$K^- d$ scattering and properties of kaonic deuterium

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K^-d elastic scattering: three-body coupled-channel AGS equations (Faddeev equations in Alt-Grassberger-Sandhas form)

$$U_{ij}^{\alpha\beta} = \delta_{\alpha\beta}\left(1-\delta_{ij}\right)\left(G_0^{\alpha}\right)^{-1} + \sum_{k,\gamma=1}^{3}\left(1-\delta_{ik}\right)T_k^{\alpha\gamma}G_0^{\gamma}U_{kj}^{\gamma\beta}, \qquad i,j=1,2,3$$

$\pi\Sigma$ channel included directly $\Rightarrow$ particle channels:

$$\alpha=1:\left|\bar{K}_1\, N_2\, N_3\right\rangle, \quad \alpha=2:\left|\pi_1\, \Sigma_2\, N_3\right\rangle, \quad \alpha=3:\left|\pi_1\, N_2\, \Sigma_3\right\rangle$$

Input: two-body T-matrices:

T^{NN}, $T^{\Sigma N}$, and $T^{\pi N}$ are one-channel (usual) T-matrices;

T^{KK} : $\bar{K}N \to \bar{K}N$, $\quad T^{K\pi}$: $\pi\Sigma \to \bar{K}N$,

$T^{\pi K}$: $\bar{K}N \to \pi\Sigma$, $\quad T^{\pi\pi}$: $\pi\Sigma \to \pi\Sigma$ $\quad$ - elements of 2-channel $T^{\bar{K}N-\pi\Sigma}$

Quantum numbers: spin $S=1$, orbital momentum $L=0$, isospin $I=1/2$

Two identical nucleons - antisymmetrization $\Rightarrow$ system of 10 integral equations

Coupled-channel $\bar{K}N - \pi\Sigma$ interaction

J. Révai, N.V.S., Phys. Rev. C 79 (2009) 035202; new fits

A phenomenological potential with one- and two-pole structure of $\Lambda(1405)$ resonance, equally properly reproducing:

- Measured 1*s* K^-p level shift and width:

$$\Delta E_{1s}^{KEK} = -323 \pm 63 \pm 11 \text{ eV}, \quad \Gamma_{1s}^{KEK} = 407 \pm 208 \pm 100 \text{ eV}$$

$$\Delta E_{1s}^{DEAR} = -193 \pm 37 \pm 6 \text{ eV}, \quad \Gamma_{1s}^{DEAR} = 249 \pm 111 \pm 30 \text{ eV}$$

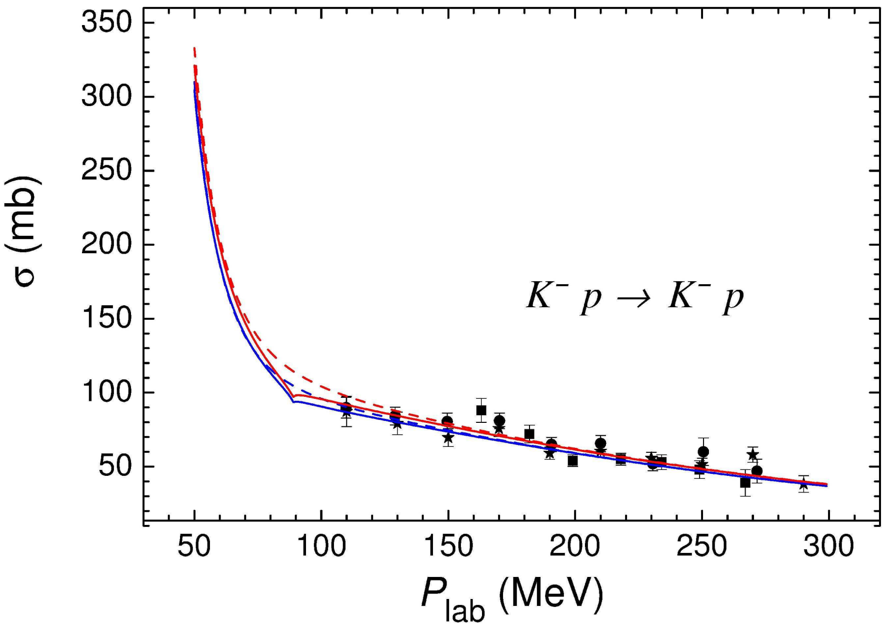
$$\Delta E_{1s}^{SIDD} = -283 \pm 36 \pm 6 \text{ eV}, \quad \Gamma_{1s}^{SIDD} = 541 \pm 89 \pm 22 \text{ eV}$$

