m_d) =>> not straight forward expansion from NN to YN or YY
- Scalar meson puzzle!!! (particle or resonance??)

Models already in use

-Jülich (extension of Bonn model, based on SU(6))

-Extended **S**oft **C**ore **M**odels (from Nijmegen)

-However whole octet sector not under a single umbrella.

- Jülich 89,Jülich 94a/b, Jülich'04
- NSC89, NSC97a/c/f,ESC04a,ESC04d

Our Aim

Valid for whole octet sector

Qualitative
(large error bars !!!)

Respects SU(3)

Single model for Free space ??

Not fitting the NN sector phenomenologically(keeping SU(3) unchanged)

SU(3)Breaking effect systematically:: physical masses, thresholds, cut-offs , interaction parameters

Interactions over wide energy/momentum ranges (unlike Chiral EFT)

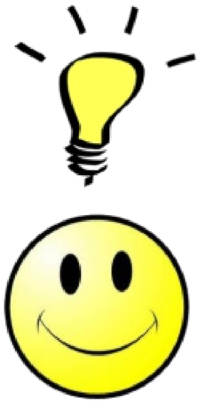
Ultimate aim : **In-medium Interactions**

”ab-initio” description of YN and YY interaction in nuclear systems

How to check the model and fix parameters

- We need to find observables :

- Cross section
- Binding energy
- Scattering length
- Phase shift
- Low energy parameters etc.



- How to find observables?

By solving a **scattering equation.**

Scattering Equation

Scattering Equation

- 4D Bethe-Salpeter (BS) equation : describes two-body scattering covariantly

$$\mathrm{T}(q';q|P)=V(q';q|P)+\int V(q';k|P)\varsigma(k|P)\mathrm{T}(k;q|P)d^4k$$

Bethe–Salpeter amplitude

Propagator

-four dimensional integration ›› difficult to solve

-Reduce to 3D without effecting the physics by using

G_{Bbs} (Blankenbecler-Sugar operator)

- Blankenbecler-Sugar reduction ⇒ 3D Lippman-Schwinger type equation

$$\breve{\mathrm{T}}(q',q)=\breve{V}(q',q)+\int \breve{V}(q',k)\frac{1}{2E_q-2E_k+i\epsilon}\breve{\mathrm{T}}(k,q)d^3k \Rightarrow T=V+\int VGT$$

-Solved in momentum space and K-matrix formalism

$$T=\frac{K}{1-iK},\ K=V+P\int VGK,\ P:Principal\ Value$$

Numerical Details

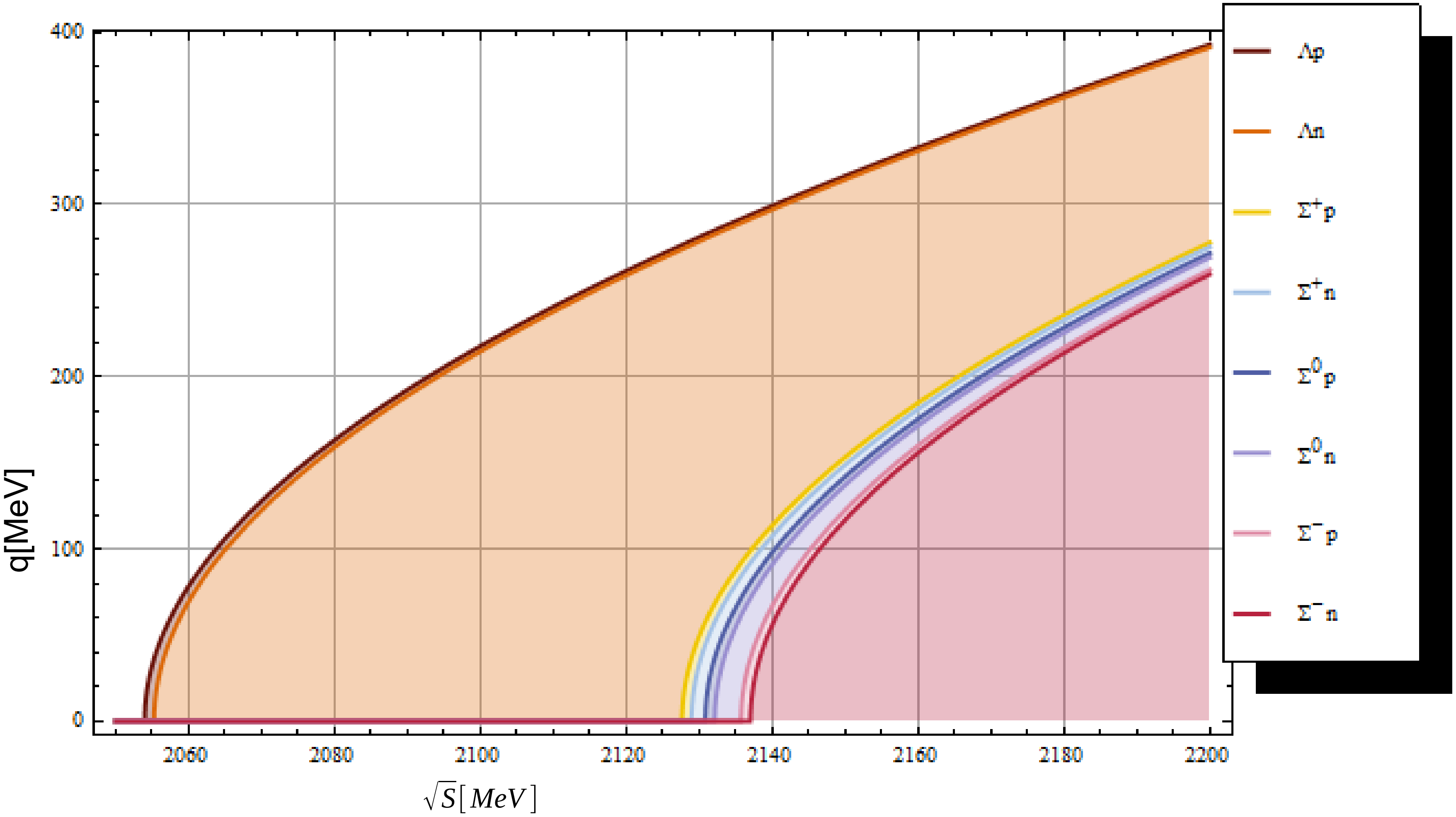
- Steps :

