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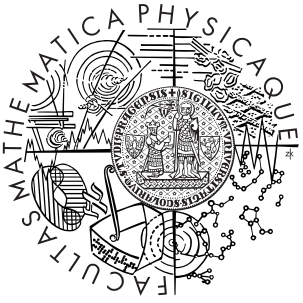

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Few-Body Systems with Non-Zero Strangeness

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I declare that I wrote my diploma thesis independently and exclusively with the use of the cited sources. I agree with lending and publishing the thesis.

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In Prague, Apr 30, 2008 Jan Donoval

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Abstract: In this work, we studied the K^-d scattering length. We formulated the Faddeev equations in the AGS form for the K^-d system. We performed three-body Faddeev calculations in the s-wave approximation in the isospin basis. We studied the sensitivity of the K^-d scattering length on the two-body inputs. Our results are reasonably close to the values of previous calculations.
Keywords: scattering length, Faddeev equations, $\bar{K}N$ interaction

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Abstrakt: Pro K^-d systém byly formulovány Fadějevovy rovnice v AGS formalismu. V isospinové bázi byla počítána rozptylová délka K^-d v s-vlnovém přiblížení a studovány její závislosti na vstupních parametrech. Výsledky práce jsou ve shodě s předchozími výpočty.
Klíčová slova: rozptylová délka, Fadějevovy rovnice, $\bar{K}N$ interakce

1 Introduction

This work is devoted to the calculations of the K^--deuteron scattering length within the Faddeev approach. The two-body $\bar{K}N$ interaction plays an essential role in our considerations.

The $\bar{K}N$ interaction near threshold is known to be strongly attractive as well as strongly absorptive. It is mainly affected by the subthreshold $I = 0$ resonance $\Lambda(1405)$, which is usually assumed a $\bar{K}N$ bound state and a resonance in the $\pi\Sigma$ channel [1]. The experimental data which have been used to constrain the $\bar{K}N$ interaction consist of cross sections of low-energy K^-p scattering and reactions [2], and of the position and width of the $\Lambda(1405)$ resonance. An important experimental information near threshold is the K^-p scattering length a_{K^-p}, which has been determined from measurements of the energy shift and width of the $1s$ state in the kaonic hydrogen [3, 4, 5]. The value of a_{K^-p} can be extracted from the kaonic hydrogen $1s$ level shift Γ and width ϵ by applying the Deser-Trueman formula [6, 7]

$$\epsilon + i\frac{\Gamma}{2} = 2\alpha^3\mu^2 a_{K^-p}, \tag{1}$$

where α is the fine structure constant and μ corresponds to the K^-p reduced mass.

The $\bar{K}$-nucleus interaction is also strongly attractive, as deduced from analysis of kaonic atoms [8]. Global density-dependent fits lead to optical potentials $150 - 200$ MeV deep, whereas coupled-channel calculations based on chiral models on the $\bar{K}N$ interaction [9, 10] yield relatively shallow potential with depth $\approx 50 - 60$ MeV. The depth of the $\bar{K}$-nucleus potential is closely related to the existence and possible width of $\bar{K}$-nuclear states. This issue has attracted considerable attention recently. Kishimoto [11] suggested to search for $\bar{K}$-nuclear states in the nuclear (K^-, p) reaction and Akaishi and Yamazaki [12] predicted a narrow $\bar{K}NNN$ $I = 0$ nuclear state bound by more than 100 MeV. Dote et al. [13] in calculations of very light nuclei predicted considerable polarization of the nuclear core caused by the strongly attractive $\bar{K}$-nucleus interaction. Calculations of the K^-pp system performed by Shevchenko et al. [14] using the three-body coupled channel $\bar{K}NN - \pi\Sigma N$ Faddeev equations yielded a quasibound K^-pp state with considerable width. The FINUDA experiment at Frascati reported

evidence for deeply bound K^-pp states [15] and later preliminary evidence for a narrow $\bar{K}NNN$ quasibound state [16]. However, an alternative, more conventional interpretation of the FINUDA events was presented in refs. [17, 18]. The issue of $\bar{K}$-nuclear quasibound states is clearly far from being resolved and more experimental as well as theoretical explorations, including the study of the $\bar{K}N$ interaction, are necessary.

This year, the SIDDHARTA experiment at Frascati is going to measure both the energy shift and the width of kaonic hydrogen with a higher precision (of several eV) than previous experiments [3, 4, 5]. Moreover, SIDDHARTA will perform measurements of the kaonic deuterium. A precise value of the K^-d scattering length from the measurements of the K^-d atomic level shift and width and a precise value of the K^-p scattering length are essential for extracting the K^-n scattering length, for our better understanding of low-energy $\bar{K}N$ interaction and for extrapolating into $\bar{K}$-nuclear systems.

The first Faddeev calculations of K^-d elastic scattering were performed by Hetherington and Schick [19]. Later the approach was refined by Schick and Gibson [20] who included explicitly hyperonic channels to account for processes $\bar{K}N \leftrightarrow \pi Y$ ($Y = \Sigma, \Lambda$). In 1980, Toker et al. [21] studied $K^-d \to \pi^-\Lambda p$ reactions, as well as other three-body K^- processes, in the Faddeev formalism for separable two-body coupled-channel interactions fitted to available low-energy data. They found that the elastic and total K^-d cross sections are quite independent of the type of the YN interaction. Bahaoui et al. [23] calculated the K^-d elastic scattering at low energies within a multi-channel three-body approach using relativistic separable parametrization for the YN and $\bar{K}N$ interactions. Input parameters were obtained by fitting the low-energy data with the extra constraint of reproducing the $\pi\Sigma$ mass spectrum. Deloff [24] studied ηd and K^-d scattering lengths within the Faddeev approach and compared one- and multi-channel Faddeev calculations. Moreover, he checked how reliable is the fixed center approximation (FCA), in which the physical deuteron is approximated by two nucleons separated by a fixed distance. This approximation, which was originally applied in atomic and molecular physics, is inadequate in K^-d calculations, because of relatively large value of the kaon mass $m_K \approx 495MeV$ compared to the nucleon

mass $m_N \approx 939MeV$. Kamalov, Oset and Ramos [25] applied the FCA of the Faddeev equations to the evaluation of the scattering length a_{K^-d}. Their input consisted of elementary $\bar{K}N$ amplitudes calculated using chiral Lagrangians and a coupled-channel unitary method. Bahaoui et al. [26] refined their own multi-channel Faddeev approach and calculated the K^-d scattering length both in isospin and particle basis. Besides π-nucleon and hyperon-nucleon interactions they considered also the D component of the deuteron wave function. They found the effect of the additional two-body inputs negligible. Meiβner, Raha and Rusetsky [27, 28] applied the FCA to the study of the $\bar{K}N$ and K^-d scattering lengths within the framework of a low-energy effective field theory. They considered the extraction of the $\bar{K}N$ scattering lengths $a_{I=0}$ and $a_{I=1}$ from a combined fit to the kaonic hydrogen and kaonic deuterium data. They concluded that with the present DEAR values for the kaonic hydrogen $1s$ level shift and width, a solution for $a_{I=0}$ and $a_{I=1}$ exists only in a restricted domain of input values of the K^-d scattering length. Gal [29] reviewed multiple-scattering approximations to the Faddeev theory of the K^-d scattering length and compared them with published $\bar{K}NN - \pi YN$ Faddeev calculations.

The three-body Faddeev equations [31, 32] are used for accurate formulation of the quantum-mechanical three-body problem. These equations exactly describe dynamics of a system of three particles. In general, Faddeev equations require as an input all potentials describing interactions between every two particles. The kernels of the Faddeev equations are Hilbert-Schmidt operators. Therefore, unlike the three-body problem in classical mechanics, the quantum three-body problem is uniquely solvable. However, these equations are too complicated for practical purposes. In particular, they are still two-dimensional after angular momentum decomposition. In the Alt-Grassberger-Sandhas (AGS) form [33], the three-body Faddeev equations become one-dimensional integral equations for the three-body transition amplitudes. In these equations separable parts of two-particle transition amplitudes are used. The three-body AGS equations have the structure of multi-channel two-particle Lippmann-Schwinger (LS) equations and are more practical than the original Faddeev equations.

In this work we use the Faddeev equations in the AGS form [33] for the calcu-

lation of the K^-d scattering length. Dealing with scattering at low-energies, we work in s-wave approximation and we do not consider any relativistic corrections. For simplicity we assume that the isospin symmetry is not broken. It means that there is no difference between the proton and neutron masses, as well as between the masses of K^- and $\bar{K}^0$. Furthermore, we do not consider Coulomb interaction. Our calculations are performed in the momentum and isospin basis, where protons and neutrons are treated as identical particles.

The details of our Faddeev calculations of the K^-d scattering length are described in the next section and attached appendices. Section 3 is devoted to the two-body $\bar{K}N$ and NN potentials used in the calculations. Our results are presented and discussed in section 4. We conclude with a brief summary in section 5.

2 Three-body Faddeev equations

Following Sandhas [30] we start from the Faddeev-type three-body equations in the Alt-Grassberger-Sandhas (AGS) form [33]

$$U_{\beta\alpha} = (1 - \delta_{\beta\alpha})G_0^{-1} + \sum_{\gamma\neq\beta} T_\gamma G_0 U_{\gamma\alpha}, \tag{2}$$

where the operator G_0 is the free three-body Green's function defined by the standard way $G_0 = (z - H_0)^{-1}$, where z stands for the three-body energy. $U_{\beta\alpha}$ are the three-body transition operators describing the elastic and rearrangement processes in our system. The Faddeev indices $\alpha, \beta = 1, 2, 3$ determine two bound particles in the initial or final state. For example, $\alpha = 1$ means a bound state of particles labeled by indices 2 and 3. Consequently,

$$\begin{aligned} U_{11} &: 1 + (23) \to 1 + (23), \\ U_{21} &: 1 + (23) \to 2 + (31), \\ U_{31} &: 1 + (23) \to 3 + (12). \end{aligned} \tag{3}$$

The operator U_{11} describes the elastic scattering. Therefore, it is directly connected to the scattering length which is the main goal of this work. In the AGS equations (2) the two-body potentials enter the two-body transition operators T_γ

which fulfill the two-body Lippmann-Schwinger (LS) equation

$$T_\gamma = V_\gamma + V_\gamma G_0^{(2)} T_\gamma. \tag{4}$$

Here the operator $G_0^{(2)}$ is the free two-body Green's function and V_γ is the two-body separable potential in the form

$$V_\gamma = |g_\gamma\rangle \lambda_\gamma \langle g_\gamma|, \tag{5}$$

where λ_γ is the strength factor describing the power of the two-body interaction. By using this form of the two-body potential we can analytically solve the LS equation (4) for the two-body transition operator T_γ as follows

$$T_\gamma(z) = |g_\gamma\rangle \tau_\gamma(z) \langle g_\gamma|, \tag{6}$$

where $\tau_\gamma(z)$ is the energy dependent part of the operator T_γ

$$\tau_\gamma = \left[\frac{1}{\lambda_\gamma} - \langle g_\gamma | G_0^{(2)} | g_\gamma \rangle \right]^{-1}. \tag{7}$$