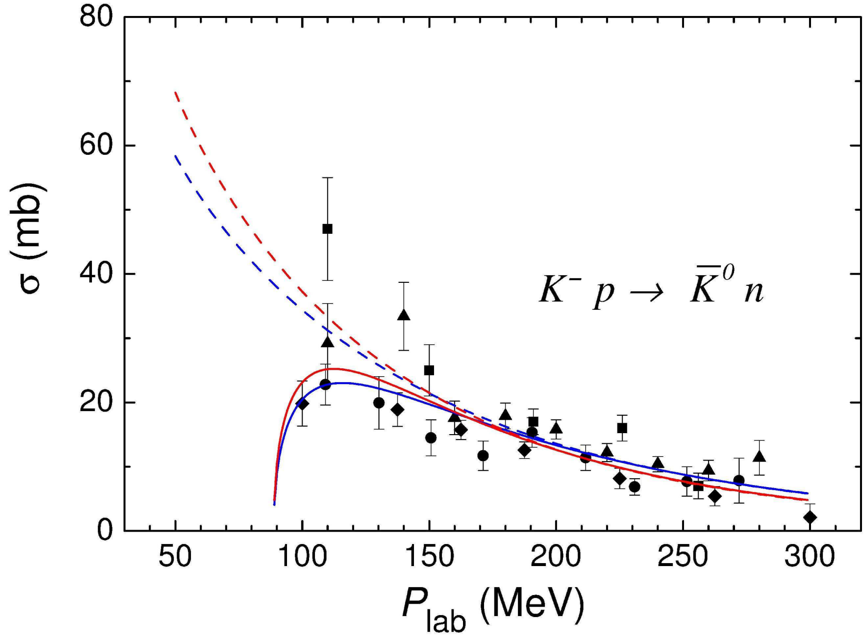
- Cross-sections of $K^- p \to K^- p$ and $K^- p \to MB$ reactions,
- Threshold branching ratios γ and $R_{\pi\Sigma} = \dfrac{R_c}{1 - R_n(1 - R_c)}$,

Isospin-breaking effects:

1. Kaonic hydrogen: direct inclusion of *Coulomb interaction*
2. Using of the *physical masses*:

$$m_{K^-}, m_{\bar{K}^0}, m_p, m_n \quad \text{instead of} \quad m_{\bar{K}}, m_N$$