1) Choose the channel according to conserved S and Q
2) Choose the appropriate mesons to be exchanged and hence the potentials
3) Solve the Lippman- Schwinger Equation
4) Find the observables

- Fortran Code

Kinematics

Different channels for S= -1



- Λ channels open at lower energy than Σ's (Σ -Λ ≈ 70 MeV)

-SU(3) is broken due to mass difference

Closer look on the history

Vertex	$g_{BB'm}/\sqrt{4\pi}$	$f_{BB'm}/\sqrt{4\pi}$	$\Lambda_{BB'm}$ (GeV)
$NN\pi$	3.795		1.3
$\Lambda\Sigma\pi$	2.629		1.3
$\Sigma\Sigma\pi$	3.036		1.3
$N\Lambda K$	−3.944		1.2
$N\Sigma K$	0.759		1.2
$NN\omega$	3.317		1.7
$\Lambda\Lambda\omega$	2.211	−2.796	1.4
$\Sigma\Sigma\omega$	2.211	2.796	1.7
$N\Lambda K^*$	−1.588	−5.175	1.2
$N\Sigma K^*$	−0.917	2.219	1.4

Jülich'04

mesons		{1}	{8}	$F/(F+D)$	angles
ps-scalar	f	0.1852	0.2631	$\alpha_{PV} = 0.4668^{*)}$	$\theta_P = -23.00^0$
vector	g	2.6218	0.7800	$\alpha_V^e = 1.0$	$\theta_V = 37.50^0$
	f	0.3845	3.4711	$\alpha_V^m = 0.2760^{*)}$	
axial	g	1.5023	2.5426	$\alpha_A = 0.2340$	$\theta_A = -23.00^{0\ *)}$
scalar	g	3.1688	0.9251	$\alpha_S = 0.8410$	$\theta_S = 40.32^{0\ *)}$
diffractive	g	1.9651	0.0000	$\alpha_D = 1.000$	$\psi_D = 0.0^{0\ *)}$

ESC'04

Vertex	$g_\alpha/\sqrt{4\pi}$	$g_t/\sqrt{4\pi}$	Λ_α (GeV)
$NN\pi$	3.795		1.3
$\Lambda\Sigma\pi$	2.629		1.4
$\Sigma\Sigma\pi$	3.036		1.2
$NN\rho$	0.917	5.591	1.4
$\Lambda\Sigma\rho$	0	4.509	1.16
$\Sigma\Sigma\rho$	1.834	3.372	1.35
$N\Lambda K$	−3.944		1.2 (1.4)
$N\Sigma K$	0.759		2.0
$N\Lambda K^*$	−1.588	−5.175	2.2 (2.1)
$N\Sigma K^*$	−0.917	2.219	1.07 (1.0)
$NN\omega$	4.472		1.5
$\Lambda\Lambda\omega$	2.981	−2.796	2.0
$\Sigma\Sigma\omega$	2.981	2.796	2.0
$NN\sigma$	2.385		1.7
$\Lambda\Lambda\sigma$	2.306 (1.845)		1.0
$\Sigma\Sigma\sigma$	3.061; 3.102 (2.516)		1.0; 1.12 (1.02)

Jülich'89

$NN\rho$	0.917		5.591	1.4
$\Lambda\Sigma\rho$	0.		4.509	1.16
$\Sigma\Sigma\rho$	1.834		3.372	1.41 (1.35)
$NN\omega$	4.472		0.	1.5
$\Lambda\Lambda\omega$	2.981		−2.796	2.0
$\Sigma\Sigma\omega$	2.981		2.796	2.0
ΛNK^*	−1.588		−5.175	2.2
ΣNK^*	−0.917		2.219	1.07 (1.0)
$NN\sigma$	2.385			1.7
$\Lambda\Lambda\sigma$	2.138	(1.635)		1.0
$\Sigma\Sigma\sigma(I = 1/2)$	3.061	(2.516)		1.0 (1.06)
$\Sigma\Sigma\sigma(I = 3/2)$	3.102	(2.516)		1.12 (1.06)

Jülich'94

(b) $SU(3)$ parameters for cases A and B.

	$g_1/\sqrt{4\pi}$	$g_8\sqrt{4\pi}$	α	θ [degree]
S(A)	5.37138	0.76202	3.21258	−5.61
(B)	7.01988	0.13417	5.41593	75.88
P(A)	0.14853	0.26600	0.49061	−23.92
(B)	0.22637	0.26600	0.39508	−23.92
V^e(A)	3.44302	0.68648	1.00000	36.44
(B)	3.07021	0.97966	1.00000	36.44
V^m(A)	4.72583	6.12176	0.43590	36.44
(B)	6.13750	5.47123	0.30512	36.44

Wada et. al, 2000

TABLE XII. Parameters to be used in the SU(3) relations for the pseudoscalar (P), direct vector (V_e), derivative vector (V_m), and scalar (S) meson coupling constants.

	$g_8/\sqrt{4\pi}$	α	θ	$g_1/\sqrt{4\pi}$
P	3.660 00	0.464 03	−10.4°	4.316 75
V_e	0.594 44	1	35.264 30°	3.403 12
V_m	4.816 96	0.334 28	35.264 30°	2.202 86
S	⋯	⋯	⋯	5.032 08

Nijmegen,1977

1^{st} Step: Choosing the Parameters

	$g_8/\sqrt{4\pi}$	$g_1/\sqrt{4\pi}$	α	θ_{mixing}(degree)	Λ_c(GeV)
pseudoscalar	3.567- 3.795	2.08 - 4.16	.355-.491	-10 or -23 (Gellmann Okubo Mass formula)	1.2-1.4
vector	.68-1.18	2.529-3.762	E:1 M: 0.275-.4447	35.26(OZI Rule) 37.56	1.07-2
scalar	.76-1.395	3.17-4.598	.841-1.285	37.05 - 54.75	.988-2

Starting point

Pseudoscalar: $g_8/\sqrt{4\pi} = \sqrt{14}$ (Phenomenology)