As pointed out earlier our calculations will be carried out in the momentum and isospin basis. The scattering amplitude is defined as the matrix element of the elastic transition operator $U_{\beta\alpha}$, $\alpha = \beta$, between the three-body wave functions. Since in the asymptotic region two particles are bound we can rewrite the three-body wave function as a product of the bound state wave function of the two-particle subsystem $|\psi_\alpha^B\rangle$, plane wave corresponding to the third particle $|\mathbf{k}_\alpha\rangle$ and the three-body isospin vector. The construction of the isospin vectors is described in Appendix B. Then, the scattering amplitude corresponding to the elastic scattering of the particle α on the subsystem composed of the particles β and γ can be expressed

$$f_\alpha(\mathbf{k}_\beta, \mathbf{k}'_\alpha; z) = -(2\pi)^2 \mu_\alpha \langle I_\beta, \mathbf{k}_\beta; \psi_\beta^B | U_{\beta\alpha}(z) | \psi_\alpha^B; \mathbf{k}'_\alpha, I_\alpha \rangle, \ \beta = \alpha, \tag{8}$$

where μ_α stands for the three-body reduced mass of the particle α relative to the particles β and γ. The definition of the three-body masses can be found in Appendix A. The scattering length of the particle α on the particles β and γ subsystem is obtained by going with the three-body energy to zero

$$a_\alpha = f_\alpha(\mathbf{k}_\beta = 0, \mathbf{k}'_\alpha = 0; z \to 0), \ \beta = \alpha. \tag{9}$$

From the stacionary Schrödinger equation we can express the bound state wave function using the Green's function and the two-body formfactor as follows

$$|\psi_\alpha^B\rangle = \lambda_\alpha N_\alpha^B G_0^{(2)}(z)|g_\alpha\rangle. \tag{10}$$

Here, N_α^B is the normalization constant, which can be obtained from the condition $\langle\psi_\alpha^B|\psi_\alpha^B\rangle = 1$. After using equation (10) and introducing the notation

$$X_{\beta\alpha}(z) \equiv \langle g_\beta|G_0(z)U_{\beta\alpha}(z)G_0(z)|g_\alpha\rangle, \tag{11}$$

$$Z_{\beta\alpha}(z) \equiv \langle g_\beta|G_0(z)|g_\alpha\rangle, \tag{12}$$

we can rewrite the original AGS equations in a more suitable form in the momentum and isospin basis. Apparently each term in derived equations will contain a combination of the constants $\lambda_\beta\lambda_\alpha N_\beta^B N_\alpha^B$, which can be omitted. Therefore we can write the AGS equations in the form

$$\begin{aligned}&\langle I_\beta, \mathbf{k}_\beta|X_{\beta\alpha}(z)|\mathbf{k}'_\alpha, I_\alpha\rangle \\ &= (1-\delta_{\beta\alpha})\langle I_\beta, \mathbf{k}_\beta|Z_{\beta\alpha}(z)|\mathbf{k}'_\alpha, I_\alpha\rangle + \sum_{\gamma\neq\beta}\langle I_\beta, \mathbf{k}_\beta|Z_{\beta\gamma}(z)\tau_\gamma(z)X_{\gamma\alpha}(z)|\mathbf{k}'_\alpha, I_\alpha\rangle.\end{aligned} \tag{13}$$

After partial wave decomposition which is described in detail in Appendix C and where we assumed that only s-wave contribution will be significant in our calculations, we get the following equations

$$\begin{aligned}\langle I_\beta, k_\beta|X_{\beta\alpha}(z)|k'_\alpha, I_\alpha\rangle &= (1-\delta_{\beta\alpha})\langle I_\beta, k_\beta|Z_{\beta\alpha}(z)|k'_\alpha, I_\alpha\rangle + \\ &+ 4\pi\sum_{\gamma\neq\beta}\sum_{I_\gamma}\int \mathrm{d}\bar{k}_\gamma \bar{k}_\gamma^2\langle I_\beta, k_\beta|Z_{\beta\gamma}(z)|\bar{k}_\gamma, I_\gamma\rangle\times \\ &\times\langle I_\gamma|\tau_\gamma\left(z-\frac{\bar{k}_\gamma^2}{2\mu_\gamma}\right)|I_\gamma\rangle\langle I_\gamma, \bar{k}_\gamma|X_{\gamma\alpha}(z)|k'_\alpha, I_\alpha\rangle,\end{aligned} \tag{14}$$

where relation (C-8) was used when dealing with the function $\tau_\gamma(z)$. If we act by the operators $X_{\mu\nu}$,$Z_{\mu\nu}$ and τ_μ on the isospin vectors, we get a two-body isospin dependence of the operators, because our two-body interactions are isospin-dependent. In addition, all these operators are defined as multiplied by the scalar products of isospin vectors which are described in detail in Appendix B:

$$\langle I_\beta, k_\beta|X_{\beta\alpha}(z)|k'_\alpha, I_\alpha\rangle = \hat{X}_{\beta\alpha}^{I_\beta, I_\alpha}(k_\beta, k'_\alpha; z)\langle I_\beta|I_\alpha\rangle \equiv X_{\beta\alpha}^{I_\beta, I_\alpha}(k_\beta, k'_\alpha; z). \tag{15}$$

Then equations (14) transform into the form

$$X^{I_\beta,I_\alpha}_{\beta\alpha}(k_\beta,k'_\alpha;z) = (1-\delta_{\beta\alpha})Z^{I_\beta,I_\alpha}_{\beta\alpha}(k_\beta,k'_\alpha;z)+ \\ + 4\pi\sum_{\gamma\neq\beta}\sum_{I_\gamma}\int_0^\infty \mathrm{d}\bar{k}_\gamma \bar{k}^2_\gamma Z^{I_\beta,I_\gamma}_{\beta\gamma}(k_\beta,\bar{k}_\gamma;z)\tau^{I_\gamma}_\gamma\left(z-\frac{\bar{k}^2_\gamma}{2\mu_\gamma}\right)X^{I_\gamma,I_\alpha}_{\gamma\alpha}(\bar{k}_\gamma,k'_\alpha;z), \tag{16}$$

where the indices I_α denote the isospin dependence of the operators.

We want to calculate the K^-d scattering length and therefore, we introduce the indices K,N_1 and N_2 instead of α,β,γ. This labeling corresponds to the kaon and two nucleons which we consider distinguishable for the moment. Using this labeling and equations (8-11) we can express the K^-d scattering length as

$$a_{K^-d} = -(2\pi)^2\mu_K(N^B_K)^2\lambda^2_K X^{0,0}_{KK}(k_K=0,k'_K=0;z\to 0), \tag{17}$$

where $I_\alpha = I_\beta = 0$ because of known deuteron isospin $I_{NN}=0$. According to (16) three Faddeev equations with the same initial states α are coupled, we thus need to solve equation (16) for X_{KK}, X_{N_1K} and X_{N_2K}. This gives us the set of five integral equations (D-1–D-5). Nucleons are identical fermions, therefore all three-body wave functions must be antisymmetric. Two-body wave function of the NN system with orbital momentum $l=0$ is antisymmetric for $(I_{NN}=0,S_{NN}=1)$ and $(I_{NN}=1,S_{NN}=0)$, which means that $|N_1N_2\rangle = -|N_2N_1\rangle$. The three-body spin conservation and zero spin of the antikaon means, that we have only $(I_{NN}=0,S_{NN}=1)$ and than $I_{NN}=1$ does not enter our three-body equations. Therefore the three-body wave function corresponding to the bound state of two nucleons and a free kaon $|K(N_1N_2)\rangle$ is automatically antisymmetric. $|N(KN)\rangle$ states should be antisymmetrized by hand. We constructed the wave function as a combination of two parts $|N_1(KN_2)\rangle - |N_2(KN_1)\rangle$. Now we can define new operators totally antisymmetric under exchange of the two nucleons

$$\begin{aligned} X_D &\equiv X^{0,0}_{KK}, \\ X_0 &\equiv X^{0,0}_{N_1K} - X^{0,0}_{N_2K}, \\ X_1 &\equiv X^{1,0}_{N_1K} - X^{1,0}_{N_2K}. \end{aligned} \tag{18}$$

With the above mentioned wave functions it is possible to derive relations for the operators $Z_{\beta\alpha}$ and τ_α:

$$\tau_{N_1} = \tau_{N_2},\ Z_{N_1N_2} = Z_{N_2N_1},\ Z_{N_1K} = -Z_{N_2K}. \tag{19}$$

Using labeling (18) and relations (19) we can rewrite the former set (D-1–D-5) as a new set of three integral equations (D-6,D-7,D-8) for the three unknown quantities X_D, X_0 and X_1, where the indices N_1 and N_2 are replaced by the common index N. Here we have used relations derived by Bahaoui et al. [26] for the antisymmetrized operators $\tilde{Z}_{\beta\alpha}$

$$\tilde{Z}_{KN} = \sqrt{2}Z_{KN_1} = -\sqrt{2}Z_{KN_2},\ \tilde{Z}_{NN} = -Z_{N_1N_2} = -Z_{N_2N_1}. \tag{20}$$

Next, we replace integration by summation using standard Gaussian quadrature from Numerical Recipes [38]

$$\int_0^L dx f(x) = \sum_{j=1}^{N} \omega_j f(x_j), \tag{21}$$

where ω_j are weight factors, N is dimension and L is the integration limit identical in all integrations in our set of equations. We have set the value of this parameter $L = 3.5\,\mathrm{fm}^{-1}$ as we have checked that integration in the region beyond this limit varies the results only insignificantly. Now, we can write just one matrix equation instead of the previous set (D-6,D-7,D-8)

$$\begin{pmatrix} \mathbf{X}_D \\ \mathbf{X}_0 \\ \mathbf{X}_1 \end{pmatrix} = \begin{pmatrix} 0 & \omega\cdot\mathbf{K}_{D0} & \omega\cdot\mathbf{K}_{D1} \\ \omega\cdot\mathbf{K}_{0D} & \omega\cdot\mathbf{K}_{00} & \omega\cdot\mathbf{K}_{01} \\ \omega\cdot\mathbf{K}_{1D} & \omega\cdot\mathbf{K}_{10} & \omega\cdot\mathbf{K}_{11} \end{pmatrix} \begin{pmatrix} \mathbf{X}_D \\ \mathbf{X}_0 \\ \mathbf{X}_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 2\mathbf{Z}^{0,0}_{NK} \\ 2\mathbf{Z}^{1,0}_{NK} \end{pmatrix}. \tag{22}$$

