On Partial Fuzzy Type Theory

NOVÁK Vilém

University of Ostrava
Institute for Research and Applications of Fuzzy Modeling
NSC IT4Innovations, 30. dubna 22, 701 03 Ostrava 1, Czech Republic
E-mail: Vilem.Novak@osu.cz

This paper is a study of fuzzy type theory (FTT) with partial functions. We introduce a special value "$*$" to all the types which represents "undefined". In the interpretation of FTT, this value lays outside of the corresponding domains. The value $*_o$ of type o is defined as the formula $\iota_{o(oo)} \cdot \lambda x_o \bot$ which means application of the description operator to the empty set. Similarly, the $*_\epsilon$ is defined as $\iota_{\epsilon(o\epsilon)} \cdot \lambda x_\epsilon *_o$, i.e., the description operator is applied to a fuzzy set on M_ϵ whose membership function is nowhere defined. For higher types we define

$$*_{\beta\alpha} \equiv \lambda x_\alpha *_\beta$$

which means that "undefined" is a nowhere defined function from the set M_α of type α to a set M_β of type β.

In the development of FTT with partial functions, we must be careful because the value "undefined" is a well formed formula. The outcome is that $T \vdash A_o \equiv *_o$ means that the formula A_o is in the theory T equal to "undefined". This cannot be true because otherwise A_o would have to be also undefined. Consequently, a formula A_o is defined if $T \vdash \neg(A_o \equiv *_o)$. We thus introduce two special predicates "?" (the given formula is undefined) and "!" (the given formula is defined) which can be extended to all types.

Important outcome of our approach is that the λ-conversion is preserved which makes our system of FTT very powerful. Among many results, we show that $T \vdash *_o$ implies that T is contradictory. We prove that any consistent theory of FTT with partial functions has a model. We can also include the theory presented in the papers [4, 5] as a special theory of partial FTT. The proposed extension of FTT works of all (so far considered) kinds of algebras of truth values.

References

[1] P. Andrews, *An Introduction to Mathematical Logic and Type Theory: To Truth Through Proof*, Kluwer, Dordrecht, 2002.

[2] L. Bhounek, V. Novk: *Towards fuzzy partial logic.* In: Proceedings of the IEEE 45th Intl. Symposium on Multiple-Valued Logics (ISMVL 2015), pp. 139-144, 2015

Acknowledgement This paper was supported by the grant 16-19170S of GAČR, Czech Republic.

[3] W. M. Farmer, *A partial functions version of Church's simple theory of types*, Journal of Symbolic Logic 55 (1990) 1269–1291.

[4] S. Lappiere, *A functional partial semantics for intensional logic,* Notre Dame Journal of Formal Logic 33 (1992) 517–541.

[5] F. Lepage, *Partial functions in type theory*, Notre Dame Journal of Formal Logic 33 (1992) 493–516.

[6] V. Novák, *On fuzzy type theory*, Fuzzy Sets and Systems 149 (2005) 235–273.

[7] V. Novák, *EQ-algebra-based fuzzy type theory and its extensions*, Logic Journal of the IGPL 19 (2011) 512–542.

[8] V. Novák, *Fuzzy type theory with partial functions.* Iranian Journal of Fuzzy Systems (submitted).