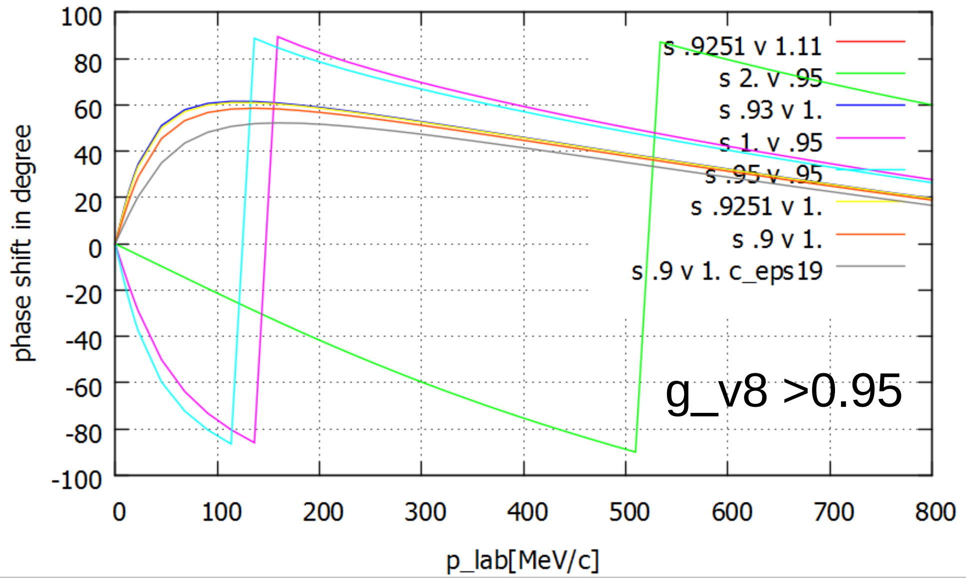
$\alpha = .35$ (Cabbibo Theory of semi leptonic decays)

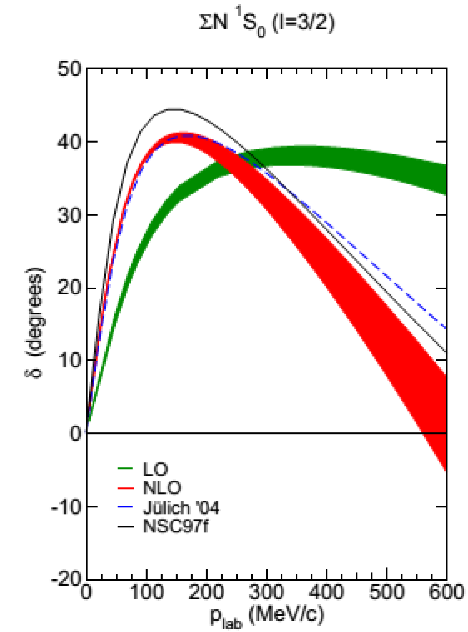
Vector: $\alpha_E = 1$ (Universal coupling)

θ_{mixing} =35.26 degree

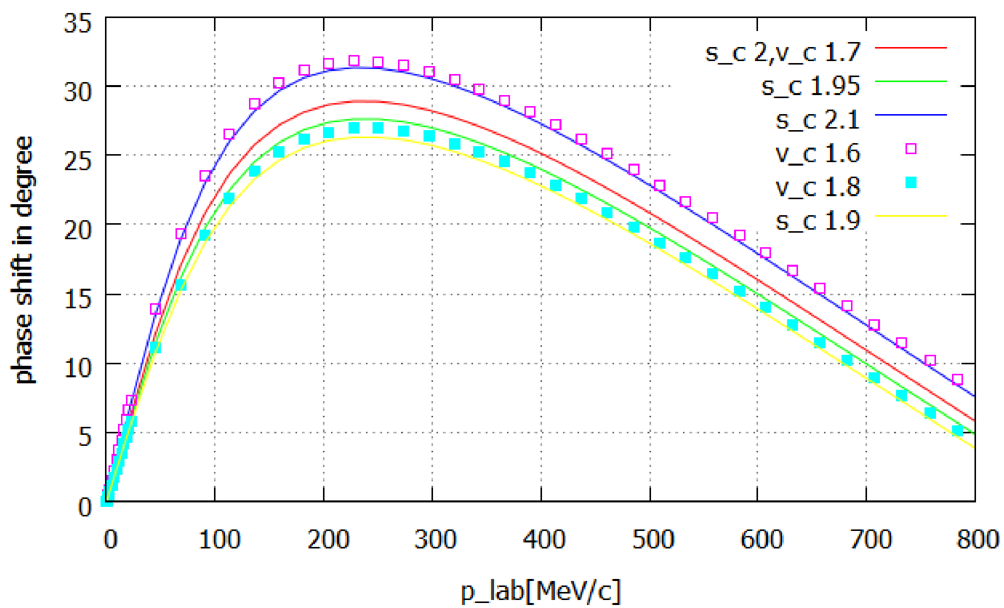
Λ_c : ps= 1.3 GeV vector= 1.7 GeV scalar= 2 GeV

