Energy Level Displacement of Excited *np* State of Kaonic Deuterium In Faddeev Equation Approach

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Mai 21 - 22, 2012 / LEANNIS2012 Prag

Content

$$K^- + d \to K^- + d$$

Complex S– and P–Wave Scattering Lengths of K^-d Scattering

$$\left.\begin{array}{c}\tilde{a}^{(0)}_{K^-d}\\ \tilde{a}^{(1)}_{K^-d}\end{array}\right\} = \frac{m_d}{m_K + m_d}\int d^3x\,|\Phi_d(\vec{r}\,)|^2\left\{\begin{array}{c}\hat{A}^{(0)}_{K^-d}(r)\\ \hat{A}^{(1)}_{K^-d}(r)\end{array}\right.$$

- $\Phi_d(\vec{r}\,)$: R. Machleidt, K. Holinde, Ch. Elster, Phys. Rep. **149**, 1 (1987)
- $\hat{A}^{(0)}_{K^-d}(r)$: S. S. Kamalov, E. Oset, A. Ramos, NPA **690**, 494 (2004)
- $\hat{A}^{(1)}_{K^-d}(r)$: M. Faber *et al.*, PRC **84**, 064314 (2011)

Energy Level Displacements of Kaonic Deuterium

Energy Level Displacement of Excited *ns* State

$$-\epsilon_{ns} + i\,\frac{\Gamma_{ns}}{2} = 2\,\frac{\alpha^3}{n^3}\left(\frac{m_K m_d}{m_K + m_d}\right)^2 \tilde{a}^{(0)}_{K^-d}$$

Energy Level Displacement of Excited *np* State

$$-\epsilon_{np} + i\,\frac{\Gamma_{np}}{2} = 2\,\frac{\alpha^5}{n^3}\left(1 - \frac{1}{n^2}\right)\left(\frac{m_K m_d}{m_K + m_d}\right)^4 \tilde{a}^{(1)}_{K^-d}$$

- T. E. O. Ericson and W. Weise, in “Pions and Nuclei”, Clarendon Press, Oxford, 1988
- A. N. Ivanov *et al.*, PRA71, 052508 (2005)

Faddeev Equations for S–Wave K^-d Scattering

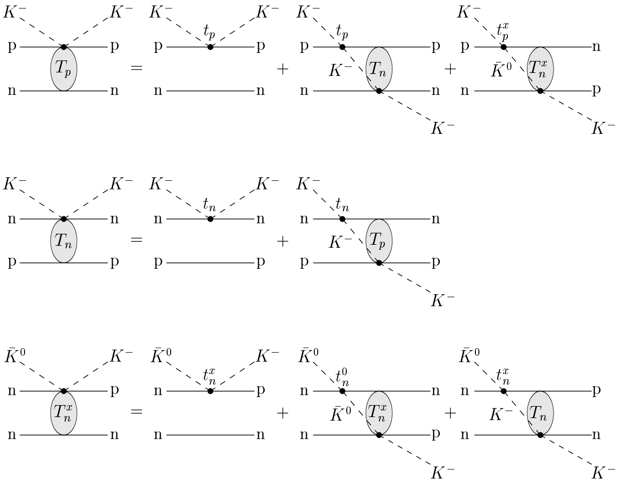
$$T_{Kd}^{(0)} = T_p^{(0)} + T_n^{(0)}$$

$$T_p^{(0)} = t_p^{(0)} + t_p^{(0)} G_0 T_n^{(0)} + t_p^{x(0)} G_0 T_n^{x(0)}$$

$$T_n^{(0)} = t_n^{(0)} + t_n^{(0)} G_0 T_p^{(0)}$$

$$T_n^{x(0)} = t_n^{x(0)} + t_n^{0(0)} G_0 T_n^{x(0)} + t_n^{x(0)} G_0 T_n^{(0)}$$