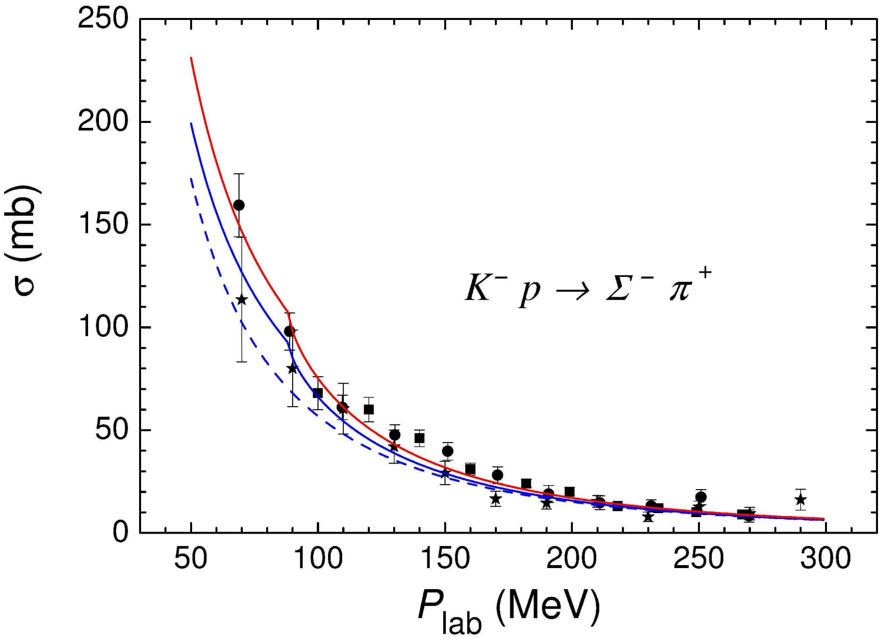


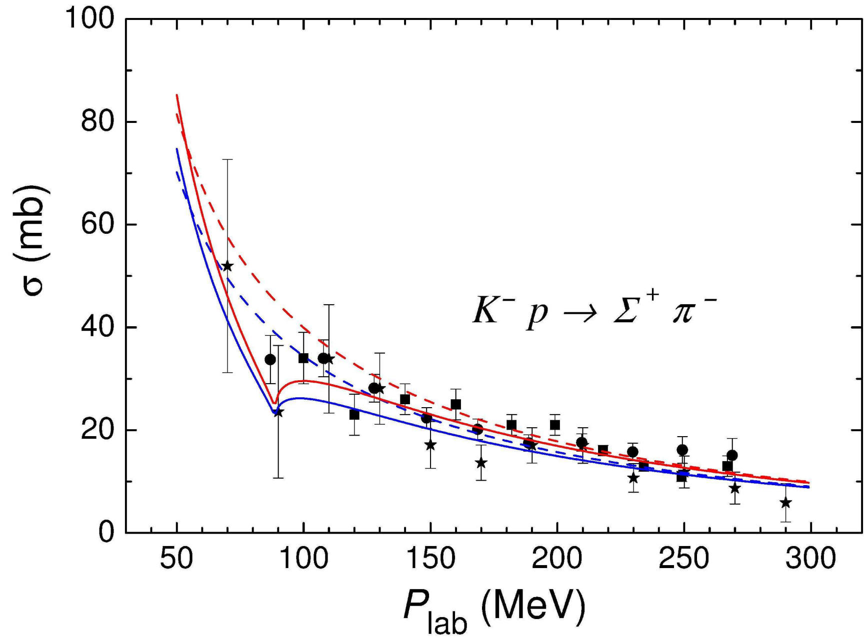


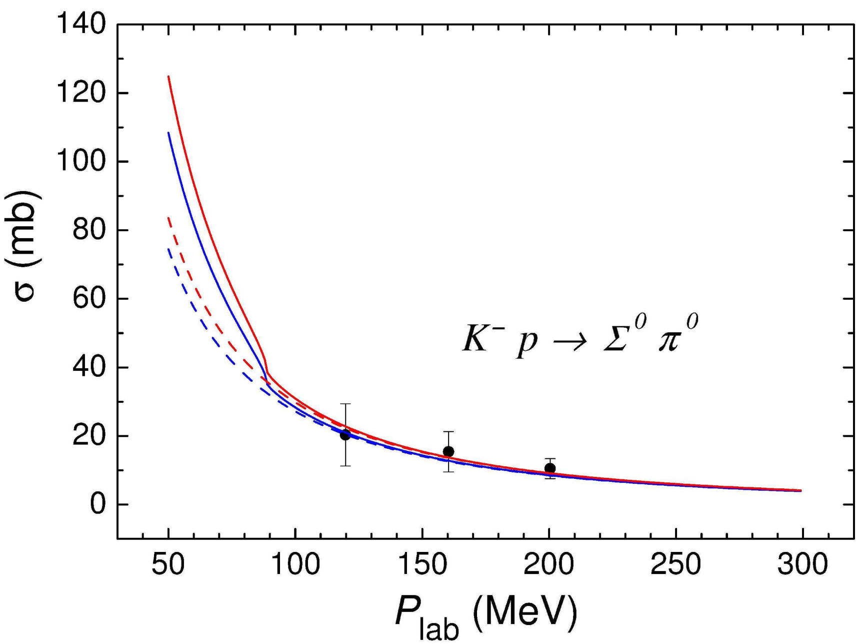
Comparison with experimental data, cross-sections:

one-pole potential: with physical masses (blue solid line)
with averaged masses (blue dashed line)

two-pole potential: with physical masses (red solid line)
with averaged masses (red dashed line)