Free Space : Fixing the Parameters





Chiral EFT, J. Haidenbauer

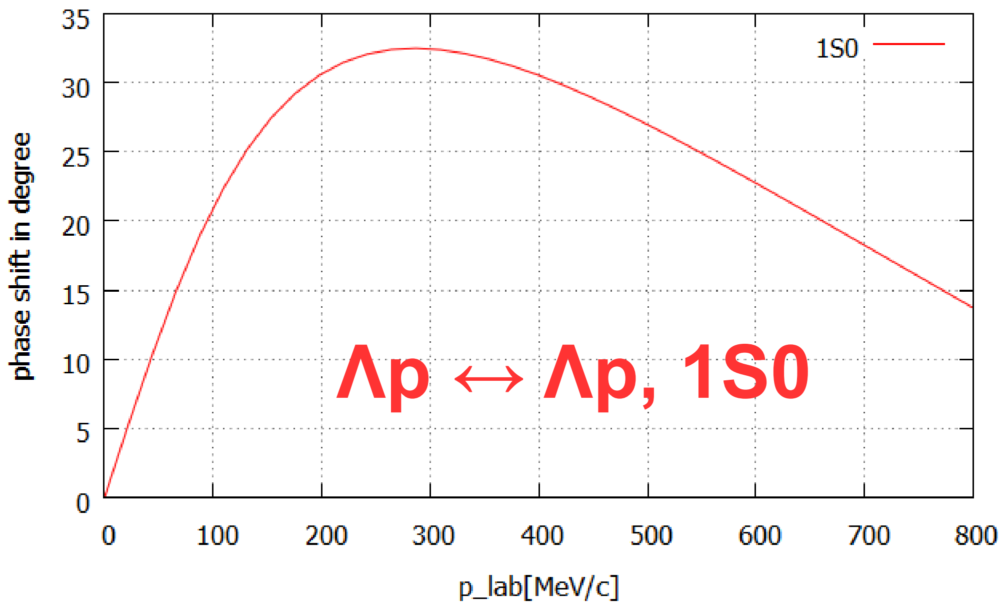


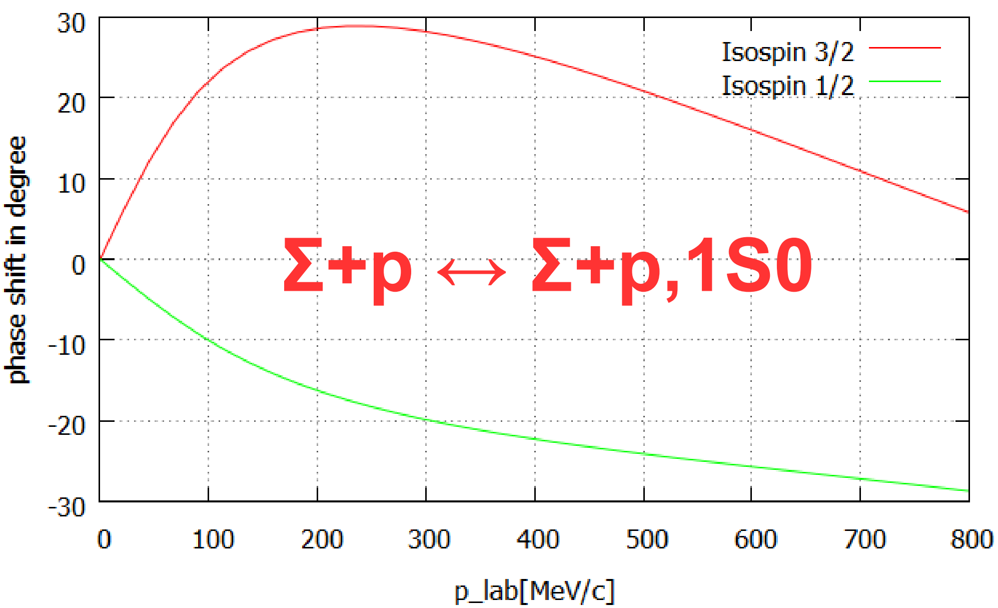
Σ+p ↔ Σ+p,1S0

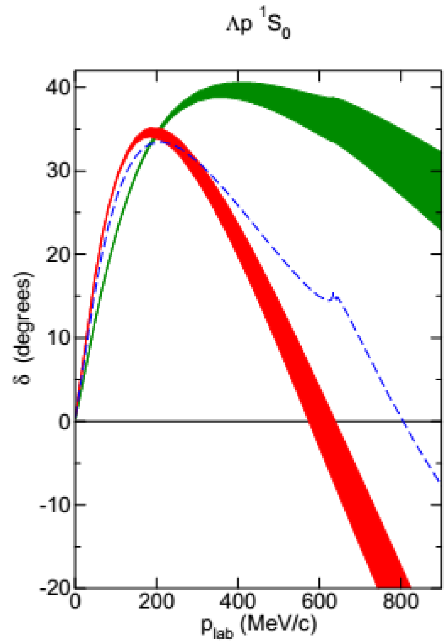
I=3/2 a_s	-2.24 fm
r_s	3.69 fm

-4.71 Jülich04,
-4.35 NSC97f,
-2.24..-2.36 EFT LO,
-3.40...-3.60 EFT NLO

Free Space S=-1







Chiral EFT, J. Haidenbauer

a_s^{1S0}	-1.83 fm
r_s	2.11 fm

For Λ_c 1.9 GeV for sigma meson

EFT LO	EFT NLO	Jülich '04	NSC97f	experiment*
550 · · · 700	500 · · · 650			
−1.90 · · · −1.91	−2.90 · · · −2.91	−2.56	−2.51	$-1.8^{+2.3}_{-4.2}$