In this equation we have defined the vectors

$$\omega\cdot\mathbf{K}_{\Lambda\Lambda'} = \begin{pmatrix} \omega_1 K_{\Lambda\Lambda'}(k^1_\Lambda, k^1_{int}) & \omega_2 K_{\Lambda\Lambda'}(k^1_\Lambda, k^2_{int}) & \cdots & \omega_N K_{\Lambda\Lambda'}(k^1_\Lambda, k^N_{int}) \\ \omega_1 K_{\Lambda\Lambda'}(k^2_\Lambda, k^1_{int}) & \omega_2 K_{\Lambda\Lambda'}(k^2_\Lambda, k^2_{int}) & \cdots & \omega_N K_{\Lambda\Lambda'}(k^2_\Lambda, k^N_{int}) \\ \vdots & \vdots & \ddots & \vdots \\ \omega_1 K_{\Lambda\Lambda'}(k^N_\Lambda, k^1_{int}) & \omega_2 K_{\Lambda\Lambda'}(k^N_\Lambda, k^2_{int}) & \cdots & \omega_N K_{\Lambda\Lambda'}(k^N_\Lambda, k^N_{int}) \end{pmatrix},$$

$$\mathbf{X}_\Lambda = \begin{pmatrix} X_\Lambda(k^1_K, k'_K) \\ X_\Lambda(k^2_K, k'_K) \\ \vdots \\ X_\Lambda(k^N_K, k'_K) \end{pmatrix},\ \mathbf{Z}^{I,0}_{NK} = \begin{pmatrix} Z^{I,0}_{NK}(k^1_N, k'_K) \\ Z^{I,0}_{NK}(k^2_N, k'_K) \\ \vdots \\ Z^{I,0}_{NK}(k^N_N, k'_K) \end{pmatrix}, \tag{23}$$

where the notation $\Lambda, \Lambda' = D, 0$ and 1 corresponds to equations (18), k^i_Λ are momenta corresponding to abscissae from the Gaussian quadrature, k'_K is an

input parameter which in our case goes to zero, k_{int}^j are momenta over which we integrate and $K_{\Lambda\Lambda'}(k_\Lambda^i, k_{int}^j)$ are the kernels of the set of integral equations, which can be expressed as a function of known quantities

$$K_{\Lambda\Lambda'}(k_\Lambda, k_{int}) = 4\pi\tilde{C}_{\Lambda,\Lambda'}\, k_{int}^2 Z_{\Lambda\Lambda'}^{I,I'}(k_\Lambda, k_{int})\tau_{\Lambda'}^{I'}\left(z - \frac{k_{int}^2}{2\mu_\Lambda}\right), \tag{24}$$

where $\tilde{C}_{\Lambda\Lambda'} = -1, +1, +2$ are constants from the equations (D-6,D-7,D-8). It is to be noted that computing of these kernels is tedious but in all respects straightforward as is shown in Appendix E. After introducing a compact notation

$$\mathcal{X} = \begin{pmatrix} \mathbf{X}_D \\ \mathbf{X}_0 \\ \mathbf{X}_1 \end{pmatrix},\ \mathcal{Z} = \begin{pmatrix} 0 \\ \mathbf{Z}_{NK}^{0,0} \\ \mathbf{Z}_{NK}^{1,0} \end{pmatrix},\ \omega\mathcal{K} = \begin{pmatrix} 0 & \omega\cdot\mathbf{K}_{D0} & \omega\cdot\mathbf{K}_{D1} \\ \omega\cdot\mathbf{K}_{0D} & \omega\cdot\mathbf{K}_{00} & \omega\cdot\mathbf{K}_{01} \\ \omega\cdot\mathbf{K}_{1D} & \omega\cdot\mathbf{K}_{10} & \omega\cdot\mathbf{K}_{11} \end{pmatrix}, \tag{25}$$

equation (22) transforms into the final form

$$(1 - \omega\mathcal{K})\mathcal{X} = 2\mathcal{Z}, \tag{26}$$

which is solved by standard linear algebraic techniques [38] using the lower-upper (LU) decomposition and numerical procedures for solving a set of linear equations. The final expression for the K^-d scattering length follows from equations (17) and (18):

$$a_{K^-d} = -(2\pi)^2\mu_K(N_K^B)^2\lambda_K^2 X_D(k_K^1, k_K' = 0; z \to 0). \tag{27}$$

Computing time required for the numerical operations grows up with the third power of the dimension N (see eq. (21)). Fortunately, the final result converges rather rapidly (for $N \sim 100$) and the calculation of the K^-d scattering length is thus quite fast.

3 Input

The use of separable potentials brings us several advantages. For example, there is a lower number of integrations in the set of integral equations. Moreover, it is possible to express the two-body transition operator T_γ analytically. We considered two-body potentials in the form

$$V = |g\rangle\lambda\langle g|, \tag{28}$$

which can be after applying corresponding two-body momentum vectors and two-body isospin vectors rewritten as follows

$$V^I(\mathbf{p}, \mathbf{p}') = \lambda_I g^I(p) g^I(p'). \tag{29}$$

The potentials used in our calculations are s-wave, isospin dependent and isospin conserving. However, there is not too many potentials of this type. Let us remark that there exists a possibility of separabilization of non-separable potentials, but we do not use such a procedure in our work. Since we work in the isospin basis instead of the particle basis and we neglect the isospin breaking effects, we use as an input the kaon-nucleon potential and the nucleon-nucleon potential for $I = 0$ or $I = 1$. All relevant masses and fundamental physical constants are taken from the Review of Particle Physics [35]. Because of the isospin symmetry we use the average mass $m_{\bar{K}}$ of K^- and $\bar{K}^0$, and the average mass m_N of proton and neutron.

3.1 $\bar{K}$-Nucleon potential

Separable $\bar{K}N$ potentials which can be found in literature are quite old and do not reproduce the present $\bar{K}N$ data sufficiently well. Therefore, we have constructed our own separable potential with the formfactors of the form (29)

$$g^I(p) = \frac{1}{p^2 + \beta_I^2}. \tag{30}$$

Since there are two possible values of the two-body isospin, we have to determine four parameters λ_I and β_I for $I = 0$ and 1, where λ_I are in general complex due to the $\bar{K}N \to \pi\Sigma$ interaction. Consequently, we have six unknown real parameters. It is possible to determine these parameters from experimental data, namely the

mass and the width of the $\Lambda(1405)$ resonance and the complex K^-p scattering length. The two remaining parameters, the range parameters β_I, were set for both isospin channels to the value $\beta = 3.5\,\mathrm{fm}^{-1}$, according to fits of the $\bar{K}N$ potentials to the low-energy K^-p data performed by Shevchenko et al. [14].

We assume that $\Lambda(1405)$ is a quasibound state of the $\bar{K}N$ subsystem for $I = 0$. We used the PDG values [35] $M_\Lambda = 1405$ MeV and $\Gamma_\Lambda = 50$ MeV, which seem to be the most plausible at present. Nevertheless, we also studied the sensitivity of the calculated K^-d scattering length to the variations of these values as shown in the next chapter.

From the PDG mass and width of $\Lambda(1405)$ we get the binding energy of the $\bar{K}N$ bound state

$$E_B^{\bar{K}N} = (-29.5 - i\,25)\,\mathrm{MeV}. \tag{31}$$

This value is much larger then the binding energy of the deuteron. This implies that at low energies the $\bar{K}N$ interaction is stronger than the NN interaction.

There are two available experimental values of the K^-p scattering length derived from the kaonic hydrogen $1s$ level shift and width in the KEK experiment [3, 4]:

$$a_{K^-p}^{KEK} = (-0.78 \pm 0.15 \pm 0.03) + i\,(0.49 \pm 0.25 \pm 0.12)\,\mathrm{fm}, \tag{32}$$

and in the DEAR collaboration experiment [5]:

$$a_{K^-p}^{DEAR} = (-0.468 \pm 0.090 \pm 0.015) + i\,(0.302 \pm 0.135 \pm 0.036)\,\mathrm{fm}. \tag{33}$$

By comparing these two scattering lengths we see rather large discrepancy between the two measurements. The result published by the DEAR collaboration is suppose to be more accurate, because of smaller error bars. However, it is impossible to find the parameters of the $\bar{K}N$ potential which reproduce simultaneously the K^-p cross sections and the $a_{K^-p}^{DEAR}$ value. Hence we used the KEK value $a_{K^-p}^{KEK} = (-0.78 + i\,0.49)\,\mathrm{fm}$ in our fits. Moreover, in order to study the sensitivity of our predictions of the K^-d scattering length we varied the a_{K^-p} KEK value within the error bars indicated in eq. (32).

In order to connect the K^-p scattering length with the potentials for $I = 0$ and $I = 1$, we make use of the relation between the K^-p scattering length and

the scattering lengths in the $I = 0$ and $I = 1$ channels:

$$a_{K^-p} = \frac{(a_{I=0} + a_{I=1})}{2}. \tag{34}$$

Using the above mentioned input values we get all required parameters. For example, for $a_{K^-p} = -0.78 + i\,0.49\,\text{fm}$, $m_\Lambda = 1405$ MeV and $\Gamma_\Lambda = 50$ MeV, we get

$$\begin{aligned} \beta_{I=0} &= \beta_{I=1} = 3.5\,\text{fm}^{-1}, \\ \lambda_{I=0} &= (-1.944 - i\,0.253)\,\text{fm}^{-2}, \\ \lambda_{I=1} &= (-0.660 - i\,0.596)\,\text{fm}^{-2}. \end{aligned} \tag{35}$$

3.2 Nucleon-Nucleon potential

There are not too many suitable separable nucleon-nucleon potentials. In our work, we have chosen the reliable PEST potential [36] and an interesting energy dependent potential created by Garcilazo in 1980 [37], which we have modified according to some newer experimental data.

3.2.1 PEST potential

The nucleon-nucleon PEST potential [36] is a separable approximation of the Paris potential. The strength parameters were set to $\lambda_{I=0} = \lambda_{I=1} = -1$ and the formfactors were defined as follows

$$g^I(p) = \frac{1}{2\sqrt{\pi}} \sum_{i=1}^{6} \frac{c_i^I}{p^2 + (\beta_i^I)^2}. \tag{36}$$

The parameters β_i^I and c_i^I are listed in ref. [36]. PEST is on-shell and off-shell equivalent of the Paris potential up to $E_{lab.} \sim 50$ MeV. It is repulsive at distances shorter than 0.8 fm. It reproduces the deuteron binding energy $E_B^d = -2.2249$ MeV, as well as the triplet and singlet NN scattering lengths $a(^3S_1) = -5.422\,\text{fm}$ and $a(^1S_0) = 17.534\,\text{fm}$, respectively.