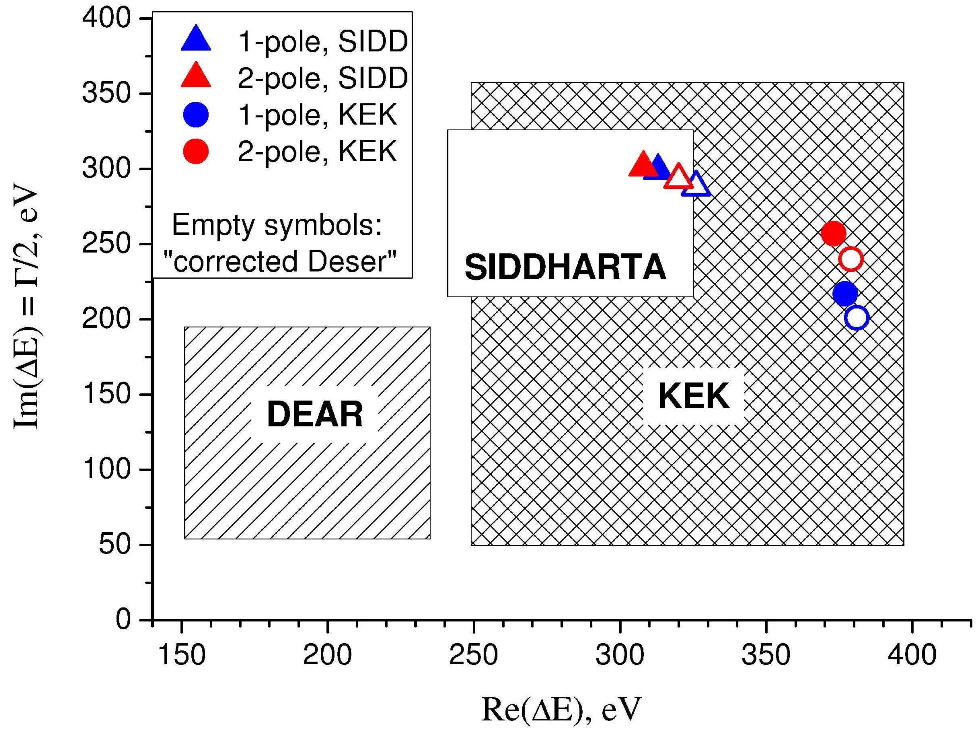






Comparison with experimental data
(continuation)

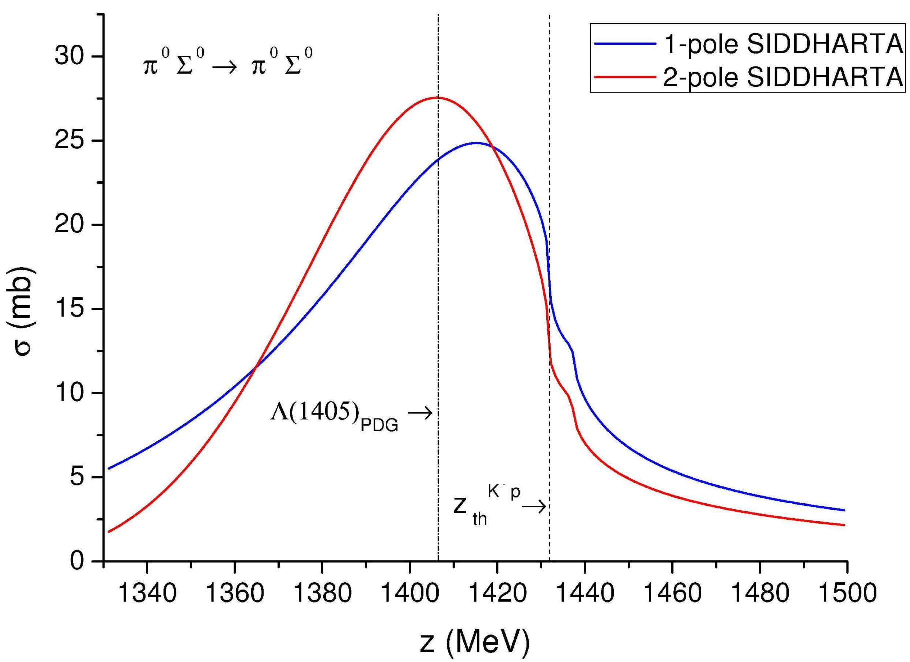
Experimental and theoretical 1s K^-p level shift and width:



Pole positions

1-pole, KEK	2-pole, KEK	1-pole, SIDD	2-pole,SIDD
1409 – *i* 36 MeV	1409 – *i* 36 MeV 1381 – *i* 105 MeV	1426 – *i* 48 MeV	1414 – *i* 58 MeV 1386 – *i* 104 MeV

Elastic $\pi\Sigma - \pi\Sigma$ cross sections for 1- and 2-pole SIDDHARTA potentials

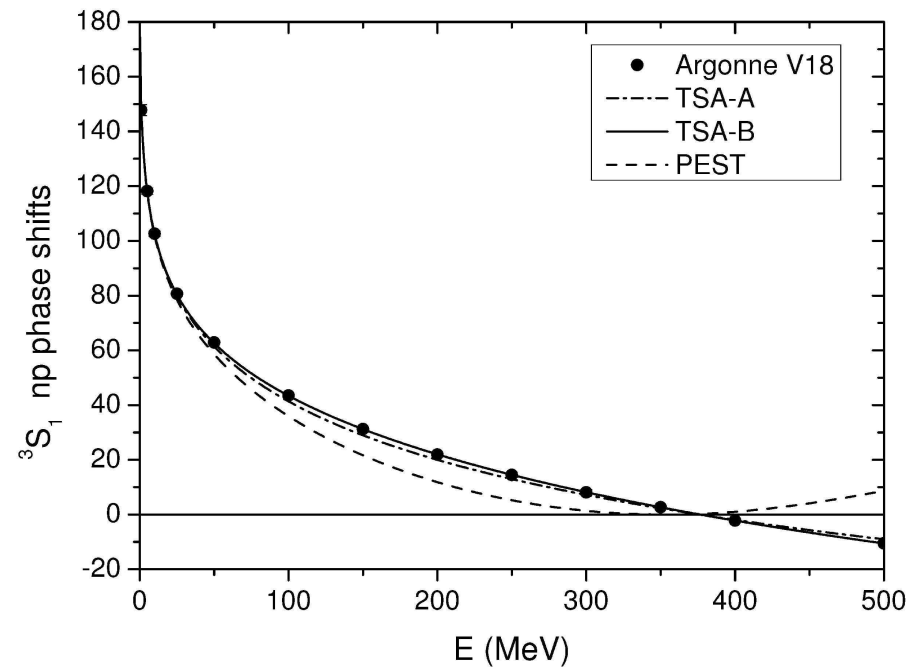


Two-term *NN* potential (TSA)