Faddeev Equations for S–Wave K^-d Scattering. Graphical illustration



Faddeev Equations for S–Wave K^-d Scattering Length in Fixed Centre Approximation

$$\hat{A}^{(0)}_{Kd}(r) = \hat{A}^{(0)}_{p}(r) + \hat{A}^{(0)}_{n}(r)$$

$$\begin{cases} \hat{A}^{(0)}_{p}(r) = \hat{a}^{(0)}_{p} + \hat{a}^{(0)}_{p}\,\frac{1}{r}\,\hat{A}^{(0)}_{n}(r) - \hat{a}^{(0)}_{x}\,\frac{1}{r}\,\hat{A}^{x(0)}_{n}(r) \\ \hat{A}^{(0)}_{n}(r) = \hat{a}^{(0)}_{n} + \hat{a}^{(0)}_{n}\,\frac{1}{r}\,\hat{A}^{(0)}_{p}(r) \\ \hat{A}^{x(0)}_{n}(r) = \hat{a}^{(0)}_{x} - \hat{a}^{0(0)}_{n}\,\frac{1}{r}\,\hat{A}^{x(0)}_{n}(r) + \hat{a}^{(0)}_{x}\,\frac{1}{r}\,\hat{A}^{(0)}_{n}(r) \end{cases}$$

- S. S. Kamalov, E. Oset, A. Ramos, NPA **690**, 494 (2004)

Faddeev Equations for P–Wave K^-d Scattering

$$T_{Kd}^{(1)} = T_p^{(1)} + T_n^{(1)}$$

$$\begin{cases} T_p^{(1)} = t_p^{(1)} + t_p^{(1)} G_0 T_n^{(0)} + t_p^{(0)} G_0 T_n^{(1)} + t_p^{x(1)} G_0 T_n^{x(0)} + t_p^{x(0)} G_0 T_n^{x(1)} \\ T_n^{(1)} = t_n^{(1)} + t_n^{(1)} G_0 T_p^{(0)} + t_n^{(0)} G_0 T_p^{(1)} \\ T_n^{x(1)} = t_n^{x(1)} + t_n^{0(1)} G_0 T_n^{x(0)} + t_n^{0(0)} G_0 T_n^{x(1)} + t_n^{x(1)} G_0 T_n^{(0)} + t_n^{x(0)} G_0 T_n^{(1)} \end{cases}$$

- M. Faber *et al.* PRC **84**, 064314 (2011)

Faddeev Equations for P–Wave K^-d Scattering Length. Fixed Centre Approximation

$$T_{Kd}^{(1)} = T_p^{(1)} + T_n^{(1)} \to A_{Kd}^{(1)}(r) = A_p^{(1)}(r) + A_n^{(1)}(r)$$

$$\begin{cases} \hat{A}_p^{(1)}(r) = \hat{a}_p^{(1)} + \frac{1}{6}\,\hat{a}_p^{(1)}\frac{1}{r}\hat{A}_n^{(0)}(r) + \frac{1}{6}\,\hat{a}_p^{(0)}\frac{1}{r}\hat{A}_n^{(1)}(r) \\ \qquad -\frac{1}{6}\,\hat{a}_x^{(1)}\frac{1}{r}\hat{A}_n^{x(0)}(r) - \frac{1}{6}\,\hat{a}_x^{(0)}\frac{1}{r}\hat{A}_n^{x(1)}(r) \\ \hat{A}_n^{(1)}(r) = \hat{a}_n^{(1)} + \frac{1}{6}\,\hat{a}_n^{(1)}\frac{1}{r}\hat{A}_p^{(0)}(r) + \frac{1}{6}\,\hat{a}_n^{(0)}\frac{1}{r}\hat{A}_p^{(1)}(r) \\ \hat{A}_n^{x(1)}(r) = \hat{a}_x^{(1)} - \frac{1}{6}\,\hat{a}_n^{0(1)}\frac{1}{r}\hat{A}_n^{x(0)}(r) - \frac{1}{6}\,\hat{a}_n^{0(0)}\frac{1}{r}\hat{A}_n^{x(1)}(r) \\ \qquad +\frac{1}{6}\,\hat{a}_x^{(1)}\frac{1}{r}\hat{A}_n^{(0)}(r) + \frac{1}{6}\,\hat{a}_x^{(0)}\frac{1}{r}\hat{A}_n^{(1)}(r) \end{cases}$$