3.2.2 Energy dependent NN potential

A separable energy dependent nucleon-nucleon potential was presented by Garcilazo in ref. [37]. This potential is defined analogously to the other potentials

used in this work

$$V^I(\mathbf{p},\mathbf{p}';E) = \lambda_I(E) g^I(p) g^I(p'), \tag{37}$$

where $g^I(p)$ are the standard Yamaguchi formfactors

$$g^I(p) = \frac{\gamma_I}{p^2+\alpha_I^2} \tag{38}$$

and $\lambda_I(E)$ are functions of the two-body energy

$$\lambda_I(E) = -\tanh\left(1-\frac{E}{E_c^I}\right), \tag{39}$$

which are negative (positive) for $E < E_c^I$ ($E > E_c^I$) and finite as E goes to $\pm\infty$. These characteristics of $\lambda_I(E)$ correspond to the required properties of the potential, which is supposed to be attractive at low energies and repulsive at high energies. E_c^I is the energy where the phase shift changes sign. In ref. [37], Garcilazo used for the determination of the parameters of the NN potential some values of the NN scattering lengths, which have been later outdated. Hence when using the original parameters of ref. [37] we got the deuteron binding energy, $E_B^d = -2.186$ MeV, which is too low. Therefore we determined our own parameters γ_I and α_I using the relevant formulae

$$\begin{aligned} \frac{m_N\pi}{2a^I} &= \frac{\alpha_I^4}{\gamma_I^2\lambda_I(0)} + \frac{m_N}{4}\pi\alpha_I, \\ -\frac{m_N}{4}\pi r^I &= \frac{2\alpha_I^2}{\gamma_I^2\lambda_I(0)} - \frac{m_N\pi}{4\alpha_I} + \frac{\alpha_I^4}{\gamma_I^2}\frac{1}{m_N}\frac{d}{dE}\left(\frac{1}{\lambda_I(E)}\right)_{E=0}, \end{aligned} \tag{40}$$

where m_N is the nucleon mass, a^I are the NN scattering lengths and r^I stand for the effective ranges for the $I=0$ and $I=1$ channels. Using the NN scattering lengths and effective ranges given by the PEST potential and the energies $E_c^{I=0} = 0.816\,\mathrm{fm}^{-1}$ and $E_c^{I=1} = 0.767\,\mathrm{fm}^{-1}$ we get the following values of the required parameters

$$\begin{aligned} \alpha_{I=0} &= 1.5659\,\mathrm{fm}^{-1},\ \gamma_{I=0}^2 = 1.7647\,\mathrm{fm}^{-2}, \\ \alpha_{I=1} &= 1.2348\,\mathrm{fm}^{-1},\ \gamma_{I=1}^2 = 0.6056\,\mathrm{fm}^{-2}. \end{aligned} \tag{41}$$

These parameters give the correct deuteron binding energy $E_B^d = -2.228$ MeV.

4 Results and discussion

In this chapter we present results of our calculations of the K^-d scattering length using the Faddeev equations in the AGS form. In particular, we demonstrate the sensitivity of the K^-d scattering length to the variations of the two-body inputs. At the end of the chapter we also compare our best results with the results of similar calculations performed so far.

4.1 Dependence of a_{K^-d} on the NN potential

We calculated the K^-d scattering length using two different NN potentials presented in section 2, namely the PEST potential and the energy dependent potential of Garcilazo (E-dep). During these calculations we considered the $\bar{K}N$ potentials with the parameters λ_I and β_I, which reproduce $m_\Lambda = 1405$ MeV, $\Gamma_\Lambda = 50$ MeV and the K^-p scattering length, for which we used as a guideline the KEK value: $a^{KEK}_{K^-p} = (-0.78 + i\,0.49)$ fm. Then we obtained the following values for the K^-d scattering length:

$$a^{PEST}_{K^-d} = (-1.39 + i\,0.96)\,\text{fm}, \tag{42}$$

$$a^{E-dep}_{K^-d} = (-1.32 + i\,1.02)\,\text{fm}. \tag{43}$$

These results are not much different from each other as can be expected from the fact that the $\bar{K}N$ potentials were kept fixed in the above calculations. The $\bar{K}N$ interaction is namely stronger than the NN interaction as can be deduced from comparsion of the binding energies of the deuteron and the $\Lambda(1405)$ resonance (see eq. (31) and text below). Even if the results are very similar we will try to trace up the origin of the difference, since both potentials give identical two-body results such as the deuteron binding energy or scattering lengths and effective radii for $I = 0$ and $I = 1$. In the two-body calculations we compute the scattering amplitude using the formula

$$f_{NN}(\mathbf{k}, \mathbf{k}', z) = -(2\pi)^2 M^{red}_{NN} \langle \mathbf{k}|T_{NN}|\mathbf{k}'\rangle, \tag{44}$$

where M^{red}_{NN} is the reduced mass of two nucleons and T_{NN} is the two-body transition operator which contains the function $\tau_{NN}(z)$, as shown in eqs. (6,7). Then while the two-body scattering amplitude is expressed as a product of the function

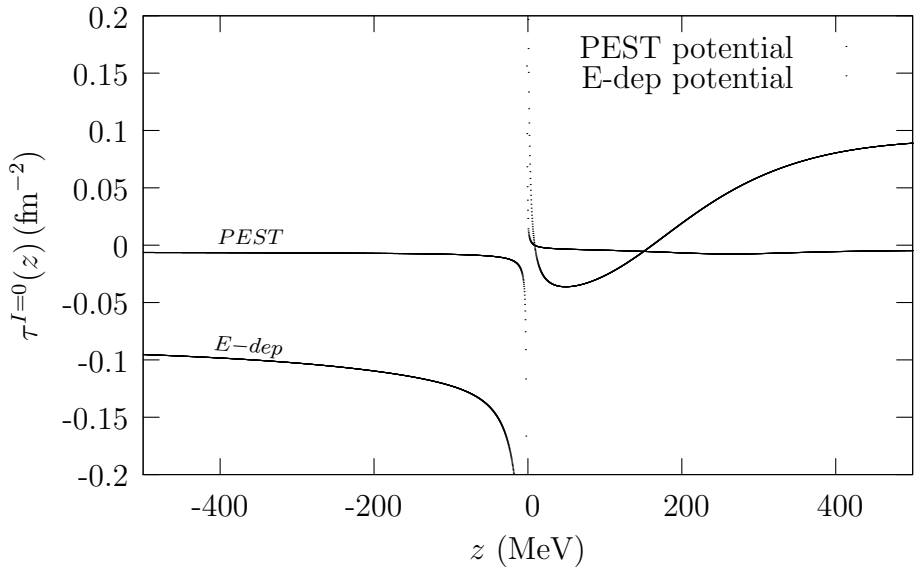

Fig. 1: The $\tau^{I=0}(z)$ for the nucleon-nucleon potentials PEST and E-dep with $I = 0$ as a function of NN energy (see text for details).

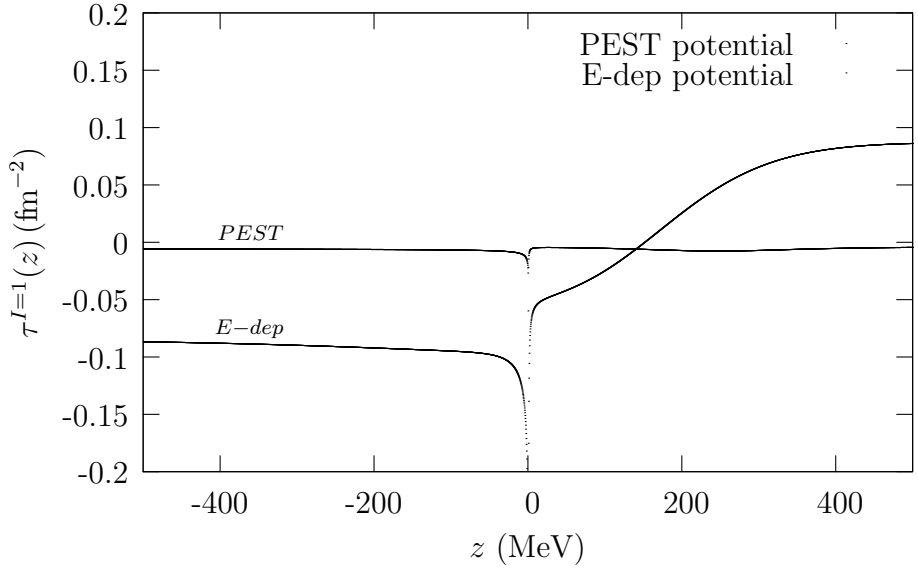

Fig. 2: The $\tau^{I=1}(z)$ for the nucleon-nucleon potentials PEST and E-dep with $I = 1$ as a function of NN energy (see text for details).

$\tau_{NN}(z)$ and two formfactors $g(k)$, $g(k')$, in the three-body Faddeev equations only the function $\tau(z)$ remains in original form (see eq. 16). We have found that the behavior of the two-body NN scattering amplitudes is identical for both NN potentials, but the behavior of the function $\tau(z)$ is much different for these two potentials as can be seen in Fig.1 and Fig.2. This could be the possible explanation of the difference between the obtained values of the K^-d scattering length $a_{K^-d}^{PEST}$ (42) and $a_{K^-d}^{E-dep}$ (43).

It is to be noted that in ref. [28] Rusetsky et al. found no difference between the K^-d scattering lengths calculated using different (Paris and Bonn) NN potentials. This result could be caused by the limited validity of the fixed-center approximation used in that work.