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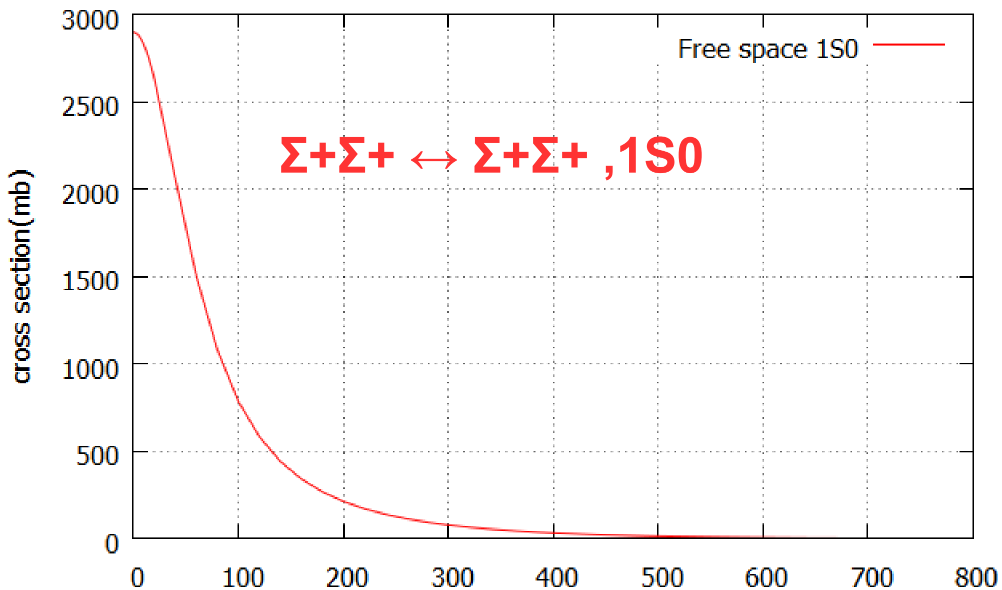
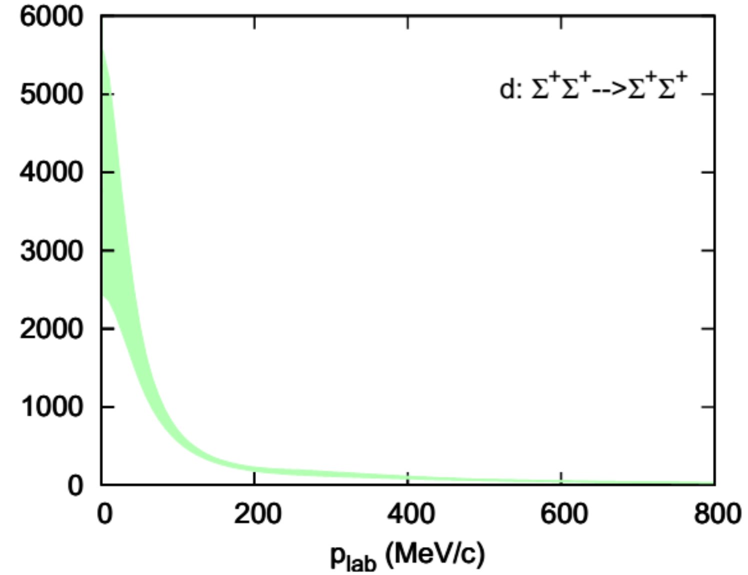
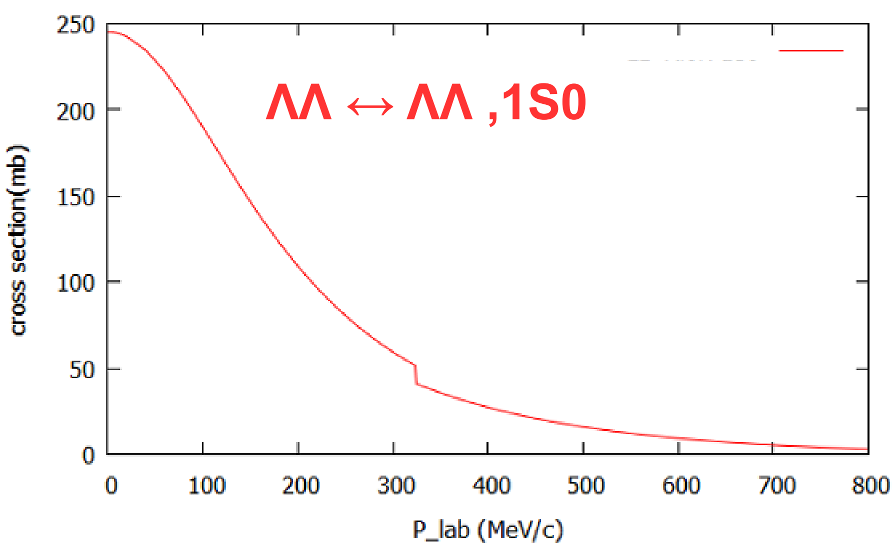
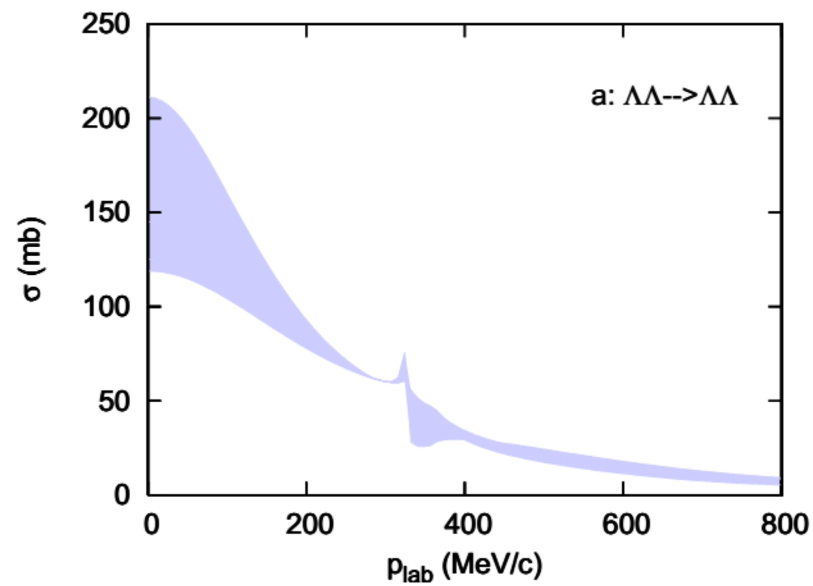
Free Space Result : S=-2





Chiral EFT, J. Haidenbauer





a_sΛΛ = -1.70 fm

Chosen Parameters Used and Details of our Model

Pseudoscalar mesons: **$g_{NN\pi}^2 / 4\pi = 13.6$** $\alpha = .36$

$\theta = -23$ degree $\Lambda = 1.3$ GeV

Vector mesons: $g_{NN\rho} / \sqrt{4\pi} = 1.11$ $\alpha_e = 1$

$\theta = 35.26$ degree $\Lambda = 1.7$ GeV

(OZI Rule)

Scalar mesons: $g_{NNa0} / \sqrt{4\pi} = .925$ $\alpha = .88$

$\theta = 37.5$ degree $\Lambda = 2$ GeV

[Still under investigation......]

Bottom Line

- set of parameters able to produce 'qualitative' result for S=-1 and S=-2

-S=-3, S=-4 ::: Work in progress

- Free Space :: Done

-->Apply medium effect.

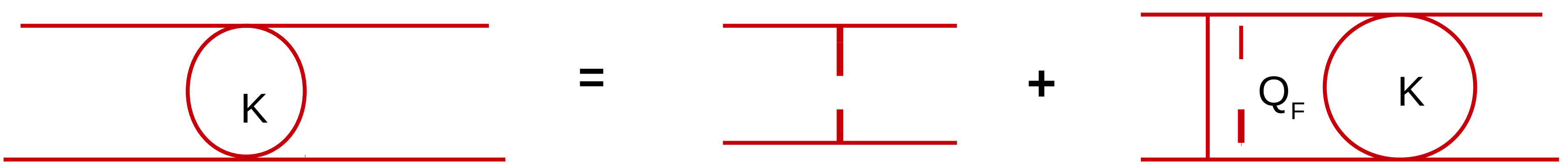
In- medium effect

- **In-medium effect** : By Multiplying each Green function in K-matrix equation by **Pauli projector operator(Q_F)**

$$Q_F = \Theta(k_1^2 - k_{F1}^2)\Theta(k_1^2 - k_{F2}^2)$$

Nuclear matter --> only one step fn.