- M. Faber *et al.* PRC **84**, 064314 (2011)

Solution of Faddeev Equations for P–Wave K^-d Scattering Length. Fixed Centre Approximation

$$\hat{A}_p^{(1)}(r)\left(1+\frac{1}{6}\frac{\hat{a}_n^{0(0)}}{r}-\frac{1}{36}\frac{\hat{a}_n^{(0)}\hat{a}_p^{(0)}}{r^2}-\frac{1}{216}\frac{\hat{a}_n^{(0)}(\hat{a}_p^{(0)}\hat{a}_n^{0(0)}-(\hat{a}_x^{(0)})^2)}{r^3}\right)=$$

$$=\hat{a}_p^{(1)}+\frac{1}{6}\frac{\hat{a}_p^{(1)}\hat{a}_n^{0(0)}}{r}+\frac{1}{6}\frac{\hat{a}_n^{(1)}\hat{a}_p^{(0)}}{r}-\frac{1}{6}\frac{\hat{a}_x^{(1)}\hat{a}_x^{(0)}}{r}$$

$$+\frac{1}{36}\frac{\hat{a}_n^{(1)}(\hat{a}_p^{(0)}\hat{a}_n^{0(0)}-(\hat{a}_x^{(0)})^2)}{r^2}+\frac{1}{6}\frac{\hat{a}_p^{(1)}}{r}\hat{A}_n^{(0)}(r)-\frac{1}{6}\frac{\hat{a}_x^{(1)}}{r}\hat{A}_n^{x(0)}(r)$$

$$+\frac{1}{36}\frac{\hat{a}_p^{(1)}\hat{a}_n^{0(0)}}{r^2}\hat{A}_n^{(0)}(r)+\frac{1}{36}\frac{\hat{a}_n^{(1)}\hat{a}_p^{(0)}}{r^2}\hat{A}_p^{(0)}(r)$$

$$+\frac{1}{36}\frac{\hat{a}_n^{0(1)}\hat{a}_x^{(0)}}{r^2}\hat{A}_n^{x(0)}(r)-\frac{1}{36}\frac{\hat{a}_x^{(1)}\hat{a}_n^{0(0)}}{r^2}\hat{A}_n^{x(0)}(r)-\frac{1}{36}\frac{\hat{a}_x^{(1)}\hat{a}_x^{(0)}}{r^2}\hat{A}_n^{(0)}(r)$$

$$+\frac{1}{216}\frac{\hat{a}_n^{(1)}(\hat{a}_p^{(0)}\hat{a}_n^{0(0)}-(\hat{a}_x^{(0)})^2)}{r^3}\hat{A}_p^{(0)}(r)$$

Solution of Faddeev Equations for P–Wave K^-d Scattering Length. Fixed Centre Approximation

$$\hat{A}_n^{(1)}(r) = \hat{a}_n^{(1)} + \frac{1}{6}\frac{\hat{a}_n^{(1)}}{r}\hat{A}_p^{(0)}(r) + \frac{1}{6}\frac{\hat{a}_n^{(0)}}{r}\hat{A}_p^{(1)}(r)$$

- M. Faber *et al.* PRC **84**, 064314 (2011)

Solution of Faddeev Equations for P–Wave K^-d Scattering Length. Effective Strong Low–Energy $\bar{K}N$ Interaction

$$\mathcal{L}_{\rm int}(x) = \mathcal{L}^{(0)}_{\rm int}(x) + \mathcal{L}^{(1)}_{\rm int}(x) =$$

$$= 4\pi[\hat{a}^{(0)}_p K^{-\dagger}(x)K^-(x)\bar{p}(x)p(x) + \hat{a}^{(0)}_x \bar{K}^{0\dagger}(x)K^-(x)\bar{n}(x)p(x)]$$