P. Doleschall, private communication, 2009

$$V_{NN} = \sum_{i=1}^{2} \left| g_i \right\rangle \lambda_i \left\langle g_i \right| \quad \rightarrow$$

$$T_{NN} = \sum_{i,j=1}^{2} \left| g_i \right\rangle \tau_{ij} \left\langle g_j \right|$$



Reproduces:

Argonne V18 NN phase shifts (with sign change)

$$a^A(np) = -5.402 \text{ fm}, \;\; r_{eff}^A(np) = 1.754 \text{ fm},$$

$$a^B(np) = -5.413 \text{ fm}, \;\; r_{eff}^B(np) = 1.760 \text{ fm},$$

$$\text{and} \;\; E_{deu} = -2.2246 \text{ MeV}.$$

$\Sigma N(-\Lambda N)$ interaction   *J. Révai, N.V.S., 2009*

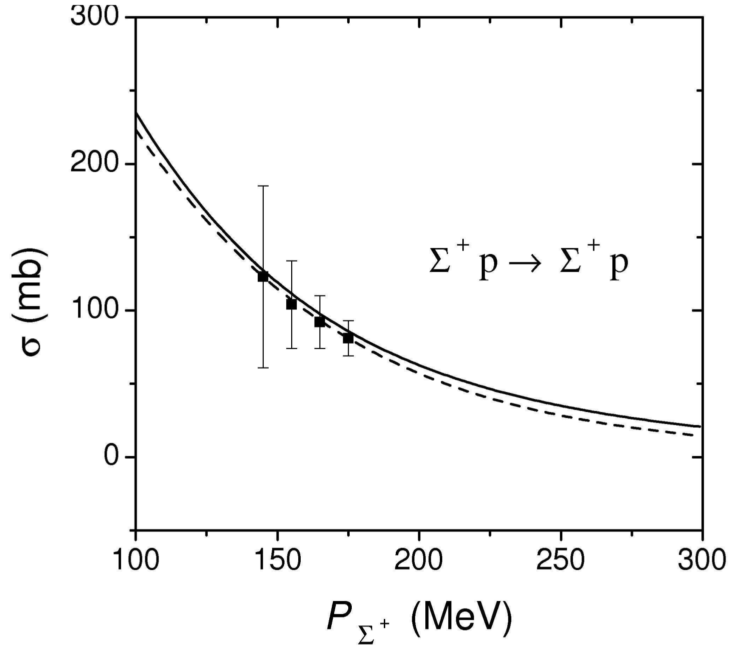
Isospin and spin-dependent $T_{I,S}^{\Sigma N}(k,k';z)$

corresponds to

$$V_{I,S}^{\Sigma N}(k,k') = \lambda_{I,S}^{\Sigma N}\; g_{I,S}^{\Sigma N}(k)\; g_{I,S}^{\Sigma N}(k')$$

with

$$g_{I,S}^{\Sigma N}(k) = \frac{1}{k^2 + \left(\beta_{I,S}^{\Sigma N}\right)^2}$$