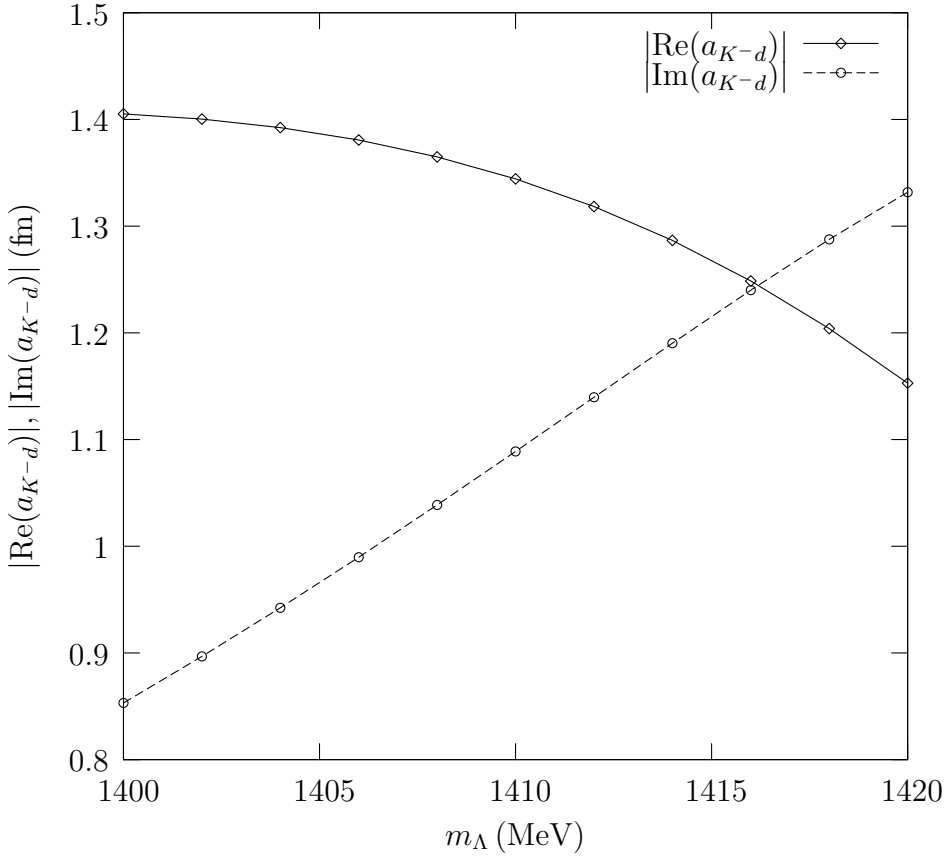

Fig. 3: The scattering length a_{K^-d} as a function of the mass m_Λ of the $\Lambda(1405)$ resonance. Here the NN potential PEST was used, $\Gamma_\Lambda = 50$ MeV and the K^-p scattering length is fixed at the KEK value (32).

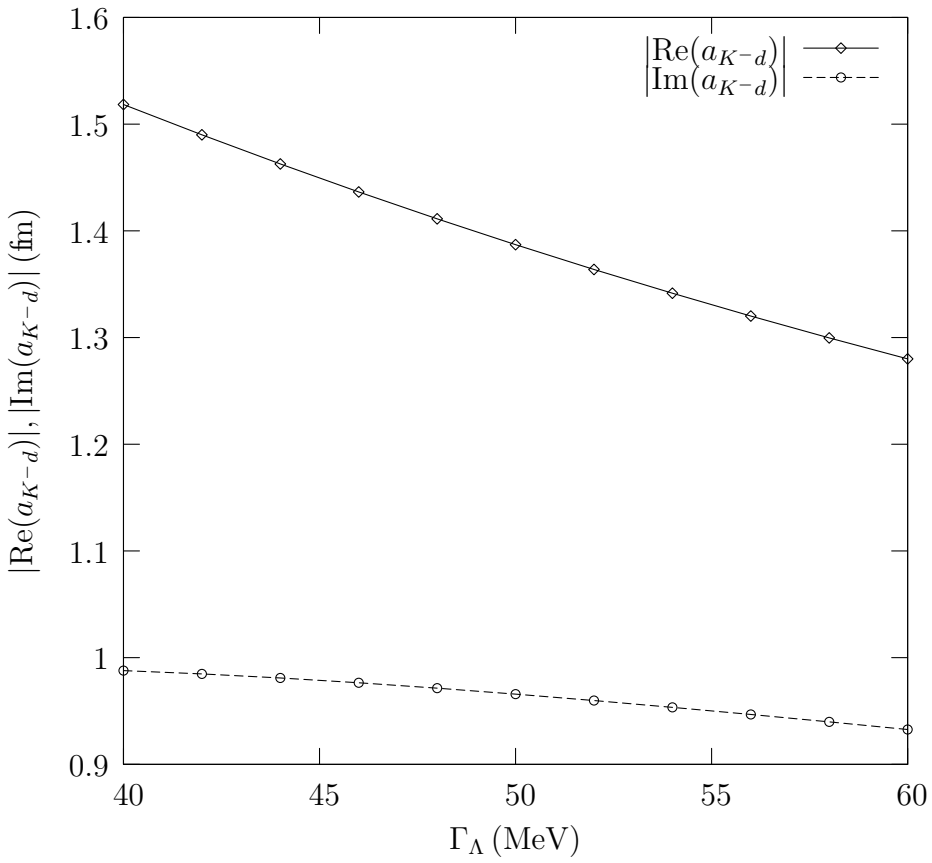

Fig. 4: The scattering length a_{K^-d} as a function of the width Γ_Λ of the $\Lambda(1405)$ resonance. Here the NN potential PEST was used, $m_\Lambda = 1405$ MeV and the K^-p scattering length is fixed at the KEK value (32).

4.2 Dependence of a_{K^-d} on the $\bar{K}N$ potential

First, we studied the sensitivity of the K^-d scattering length to the variations of the parameters of the $\Lambda(1405)$ resonance, namely the mass m_Λ and the width Γ_Λ. The K^-p scattering length was kept fixed at the $a^{KEK}_{K^-p}$ value and we used the PEST NN potential throughout the calculations. In Fig.3 and Fig.4, we present the K^-d scattering length as a function of the mass m_Λ and width Γ_Λ, respectively. It is to be noted that the real part $\mathrm{Re}(a_{K^-d})$ is negative but in the figures we plotted its absolute value $|\mathrm{Re}(a_{K^-d})|$. While the real part of the K^-d scattering length is rather sensitive to the variations of both m_Λ and Γ_Λ, the imaginary part is almost insensitive to the value of the width of $\Lambda(1405)$ (see Fig.4). As stated before, the $\Lambda(1405)$ resonance is assumed $I = 0$ quasibound state of antikaon and nucleon. Therefore the higher value of the $\Lambda(1405)$ mass corresponds to smaller binding energy of the K^-p system, which is consistent

with the lower absolute value of the real part of the K^-d scattering length as we witness in Fig.3.

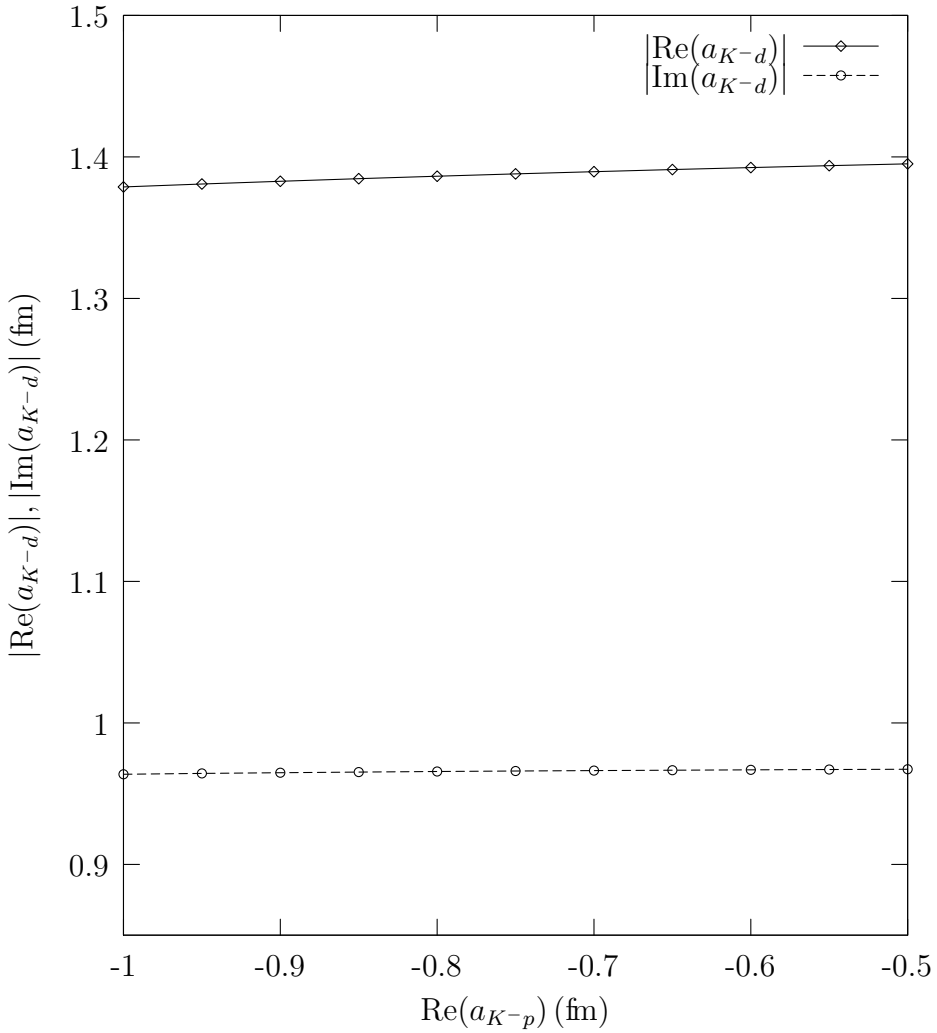

Fig. 5: The scattering length a_{K^-d} as a function of the real part of the two-body K^-p scattering length a_{K^-p}. The imaginary part of a_{K^-p} is fixed at the KEK value, the NN potential PEST was used, $m_\Lambda = 1405$ MeV and $\Gamma_\Lambda = 50$ MeV were considered.