$$+4\pi[\hat{a}^{(0)}_n K^{-\dagger}(x)K^-(x)\bar{n}(x)n(x) + \hat{a}^{(0)}_x K^{-\dagger}(x)\bar{K}^0(x)\bar{p}(x)n(x)]$$

$$+4\pi[\hat{a}^{0(0)}_n \bar{K}^{0\dagger}(x)\bar{K}^0(x)\bar{n}(x)n(x)]$$

$$+12\pi[\hat{a}^{(1)}_p \bigtriangledown K^{-\dagger}(x)\cdot\bigtriangledown K^-(x)\bar{p}(x)p(x) + \hat{a}^{(1)}_x \bigtriangledown \bar{K}^{0\dagger}(x)\cdot\bigtriangledown K^-(x)\bar{n}(x)p(x)]$$

$$+12\pi[\hat{a}^{(1)}_n \bigtriangledown K^{-\dagger}(x)\cdot\bigtriangledown K^-(x)\bar{n}(x)n(x) + \hat{a}^{(1)}_x \bigtriangledown K^{-\dagger}(x)\cdot\bigtriangledown \bar{K}^0(x)\bar{p}(x)n(x)]$$

$$+12\pi[\hat{a}^{0(1)}_n \bigtriangledown \bar{K}^{0\dagger}(x)\cdot\bigtriangledown \bar{K}^0(x)\bar{n}(x)n(x)]$$

- M. Faber *et al.* PRC **84**, 064314 (2011)

Solution of Faddeev Equations for P–Wave $K^- d$ Scattering Length. Wave Function of The Deuteron

$$|d(-\vec{k},\lambda)\rangle =$$

$$= \frac{\sqrt{2E_d(\vec{k}\,)}}{(2\pi)^3}\int \frac{d^3k_p}{\sqrt{2E_N(\vec{k}_p)}}\frac{d^3k_n}{\sqrt{2E_N(\vec{k}_n)}}\,\delta^{(3)}(\vec{k}+\vec{k}_p+\vec{k}_n)$$

$$\times\,\tilde{\Phi}_d\left(\frac{\vec{k}_p-\vec{k}_n}{2}\right)[a^\dagger_p(\vec{k}_p,\sigma_p)a^\dagger_n(\vec{k}_n,\sigma_n)]_{\sigma_p+\sigma_n=\lambda}|0\rangle$$

$$\langle d(-\vec{k}\,',\lambda'|d(-\vec{k},\lambda)\rangle = (2\pi)^3 2E_d(\vec{k}\,)\delta^{(3)}(\vec{k}\,'-\vec{k}\,)\,\delta_{\lambda'\lambda}$$

- M. Faber *et al.* PRC **84**, 064314 (2011)

Solution of Faddeev Equations for P–Wave K^-d Scattering Length. Wave Function of The Deuteron

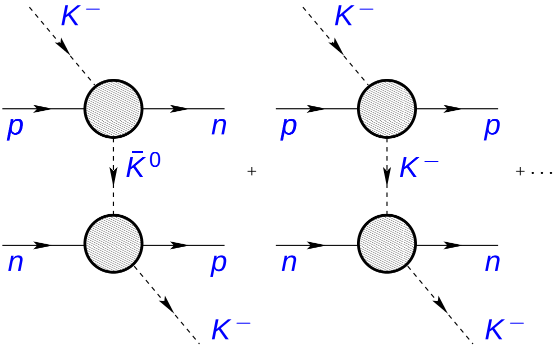


Figure: Feynman diagrams of the amplitude of the double–scattering contribution to the P–wave scattering length of K^-d scattering

Complex P–wave Scattering Length

in Single (Impulse) and Double Scattering Approximation

$$\tilde{a}^{(1)}_{K^-d} = \frac{m_d}{m_K + m_d}$$

$$\times\left(\hat{a}^{(1)}_p + \hat{a}^{(1)}_n + \frac{1}{3}\left(\hat{a}^{(0)}_p \hat{a}^{(1)}_n + \hat{a}^{(0)}_n \hat{a}^{(1)}_p - \hat{a}^{(0)}_x \hat{a}^{(1)}_x\right) \int \frac{d^3x}{r} |\Phi_d(\vec{r}\,)|^2\right) =$$

$$= -0.262 + i\,0.548\ \mathrm{fm}^3$$

- M. Faber *et al.* PRC **84**, 064314 (2011)