- Q_F prevents scattering in those states which are not allowed according to Pauli exclusion principle



$$K = V + P\int VGQ_F K$$

- Energy modification via self-energy correction >>Effect on Propagator

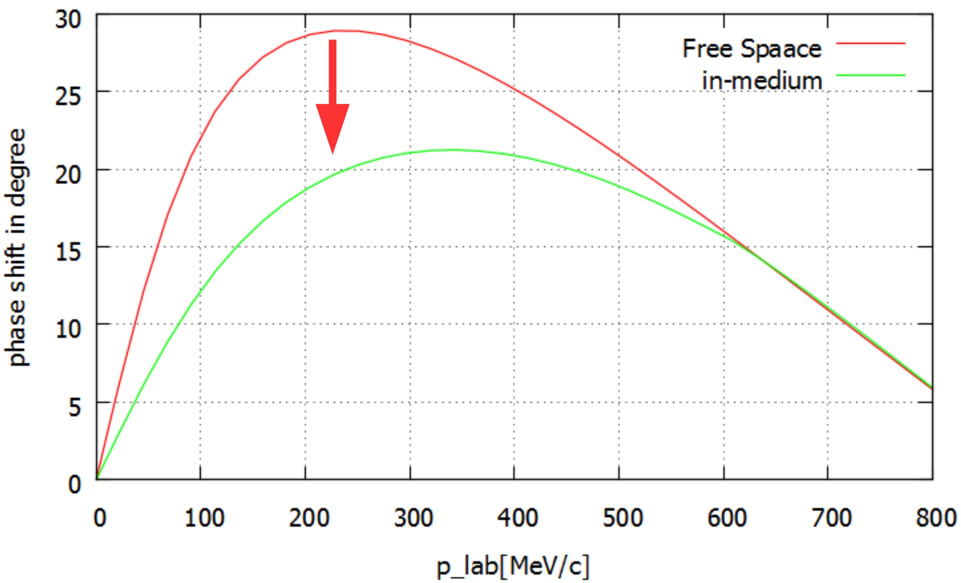
$$E \rightarrow E + \Sigma(E, q, k_F)$$

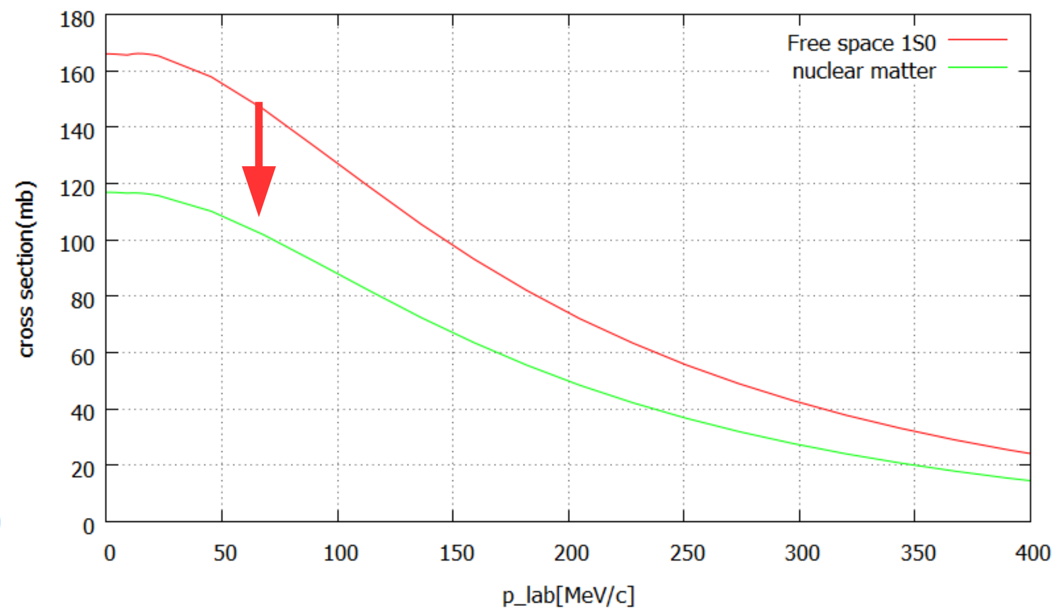
$$k_F = \sqrt[3]{3\pi^2\rho}$$

$\rho(fm^{-3})$	$k_F(MeV)$