Next we kept the position and the width of $\Lambda(1405)$ fixed, we used the PEST NN potential and studied the dependence of the a_{K^-d} on the real and imaginary part of the two-body K^-p scattering length. We varied these two input parameters within the range of the KEK experimental error bars (eq. (32)). The scattering length a_{K^-d} as a function of the real and imaginary part of a_{K^-p} are presented in Fig.5 and Fig.6, respectively. The figures demonstrate that the dependence of a_{K^-d} on the two-body K^-p scattering length is very weak. It is much weaker that the dependence of a_{K^-d} on the position and width of $\Lambda(1405)$. Since

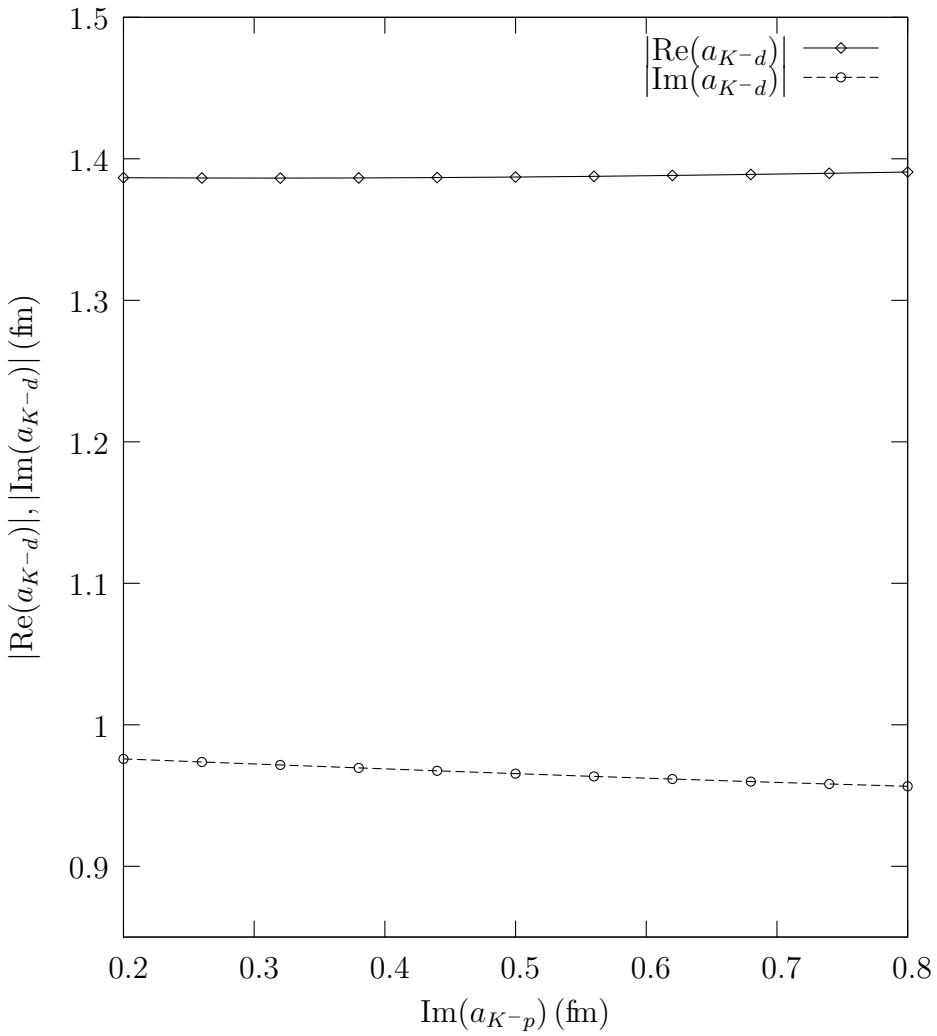

Fig. 6: The scattering length a_{K^-d} as a function of the imaginary part of the two-body K^-p scattering length a_{K^-p}. The real part of a_{K^-p} is fixed at the KEK value, the NN potential PEST was used, $m_\Lambda = 1405$ MeV and $\Gamma_\Lambda = 50$ MeV were considered.

$\Lambda(1405)$ is a quasibound state in the $I = 0$ $\bar{K}N$ channel, its mass and width are directly related to the $I = 0$ $\bar{K}N$ interaction. On the other hand the K^-p scattering length is a combination of the $I = 0$ ($a_{I=0}$) and $I = 1$ ($a_{I=1}$) scattering lengths (see eq. (34)). Figures 3 and 4 thus illustrate strong dependence of the K^-d scattering length on the $I = 0$ interaction. In Figs. 5 and 6, strength of the $I = 0$ interaction was held fixed (via the fixed $\Lambda(1405)$ characteristics) and the variations of a_{K^-p} thus represent the variations of only the $I = 1$ interaction. Consequently, Figs.3–6 confirm that the $I = 0$ $\bar{K}N$ interaction is stronger and more important in the K^-d system than the $I = 1$ interaction. This conclusion is consistent with the fact that the two-body $\bar{K}N$ in the $I = 0$ channel is stronger

than in the $I = 1$ channel, as can be also deduced from the existence of the $I = 0$ $\Lambda(1405)$ resonance while there is no resonance in the $I = 1$ channel.

For completeness, we also performed calculations for the DEAR value of the K^-p scattering length (45), the PEST NN potential and the PDG values of the $\Lambda(1405)$ parameters. The resulting value of the K^-d scattering length

$$a_{K^-d}^{DEAR} = (-1.40 + i\,0.97)\,\text{fm}, \tag{45}$$

is very close to the $a_{K^-d}^{KEK}$ value (42) and is in agreement also with Figs.5–6. This again confirms our conclusion that in the K^-d system the $I = 1$ $\bar{K}N$ interaction is much weaker and less important than the $I = 0$ $\bar{K}N$ interaction.

4.3 Comparison with other calculations

In Figure 7 we compare our results (42,43, and 45) with other calculations of the K^-d scattering length. It is to be noted that these calculations were performed within different approaches and, moreover, using different two-body inputs. This is the reason of the differences between the calculated a_{K^-d} values. The values calculated by Torres et al. (TDD) [22] and Toker and Gal (TG) [21] were obtained within multi-channel Faddeev equations. Nevertheless they are very close to our results. The multi-scattering approximation to Faddeev equations by Gal (G) [29] yielded the a_{K^-d} value which is also "perhaps fortuitously" in agreement with our results.

The calculations performed by Kamalov et al. (KOR) [25] are based on the fixed-center approximation to the Fadeev equations which is improper in the K^-d system. Their a_{K^-d} value differs significantly from the other presented values, as well as the result of Bahaoui et al. (B) [26], who performed the multi-channel Faddeev calculations and included also relativistic corrections and some other effects like d-wave contribution to the NN interaction. The real part of the scattering length calculated by Deloff (D) [24] is rather small compared to the real parts of all other results. However, the mass and the width of the K^-p resonance in the Deloff's calculations are too high ($m \approx 1440$ MeV and $\Gamma \approx 120$ MeV) and differ significantly from the values used in other calculations. It is shown in Fig.3 that the absolute value of the real part of the K^-d scattering length

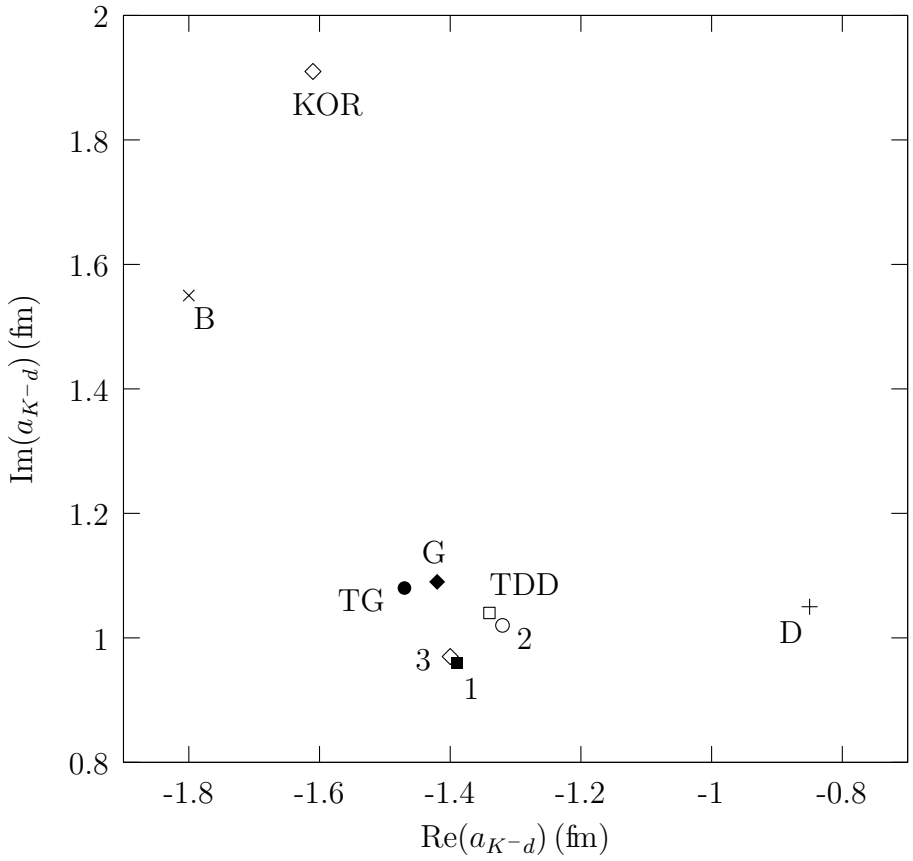

Fig. 7: The K^-d scattering lengths calculated using various two-body inputs: the PEST potential and $a^{KEK}_{K^-p}$ (1), the E-dep potential and $a^{KEK}_{K^-p}$ (2), the PEST potential and $a^{DEAR}_{K^-p}$ (3) (see text for details). For comparison we present also results of previous calculations: Bahaoui et al. (B) [26], Torres & Dalitz & Deloff (TDD) [22], Deloff (D) [24], Gal (G) [29], Kamalov & Oset & Ramos (KOR) [25], Toker & Gal (TG) [21].

decreases with the mass of the $\Lambda(1405)$ resonance. Therefore, this could be a possible explanation of the discrepancy between the the Deloff's value and other calculations.

Out of our results, we consider the most plausible value for the K^-d scattering length $a^{KEK,PEST}_{K^-d} = (-1.39 + i\,0.96)$ fm.

5 Conclusion

In this work, we studied the K^-d scattering length. We formulated the Faddeev equations in the AGS form for the K^-d system. We performed three-body Faddeev calculations in the isospin basis. Due to the known deuteron isospin $I_d = 0$, the three-body isospin of the studied system is $I = \frac{1}{2}$. Moreover, since computing of the scattering length is a low-energy calculation, we restricted our considerations to s-wave two-body interactions. In our calculations we used two separable NN potentials, PEST [36] and energy dependent (E-dep) potential [37]. For the $\bar{K}N$ interaction we applied our own separable potential, which reproduces the K^-p scattering length and the position and the width of $\Lambda(1405)$. We varied the parameters of the $\Lambda(1405)$ resonance as well as the value of the K^-p scattering length within experimental error bars in order to study the sensitivity of the K^-d scattering length on the two-body inputs. We found rather weak dependence of the K^-d scattering length on the NN potential. We observed strong dependence of a_{K^-d} on the $I = 0$ $\bar{K}N$ interaction and very weak dependence on the $I = 1$ $\bar{K}N$ interaction. Our study thus confirmed that the $I = 0$ $\bar{K}N$ interaction is much stronger and more important for the K^-d system. Our calculations yield the K^-d scattering lengths, which are reasonably close to the values of previous calculations of Torres et al. [22], Gal [29] and Toker & Gal [21]. Our most plausible value is

$$a_{K^-d}^{KEK,PEST} = (-1.39 + i\,0.96)\,\text{fm}. \tag{46}$$

The next step in the Faddeev calculations of the K^-d scattering length should be the extension to multi-channel formalism, i.e. the inclusion of $\pi\Sigma$, $\pi\Lambda$ into the coupled-channel $\bar{K}N$ interaction.