K^-d calculation: spin-triplet only

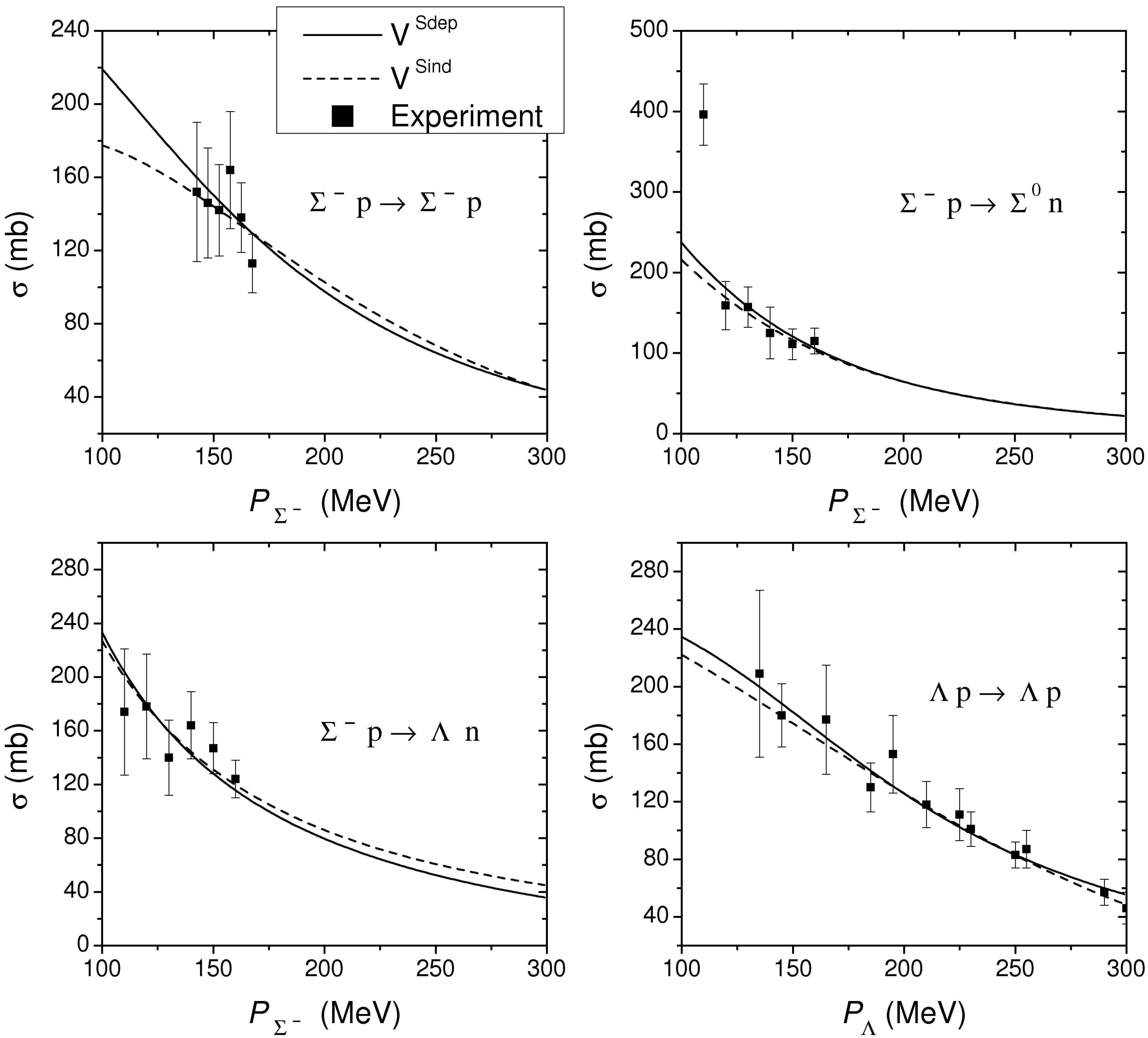
All parameters were <u>fitted</u> to reproduce experimental cross-sections

<u>I=3/2</u>

Real parameters, one-channel case

<u>I=1/2</u>

1. Two-channel $\Sigma N - \Lambda N$ potential, real parameters
2. One-channel (exact) optical ΣN potential, complex energy-dependent strength