.08	263.043
.16	331.414
.32	417.555

In medium results S=-1

Uncoupled

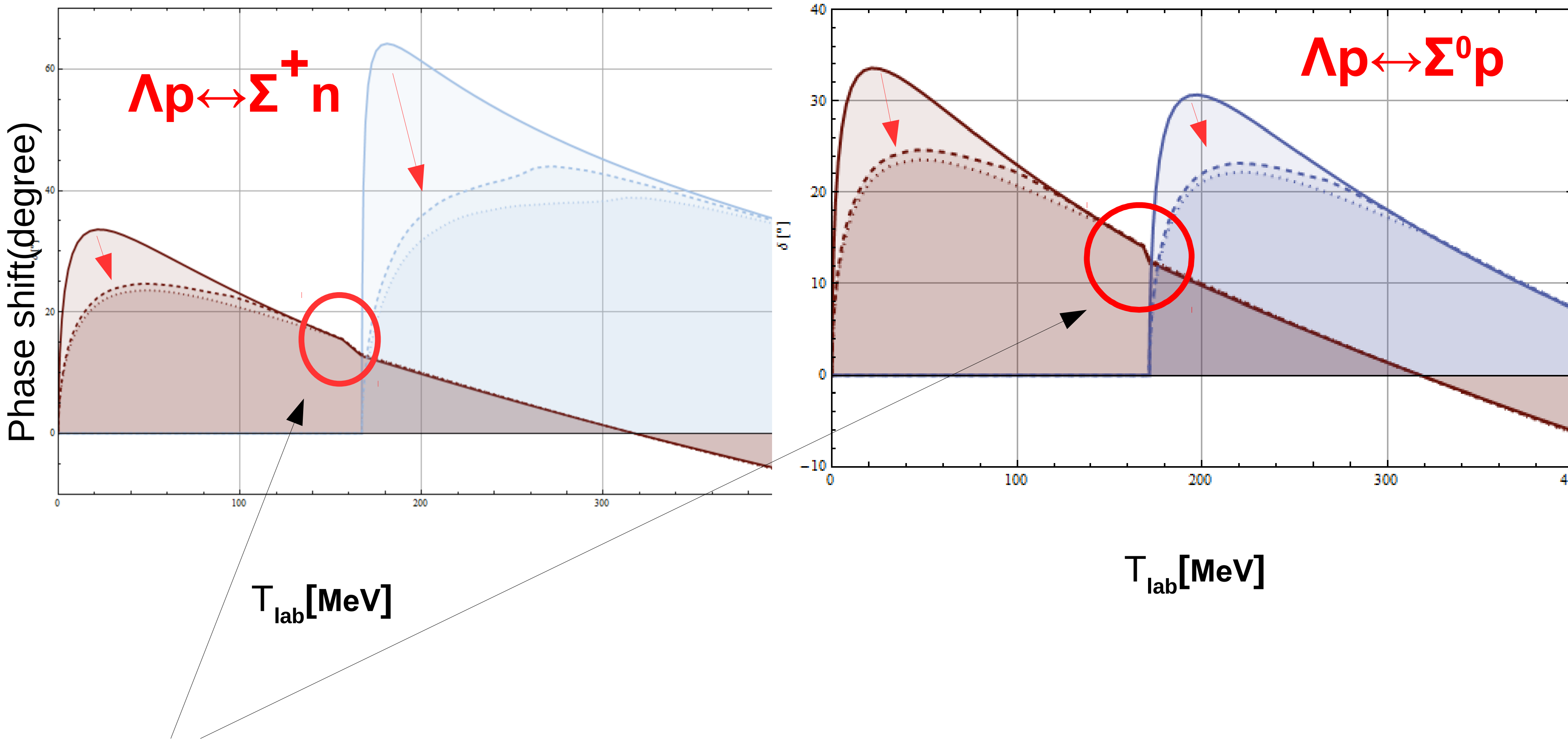




$\Sigma + p \leftrightarrow \Sigma + p$, $^1S_0$

→ Presence of medium decreases phase shift

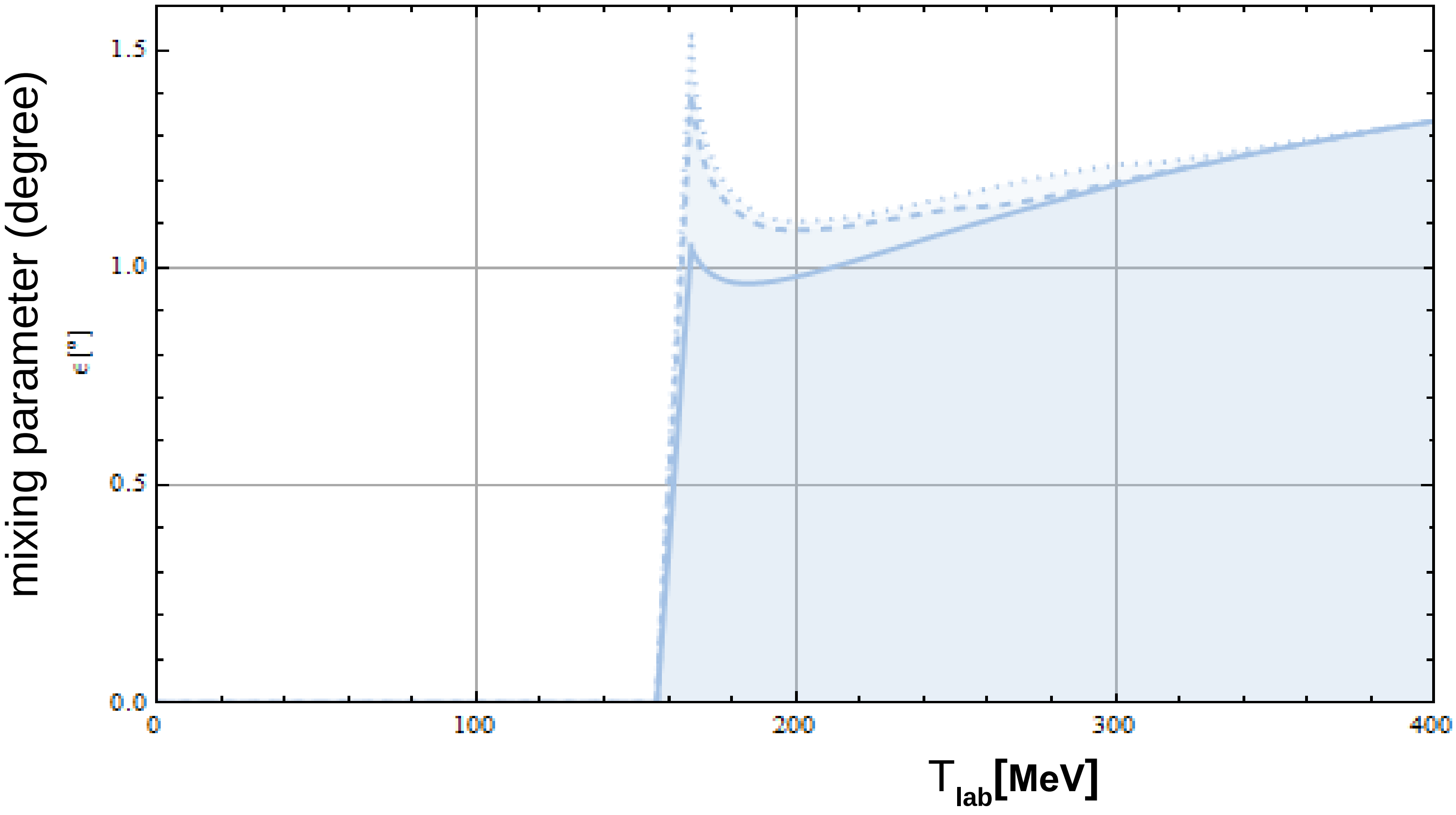
CC Phase shifts(S=0,L=0)



- Kink at the threshold(ΛN↔ΣN) in free space , suppressed in-medium
- Medium effects the 'cusp'