Appendices

A Momentum relations

The Hamiltonian of the three-body system in the rest frame is expressed in the standard form using the momenta and masses of the three participating particles

$$H = \sum_{\gamma} \left(\frac{\mathbf{q}_{\gamma}^2}{2m_{\gamma}} + V_{\gamma} \right), \tag{A-1}$$

where V_{γ}, $\gamma = 1, 2, 3$, are two-body potentials. In the center of mass system, we can rewrite the Hamiltonian as follows

$$H = \frac{\mathbf{p}_{\xi}^2}{2M_{\xi}} + \frac{\mathbf{k}_{\xi}^2}{2\mu_{\xi}} + \sum_{\gamma} V_{\gamma}, \tag{A-2}$$

where the particle labeled by index ξ is free and the two remaining particles are bound, M_{ξ} is the two-body reduced mass in the two-particle subsystem built-up without particle labeled by ξ and μ_{ξ} is the three-body reduced mass between the two-particle subsystem and the third particle labeled by ξ. The Hamiltonian can be thus expressed in three ways for $\xi = 1, 2, 3$. All reduced masses are defined in the standard way

$$M_1 = \frac{m_2 m_3}{m_2 + m_3}, \; M_2 = \frac{m_3 m_1}{m_3 + m_1}, \; M_3 = \frac{m_1 m_2}{m_1 + m_2}, \tag{A-3}$$

$$\mu_1 = \frac{m_1(m_2 + m_3)}{m_1 + m_2 + m_3}, \; \mu_2 = \frac{m_2(m_3 + m_1)}{m_1 + m_2 + m_3}, \; \mu_3 = \frac{m_3(m_1 + m_2)}{m_1 + m_2 + m_3}. \tag{A-4}$$

Both the two-body impulse $\mathbf{p}_{\xi}$ and the impulse of the third particle relative to the two-body subsystem $\mathbf{k}_{\xi}$ are functions of the momenta $\mathbf{q}_{\xi}$ relative to the rest frame

$$\mathbf{p}_1 = \frac{m_3\mathbf{q}_2 - m_2\mathbf{q}_3}{m_2 + m_3}, \; \mathbf{p}_2 = \frac{m_1\mathbf{q}_3 - m_3\mathbf{q}_1}{m_3 + m_1}, \; \mathbf{p}_3 = \frac{m_2\mathbf{q}_1 - m_1\mathbf{q}_2}{m_1 + m_2}, \tag{A-5}$$

$$\begin{aligned} \mathbf{k}_1 &= \frac{(m_2 + m_3)\,\mathbf{q}_1 - m_1\,(\mathbf{q}_2 + \mathbf{q}_3)}{m_1 + m_2 + m_3}, \\ \mathbf{k}_2 &= \frac{(m_3 + m_1)\,\mathbf{q}_2 - m_2\,(\mathbf{q}_3 + \mathbf{q}_1)}{m_1 + m_2 + m_3}, \\ \mathbf{k}_3 &= \frac{(m_1 + m_2)\,\mathbf{q}_3 - m_3\,(\mathbf{q}_1 + \mathbf{q}_2)}{m_1 + m_2 + m_3}. \end{aligned} \tag{A-6}$$

Due to the above relations it is possible to connect the three pairs of the relative momenta $(\mathbf{p}_\xi, \mathbf{k}_\xi)$ with each other by the following expressions

$$\begin{aligned} \mathbf{p}_\xi &= A_{(\xi\eta)}\mathbf{p}_\eta + B_{(\xi\eta)}\mathbf{k}_\eta, \\ \mathbf{k}_\xi &= C_{(\xi\eta)}\mathbf{p}_\eta + D_{(\xi\eta)}\mathbf{k}_\eta, \end{aligned} \tag{A-7}$$

for $\xi \neq \eta$ and $\xi, \eta = 1, 2, 3$. Coefficients in (A-7) can be easily determined as functions of the corresponding masses. Using (A-7) we can directly express the scalar products of the impulse eigenvectors

$$\langle \mathbf{p}_\xi, \mathbf{k}_\xi | \mathbf{p}_\eta, \mathbf{k}_\eta \rangle = \delta^{(3)}\left(\mathbf{p}_\xi - A_{(\xi\eta)}\mathbf{p}_\eta - B_{(\xi\eta)}\mathbf{k}_\eta\right) \delta^{(3)}\left(\mathbf{k}_\xi - C_{(\xi\eta)}\mathbf{p}_\eta - D_{(\xi\eta)}\mathbf{k}_\eta\right). \tag{A-8}$$

B Scalar products of the isospin vectors

In this work one-particle isospins are labeled by the symbol $\mathbf{I}^{(j)}$, where $j = 1, 2, 3$ defines the relevant particle. The common eigenvector of the operators $(\mathbf{I}^{(j)})^2, I_3^{(j)}$ is surely $|I_j m_j\rangle$, where $j = 1, 2, 3$. This means that in a three-particle system, where we know one-particle isospins and isospin projections for each particle, the eigenvector of the three particles in particle basis has a form

$$|I_1 m_1\rangle|I_2 m_2\rangle|I_3 m_3\rangle. \tag{B-1}$$

In isospin basis, where we are working, two-body isospins are known. Due to this we need to construct three-body eigenvectors in isospin basis and express, how they are connected with the particle ones. The three-particle isospin $\mathbf{I}$ can be constructed in three ways. For example, after adding the isospins of the second and third particle we get the two-particle operator $\mathbf{I}^{(2,3)}$, which we then put together with the isospin of the first particle

$$\begin{aligned} \mathbf{I}^{(2,3)} &= \mathbf{I}^{(2)} + \mathbf{I}^{(3)}, \\ \mathbf{I} &= \mathbf{I}^{(1)} + \mathbf{I}^{(2,3)}. \end{aligned} \tag{B-2}$$

In this way we obtain vector

$$\begin{aligned} &|I_1(I_2 I_3)I_{23} I m\rangle \\ &\equiv \sum_{m_1 m_2 m_3 m_{23}} (I_1 I_{23} m_1 m_{23}|I m)(I_2 I_3 m_2 m_3|I_{23} m_{23})|I_1 m_1\rangle|I_2 m_2\rangle|I_3 m_3\rangle, \end{aligned} \tag{B-3}$$

which is the common eigenvector of the set of the operators

$$(\mathbf{I}^{(1)})^2, (\mathbf{I}^{(2)})^2, (\mathbf{I}^{(3)})^2, (\mathbf{I}^{(2,3)})^2, (\mathbf{I})^2, I_3. \tag{B-4}$$

The symbols with round brackets in equation (B-3) are the Clebsch-Gordan coefficients. The scalar product of the two isospin vectors can be expressed by the Wigner $6j$-symbol (see e.g. ref. [34]).

$$\langle (I_1 I_2) I_{12} I_3 I m | I_1 (I_2 I_3) I_{23} I m\rangle = \frac{(-1)^{-I_1 - I_2 - I_3 - I}}{\sqrt{(2I_{12}+1)(2I_{23}+1)}} \begin{Bmatrix} I_1 & I_2 & I_{12} \\ I_3 & I & I_{23} \end{Bmatrix}. \tag{B-5}$$

In our case all isospins and their projections are conserved except the two-body isospins $\mathbf{I}^{(i,j)}$. Therefore our separable potentials are function of only the isospins

$\mathbf{I}^{(i,j)}$. Hence we can use the shortened notation for the isospin vectors $|I_\gamma\rangle$ as can be seen in eq. (8). Here index γ labels the spectator particle and the eigenvalue of this vector corresponds to the two-body isospin of the two others particles. In our calculations we encountered just three types of the scalar products of the isospin vectors, which are evaluated below according to (B-5):

$$\begin{aligned} \langle 0|0\rangle &= -\frac{1}{2}, \\ \langle 0|1\rangle = \langle 1|0\rangle &= \frac{1}{2\sqrt{3}}, \\ \langle 1|1\rangle &= \frac{1}{18}. \end{aligned} \tag{B-6}$$

C Partial wave decomposition

We start with the AGS equations (13) rewritten by using the notation (11,12).

$$\begin{aligned}&\langle I_\beta, \mathbf{k}_\beta | X_{\beta\alpha}(z) | \mathbf{k}'_\alpha, I_\alpha \rangle \\ &= (1-\delta_{\beta\alpha}) \langle I_\beta, \mathbf{k}_\beta | Z_{\beta\alpha}(z) | \mathbf{k}'_\alpha, I_\alpha \rangle + \sum_{\gamma \neq \beta} \langle I_\beta, \mathbf{k}_\beta | Z_{\beta\gamma}(z) \tau_\gamma(z) X_{\gamma\alpha}(z) | \mathbf{k}'_\alpha, I_\alpha \rangle .\end{aligned} \tag{C-1}$$

Using the closure relation

$$1 = \int \mathrm{d}k \sum_{l=0}^{\infty} \sum_{m=-l}^{l} |klm\rangle\langle klm| \tag{C-2}$$

we can express the matrix element of the operator $X_{\beta\alpha}$ between the plane waves corresponding to the spectator particle by the same operator in the basis of the partial waves

$$\begin{aligned}&\langle \mathbf{k} | X_{\beta\alpha}(z) | \mathbf{k}' \rangle \\ &= \int \mathrm{d}\bar{k} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \int \mathrm{d}\bar{k}' \sum_{l'=0}^{\infty} \sum_{m'=-l'}^{l'} \langle \mathbf{k} | \bar{k} l m \rangle \langle \bar{k}' l' m' | \mathbf{k}' \rangle \langle \bar{k} l m | X_{\beta\alpha}(z) | \bar{k}' l' m' \rangle .\end{aligned} \tag{C-3}$$