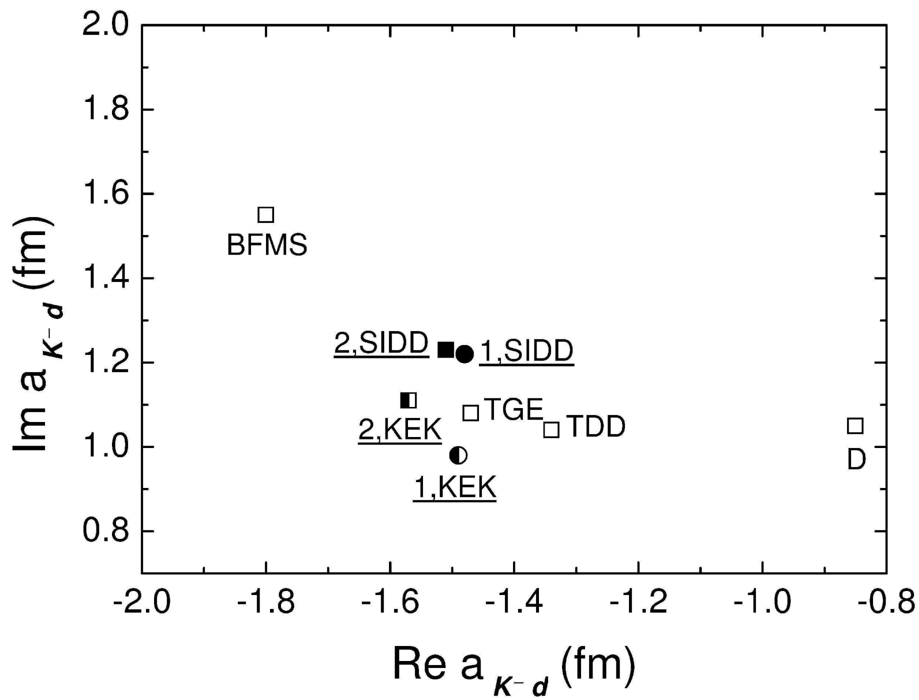


V^{Sdep}
V^{Sind}
Experiment
$\Sigma^- p \rightarrow \Sigma^- p$
$\Sigma^- p \rightarrow \Sigma^0 n$
$\Sigma^- p \rightarrow \Lambda n$
$\Lambda p \rightarrow \Lambda p$
σ (mb)
P_{Σ^-} (MeV)
P_{Λ} (MeV)

$K^{-}d$ scattering lengths (fm)

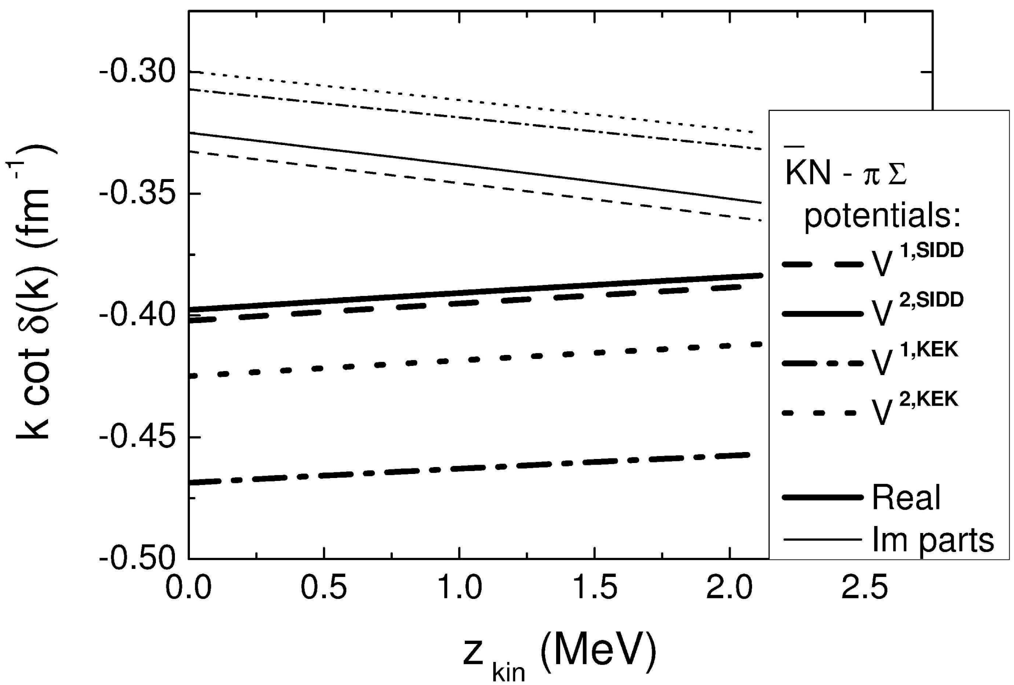
from coupled-channels Faddeev-type (AGS) calculations

N.V. Shevchenko, Phys. Rev. C 85, 034001 (2012); arXiv:1201.3173, v2 [nucl-th]



Experiment: measuring of kaonic deuterium atom 1s level shift and width (SIDDHARTA, SIDDHARTA-2) $\longrightarrow$
Calculation of the parameters, corresponding to the obtained K^{-} d scattering length

$K^- d$ scattering amplitudes as a $k \cot \delta(k)$ function



$K^- d$ effective ranges (fm)

1-pole, KEK	2-pole, KEK	1-pole, SIDD	2-pole,SIDD
$0.55 - i\,1.15$	$0.62 - i\,1.19$	$0.68 - i\,1.33$	$0.67 - i\,1.35$

Strong characteristics of kaonic deuterium

Step 1. Optical two-body K^- d potential is constructed, it reproduces near-threshold K^-d scattering amplitudes, obtained in the three-body calculation (scattering length and effective range are reproduced as well)

$$V^S_{K^-d}(k,k') = \lambda_{1,K^-d}\, g_1(k) g_1(k') + \lambda_{2,K^-d}\, g_2(k) g_2(k')$$