Solution of Faddeev Equations for P–Wave K^-d Scattering Length. Fixed Centre Approximation

Triple Scattering Contribution

$$(\tilde{a}^{(1)}_{K^-d})_{\rm tr.sc.} = \frac{m_d}{m_K + m_d}$$

$$\times \frac{1}{36}\Big[\hat{a}^{(1)}_p\Big(7\hat{a}^{(0)}_p\hat{a}^{(0)}_n + (\hat{a}^{(0)}_n)^2 - (\hat{a}^{0(0)}_n)^2\Big)$$

$$+\hat{a}^{(1)}_n\Big(7(\hat{a}^{(0)}_p\hat{a}^{(0)}_n - (\hat{a}^{(0)}_x)^2) + \hat{a}^{(0)}_p(\hat{a}^{(0)}_n + \hat{a}^{0(0)}_n) - 2\hat{a}^{(0)}_n\hat{a}^{(0)}_x\Big)$$

$$+\hat{a}^{(1)}_x\hat{a}^{(0)}_x)\Big(\hat{a}^{0(0)}_n - \hat{a}^{(0)}_n\Big) + \hat{a}^{0(1)}_n(\hat{a}^{(0)}_x)^2\Big]\int\frac{d^3x}{r^2}\,|\Phi_d(\vec{r}\,)|^2 =$$

$$= -0.015 - i\,0.023\,{\rm fm}^3$$

- M. Faber *et al.* PRC **84**, 064314 (2011)

Chiral and $SU(3)$ Coupled–Channel Approach for $\bar{K}N \to PB$: $M^{-1} = M_0^{-1} - G$

$$\mathcal{L}_{\chi D}(x) = \langle \bar{B}(x) i\gamma^\mu [s_\mu(x), B(x)]\rangle - g_A(1-\alpha_D)\langle \bar{B}(x)\gamma^\mu [p_\mu(x), B(x)]\rangle$$

$$+\alpha_D\langle \bar{B}(x)\gamma^\mu\{p_\mu(x), B(x)\}\rangle + \frac{1}{4} b_D\langle \bar{B}(x)\{\chi_+(x), B(x)\}\rangle$$

$$+\frac{1}{4} b_F\langle \bar{B}(x)[\chi_+(x), B(x)]\rangle + \frac{1}{4} b_0\langle \bar{B}(x)\langle\chi_+(x)\rangle B(x)\rangle$$

$$+\frac{1}{2} d_1\langle \bar{B}(x)\{p_\mu(x), [p^\mu(x), B(x)]\}\rangle + \frac{1}{2} d_2\langle \bar{B}(x)[p_\mu(x), [p^\mu(x), B(x)]]\rangle$$

$$+\frac{1}{2} d_3\langle \bar{B}(x)p_\mu(x)\rangle\langle p^\mu(x)B(x)\rangle + \frac{1}{2} d_4\langle \bar{B}(x)\langle p_\mu(x)p^\mu(x)\rangle B(x)\rangle$$

$$+g_{\Lambda^*}\bar{\Lambda}^*(x)\gamma^\mu\gamma^5\langle p_\mu(x)B(x)\rangle + \sqrt{2}\, g_\Delta \bar{D}^{abc}_\mu(x)\Theta^{\mu\nu}\gamma^5(p_\nu(x))^d_a B^e_b(x)\,\varepsilon_{cde} + \ldots$$

$$\epsilon^{(\mathrm{exp})}_{1s} = 283(37)\,\mathrm{eV} \quad \Gamma^{(\mathrm{exp})}_{1s} = 541(92)\,\mathrm{eV}$$