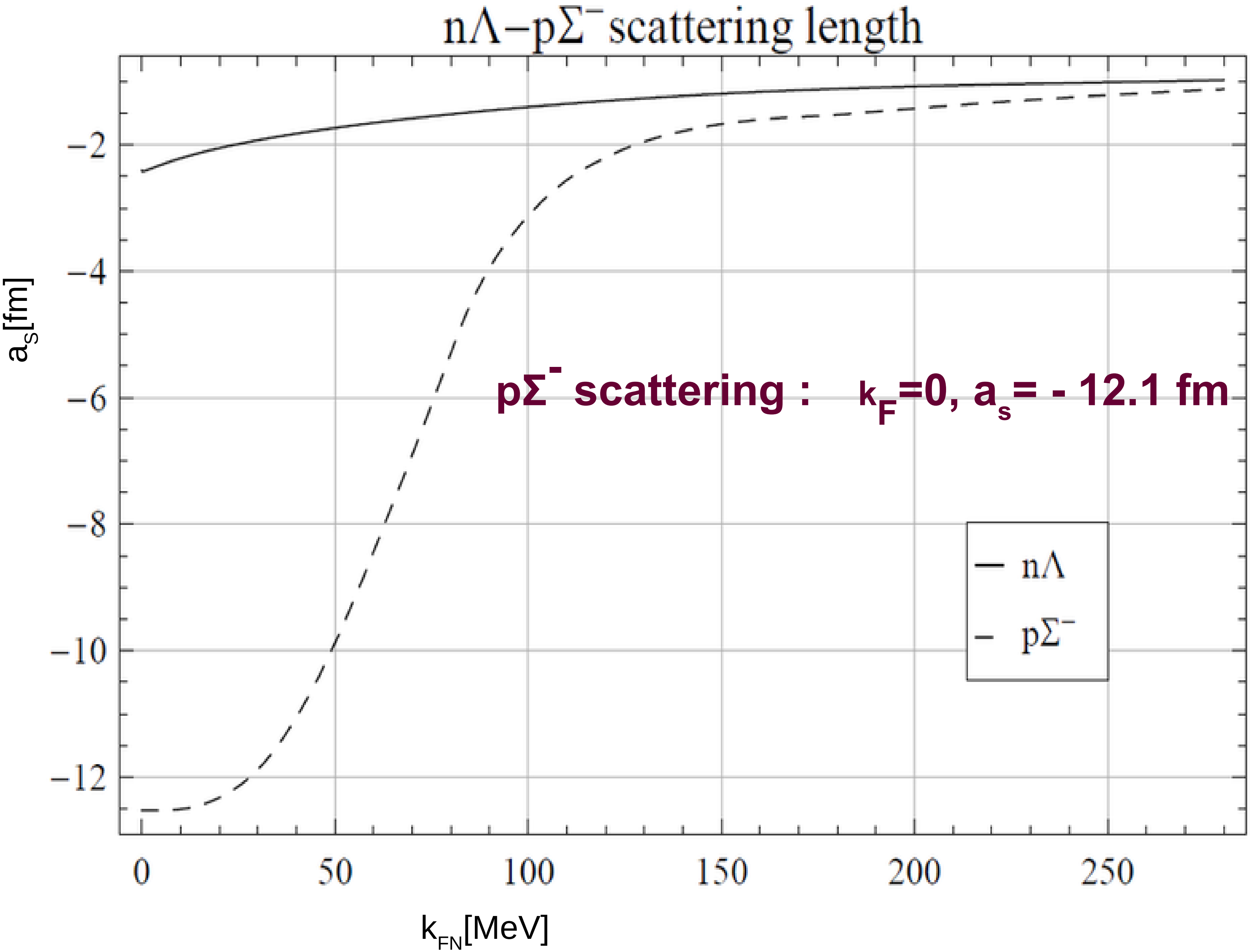
Mixing Parameter

Λp↔Σ⁰p, 1S0



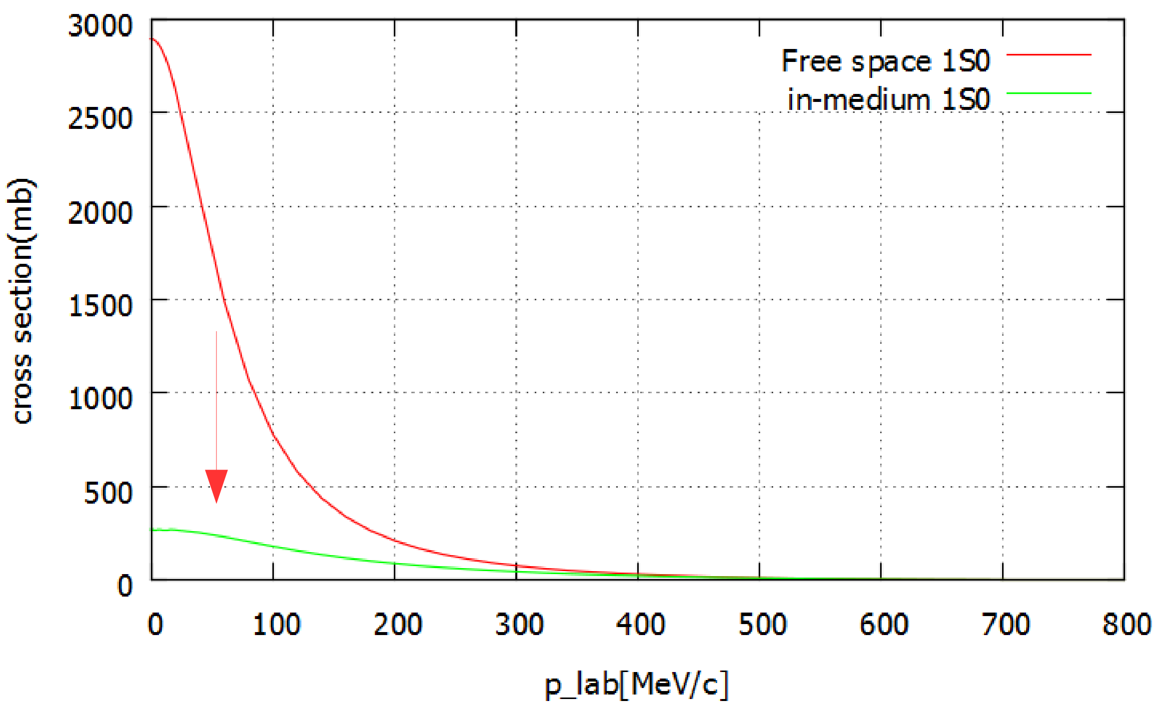
-small coupling between ΛN and ΣN channels

→ Λ-Σ^0 mixing gets affected by the medium



→ Medium results into a density dependence interaction

In-Medium: S=-2



$\Sigma^+ \Sigma^+ \rightarrow \Sigma^+ \Sigma^+$

-Decrease in cross section

-Similar effect as in S=-1

Other channels: Work in Progress

Summary

-Combined approach to BB' interaction in free-space and in-medium

-Channel coupling for fixed total S and Q

- In-medium effect by Pauli projector and self-energies

-Density dependent interactions

-In-medium effect causes

decrease in phase shift and mixing

Work in Progress

- YY interactions

- search for YN and YY bound states

-investigation of high density behaviour

-neutron star matter

- Medium effect on chiral EFT potential

- Application to Shell model calculations

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Thank you