$$\text{with} \quad g_i(k) = \frac{1}{\beta^2_{i,K^-d} + k^2}, \quad i = 1,2$$

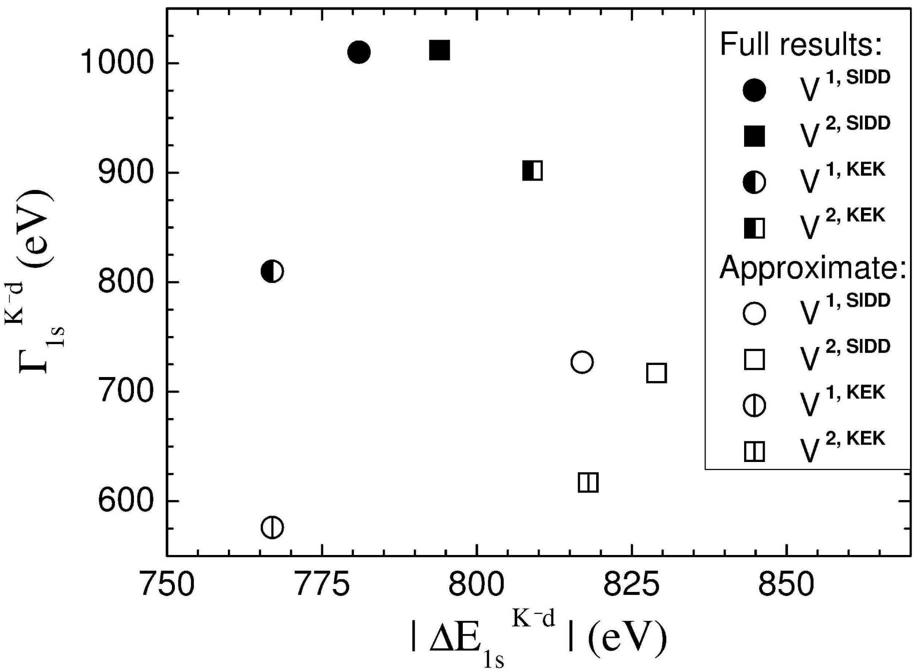
Step 2. Energy of the *1s* level of kaonic deuterium is calculated, Coulomb interaction was directly included into the Lippmann-Schinger equation:

$$H = H_0 + V^S_{K^-d}(k,k') + V^{Coul} \quad \Rightarrow \quad E^{S+Coul}_{1s}$$

Step 3. *1s* level shift and width caused by strong interaction are obtained:

$$\Delta E_{1s} = E^{Coul}_{1s} - \mathrm{Re}(E^{S+Coul}_{1s})$$

Strong interaction shift and width of the kaonic deuterium atom 1*s* level state



Corrected Deser formula $(\mu_{K^-p} \to \mu_{K^-d}, a_{K^-p} \to a_{K^-d})$: ~ 30% error in width!

$$\Delta E_{1s}^{K^-d} - i\frac{\Gamma_{1s}^{K^-d}}{2} = -2\alpha^3 \mu_{K^-d}^2 a_{K^-d}[1 - 2\alpha\, \mu_{K^-d} a_{K^-d}\ (\ln\alpha - 1)]$$

$\bar{K}N - \pi\Sigma$ pole positions (MeV)

1-pole, KEK	2-pole, KEK	1-pole, SIDD	2-pole,SIDD
1409 – *i* 36	1409 – *i* 36 1381 – *i* 105	1426 – *i* 48	1414 – *i* 58 1386 – *i* 104

$K^- d$ scattering lengths (fm)

1-pole, KEK	2-pole, KEK	1-pole, SIDD	2-pole,SIDD
– 1.49 + *i* 0.98	– 1.57 + *i* 1.11	– 1.48 + *i* 1.22	– 1.51 + *i* 1.23

Kaonic deuterium *1s* level shifts (*Re*) and half-widths (*Im*) (eV)

1-pole, KEK	2-pole, KEK	1-pole, SIDD	2-pole,SIDD
– 767 + *i* 405	– 809 + *i* 451	– 781 + *i* 505	– 794 + *i* 506

Summary

- $K^- d$ scattering amplitudes were calculated for the four versions of $\bar{K}N - \pi\Sigma$ potential up to three-body $\bar{K}NN$ break-up threshold

- The potentials reproduce SIDDHARTA (and KEK) data on kaonic hydrogen *1s* level shift and width and have one or two poles for the $\Lambda(1405)$ resonance

- The corresponding to the $K^- d$ scattering amplitudes *1s* level shift and width of kaonic deuterium are calculated:

$$\Delta E_{1s}^{1,SIDD} = -781\ eV, \quad \Gamma_{1s}^{1,SIDD} = 1010\,\text{eV}$$
$$\Delta E_{1s}^{2,SIDD} = -794\ eV, \quad \Gamma_{1s}^{2,SIDD} = 1012\,\text{eV}$$

- Kaonic deuterium characteristics are strongly correlated to the K^- d scattering length, they seems to be insensitive to the number of poles forming $\Lambda(1405)$ resonance