M. Bazzi *et al.* (SIDDHARTA), PLB **704**, 113 (2011)

Complex S– and P–Wave Scattering Lengths

$$\tilde{a}^{(0)}_{K^-d} = -1.273 + i\,2.435\,\mathrm{fm} \quad \tilde{a}^{(1)}_{K^-d} = -0.352 + i\,0.432\,\mathrm{fm}^3$$

Energy Level Displacements of Kaonic Deuterium

$$\epsilon_{1s} = 0.766\,\mathrm{keV} \qquad \Gamma_{1s} = 2.933\,\mathrm{keV}$$

$$\epsilon_{2p} = 4.158\,\mathrm{meV} \qquad \Gamma_{2p} = 10.203\,\mathrm{meV}$$

Yield of X–Rays of K_α emission line

$$Y_{K^-p} = 1.80\,\%\,, \qquad \Gamma_{1p} = 1.979\,\mathrm{meV}$$

$$Y^{(\mathrm{exp})}_{K^-p} = 1.5(5)\,\% \quad \mathrm{(KEK)\ PRC58,\,2366\,(1998)}$$

$$Y_{K^-d} = 0.27\,\%\,, \qquad \Gamma_{2p} = 10.203\,\mathrm{meV}$$

$\bar{K}N$ Data by W. Weise *et al.* and Kaonic Deuterium

- EPJA **25**, 79 (2005): S–Wave $\bar{K}N$ Scattering Lengths
- NPA **804**, 173 (2008): P–Wave $\bar{K}N$ Scattering Lengths

Complex S– and P–Wave Scattering Lengths
of K^-d Scattering

$$\tilde{a}^{(0)}_{K^-d} = -1.951 + i\,0.996\,\mathrm{fm} \qquad \tilde{a}^{(1)}_{K^-d} = -0.174 + i\,0.113\,\mathrm{fm}^3$$

Energy Level Displacements of Kaonic Deuterium

$$\epsilon_{1s} = 1.175\,\mathrm{keV} \qquad \Gamma_{1s} = 1.200\,\mathrm{keV}$$

$$\epsilon_{2p} = 2.053\,\mathrm{meV} \qquad \Gamma_{2p} = 2.675\,\mathrm{meV}$$

Yield of X–Rays of K_α emission line

$$Y_{K^-d} = 1.9\,\% \qquad \Gamma_{2p} = 2.675\,\mathrm{meV}$$

Accuracy of Obtained Solution of Faddeev Equations

Assumption

- We assume that the accuracy of our solution of the Faddeev equations in the fixed centre approximation for the complex P–wave scattering length of K^-d scattering is of about $15\,\%$

Ground of Assumption

- A. Gal, arXiv: nucl–th/0607067: Accuracy of S–wave scattering length of K^-d scattering as a solution of Faddeev equations in the fixed centre approximation is $(10-25)\,\%$
- V. Baru, E. Epelbaum, A. Rusetsky, EPJA **42**, 111 (2009): Nucleon recoil corrections to the double-scattering contribution to S–wave scattering length of K^-d scattering is $(10-15)\,\%$

Summary

- We have proposed the solution of the Faddeev coupled–channel equations in the fixed centre approximation for the P–wave scattering length of K^-d scattering within chiral $SU(3) \times SU(3)$ dynamics and $SU(3)$ coupled channel approach for low–energy $\bar{K}N$ scattering
- We have calculated the energy level displacements for the ground $1s$ and the excited $2p$ states of kaonic deuterium and the yield of X–rays of K_α emission line
- The obtained results can be used for the planning of experiments on the measurements of the energy level displacement of the ground state of kaonic deuterium and for SIDDHARTA Collaboration, measuring currently the energy level displacement of the ground state of kaonic deuterium

FWF Project on the Contract P 19487-N16

- The work on the theoretical analysis of $K^- d$ scattering in the P–wave state and energy level displacements of kaonic deuterium in the excited np state was supported by the Austrian “Fonds zur Förderung der Wissenschaftlichen Forschung” (FWF) under the contract P 19487-N16

Thank You for Attention