Using the relation

$$\langle \mathbf{k} | \bar{k} l m \rangle = \delta(k - \bar{k}) \mathrm{Y}_{lm}(\hat{\mathbf{k}}) \sqrt{4\pi}, \tag{C-4}$$

the formula for the product of two spherical harmonics

$$\sum_{m=-l}^{l} \mathrm{Y}^*_{lm}(\mathbf{n_1}) \mathrm{Y}_{lm}(\mathbf{n_2}) = \frac{2l+1}{4\pi} \mathrm{P}_l(\mathbf{n_1}, \mathbf{n_2}), \tag{C-5}$$

and the normalization of plane waves $\langle \mathbf{k} | \mathbf{k}' \rangle = \delta^{(3)}(\mathbf{k} - \mathbf{k}')$, we get the expression

$$\langle \mathbf{k} | X_{\beta\alpha}(z) | \mathbf{k}' \rangle = \sum_{l=0}^{\infty} \mathrm{P}_l(\hat{\mathbf{k}} \cdot \hat{\mathbf{k}}') \langle klm | X_{\beta\alpha}(z) | k' l m \rangle . \tag{C-6}$$

Similarly, we obtain the expression for the operator $Z_{\beta\alpha}$

$$\langle \mathbf{k} | Z_{\beta\alpha}(z) | \mathbf{k}' \rangle = \sum_{l=0}^{\infty} \mathrm{P}_l(\hat{\mathbf{k}} \cdot \hat{\mathbf{k}}') \langle klm | Z_{\beta\alpha}(z) | k' l m \rangle . \tag{C-7}$$

Now we insert the closure relations for plane waves $1 = \int \mathrm{d}\mathbf{k}_\gamma |\mathbf{k}_\gamma\rangle\langle\mathbf{k}_\gamma|$ and isospins $1 = \sum_{I_\gamma} |I_\gamma\rangle\langle I_\gamma|$ into equations (C-1). Moreover, we use the formula for the operator $\tau_\gamma(z)$ in the momentum basis

$$\langle \mathbf{k} | \tau_\gamma(z) | \mathbf{k}' \rangle = \delta^{(3)}(\mathbf{k} - \mathbf{k}') \tau_\gamma \left(z - \frac{k_\gamma^2}{2\mu_\gamma} \right) . \tag{C-8}$$

Since $\tau_\gamma(z)$ is the two-body operator, z is the three-body energy and the momenta $\mathbf{k}$ and $\mathbf{k}'$ of the third particle are not connected with the two-body interaction, it is necessary to change the energy dependence of the operator $\tau_\gamma(z)$ as shown in (C-8). After using formula (C-8) and the expressions for $X_{\beta\alpha}$ and $Z_{\beta\alpha}$ (C-6,C-7) we obtain

$$\begin{aligned}
&\sum_{l=0}^{\infty}(2l+1)\mathrm{P}_l(\hat{\mathbf{k}}_\beta\cdot\hat{\mathbf{k}}_\alpha)\langle I_\beta,k_\beta lm|X_{\beta\alpha}(z)|k_\alpha lm,I_\alpha\rangle = \\
&\qquad(1-\delta_{\beta\alpha})\sum_{l=0}^{\infty}(2l+1)\mathrm{P}_l(\hat{\mathbf{k}}_\beta\cdot\hat{\mathbf{k}}_\alpha)\langle I_\beta,k_\beta lm|Z_{\beta\alpha}(z)|k_\alpha lm,I_\alpha\rangle+ \\
&\qquad+\sum_{\gamma\neq\beta}\sum_{I_\gamma}\int\mathrm{d}\mathbf{k}_\gamma\sum_{l=0}^{\infty}\sum_{l'=0}^{\infty}(2l+1)(2l'+1)\mathrm{P}_l(\hat{\mathbf{k}}_\beta\cdot\hat{\mathbf{k}}_\gamma)\mathrm{P}_{l'}(\hat{\mathbf{k}}_\gamma\cdot\hat{\mathbf{k}}_\alpha)\times \\
&\times\langle I_\beta,k_\beta lm|Z_{\beta\alpha}(z)|k_\gamma lm,I_\gamma\rangle\langle I_\gamma|\tau_\gamma\left(z-\frac{k_\gamma^2}{2\mu_\gamma}\right)|I_\gamma\rangle\langle I_\gamma,k_\gamma l'm'|X_{\beta\alpha}(z)|k_\alpha l'm',I_\alpha\rangle.
\end{aligned} \tag{C-9}$$

Then we substitute $\mathrm{d}\mathbf{k}_\gamma = k_\gamma^2\mathrm{d}k_\gamma\mathrm{d}\Omega$, use the orthogonality relation for the Legendre polynomials

$$\int\mathrm{d}\Omega\,\mathrm{P}_l(\mathbf{n}\cdot\mathbf{n}_1)\mathrm{P}_k(\mathbf{n}\cdot\mathbf{n}_2) = \frac{4\pi}{2l+1}\mathrm{P}_l(\mathbf{n}_1\cdot\mathbf{n}_2)\delta_{lk} \tag{C-10}$$

and remove the Legendre polynomials in equations (C-9). Finally, we put $l=0$ and $m=0$ (s-wave approximation) and write the equations (C-9) in a following form

$$\begin{aligned}
\langle I_\beta,k_\beta|X_{\beta\alpha}(z)|k_\alpha,&I_\alpha\rangle = (1-\delta_{\beta\alpha})\langle I_\beta,k_\beta|Z_{\beta\alpha}(z)|k_\alpha,I_\alpha\rangle+ \\
&+4\pi\sum_{\gamma\neq\beta}\sum_{I_\gamma}\int\mathrm{d}k_\gamma k_\gamma^2\langle I_\beta,k_\beta|Z_{\beta\gamma}(z)|k_\gamma,I_\gamma\rangle\times \\
&\times\langle I_\gamma|\tau_\gamma\left(z-\frac{k_\gamma^2}{2\mu_\gamma}\right)|I_\gamma\rangle\langle I_\gamma,k_\gamma|X_{\gamma\alpha}(z)|k_\alpha,I_\alpha\rangle.
\end{aligned} \tag{C-11}$$

D Sets of integral equations

There are two sets of integral equations mentioned in the text, which we derived during manipulations with the Faddeev equations. The first set:

$$
\begin{aligned}
X^{0,0}_{KK}(k_K,k'_K) = \\
&4\pi\int_0^\infty \mathrm{d}\bar{k}_{N_1}\bar{k}^2_{N_1} Z^{0,0}_{KN_1}(k_K,\bar{k}_{N_1})\tau^0_{N_1}\left(z-\frac{\bar{k}^2_{N_1}}{2\mu_{N_1}}\right) X^{0,0}_{N_1K}(\bar{k}_{N_1},k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_{N_1}\bar{k}^2_{N_1} Z^{0,1}_{KN_1}(k_K,\bar{k}_{N_1})\tau^1_{N_1}\left(z-\frac{\bar{k}^2_{N_1}}{2\mu_{N_1}}\right) X^{1,0}_{N_1K}(\bar{k}_{N_1},k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_{N_2}\bar{k}^2_{N_2} Z^{0,0}_{KN_2}(k_K,\bar{k}_{N_2})\tau^0_{N_2}\left(z-\frac{\bar{k}^2_{N_2}}{2\mu_{N_2}}\right) X^{0,0}_{N_2K}(\bar{k}_{N_2},k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_{N_2}\bar{k}^2_{N_2} Z^{0,1}_{KN_2}(k_K,\bar{k}_{N_2})\tau^1_{N_2}\left(z-\frac{\bar{k}^2_{N_2}}{2\mu_{N_2}}\right) X^{1,0}_{N_2K}(\bar{k}_{N_2},k'_K),
\end{aligned}
\tag{D-1}
$$

$$
\begin{aligned}
X^{0,0}_{N_1K}(K_{N_1},k'_K) = Z^{0,0}_{N_1K}(K_{N_1},k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_K\bar{k}^2_K Z^{0,0}_{N_1K}(K_{N_1},\bar{k}_K)\tau^0_K\left(z-\frac{\bar{k}^2_K}{2\mu_K}\right) X^{0,0}_{KK}(\bar{k}_K,k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_{N_2}\bar{k}^2_{N_2} Z^{0,0}_{N_1N_2}(k_{N_1},\bar{k}_{N_2})\tau^0_{N_2}\left(z-\frac{\bar{k}^2_{N_2}}{2\mu_{N_2}}\right) X^{0,0}_{N_2K}(\bar{k}_{N_2},k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_{N_2}\bar{k}^2_{N_2} Z^{0,1}_{N_1N_2}(k_{N_1},\bar{k}_{N_2})\tau^1_{N_2}\left(z-\frac{\bar{k}^2_{N_2}}{2\mu_{N_2}}\right) X^{1,0}_{N_2K}(\bar{k}_{N_2},k'_K),
\end{aligned}
\tag{D-2}
$$

$$
\begin{aligned}
X^{1,0}_{N_1K}(K_{N_1},k'_K) = Z^{1,0}_{N_1K}(K_{N_1},k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_K\bar{k}^2_K Z^{1,0}_{N_1K}(K_{N_1},\bar{k}_K)\tau^0_K\left(z-\frac{\bar{k}^2_K}{2\mu_K}\right) X^{0,0}_{KK}(\bar{k}_K,k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_{N_2}\bar{k}^2_{N_2} Z^{1,0}_{N_1N_2}(k_{N_1},\bar{k}_{N_2})\tau^0_{N_2}\left(z-\frac{\bar{k}^2_{N_2}}{2\mu_{N_2}}\right) X^{0,0}_{N_2K}(\bar{k}_{N_2},k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_{N_2}\bar{k}^2_{N_2} Z^{1,1}_{N_1N_2}(k_{N_1},\bar{k}_{N_2})\tau^1_{N_2}\left(z-\frac{\bar{k}^2_{N_2}}{2\mu_{N_2}}\right) X^{1,0}_{N_2K}(\bar{k}_{N_2},k'_K),
\end{aligned}
\tag{D-3}
$$

$$
\begin{aligned}
X^{0,0}_{N_2K}(K_{N_2},k'_K) = Z^{0,0}_{N_2K}(K_{N_2},k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_K\bar{k}^2_K Z^{0,0}_{N_2K}(K_{N_2},\bar{k}_K)\tau^0_K\left(z-\frac{\bar{k}^2_K}{2\mu_K}\right) X^{0,0}_{KK}(\bar{k}_K,k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_{N_1}\bar{k}^2_{N_1} Z^{0,0}_{N_2N_1}(k_{N_2},\bar{k}_{N_1})\tau^0_{N_1}\left(z-\frac{\bar{k}^2_{N_1}}{2\mu_{N_1}}\right) X^{0,0}_{N_1K}(\bar{k}_{N_1},k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_{N_1}\bar{k}^2_{N_1} Z^{0,1}_{N_2N_1}(k_{N_2},\bar{k}_{N_1})\tau^1_{N_1}\left(z-\frac{\bar{k}^2_{N_1}}{2\mu_{N_1}}\right) X^{1,0}_{N_1K}(\bar{k}_{N_1},k'_K),
\end{aligned}
\tag{D-4}
$$

$$
\begin{aligned}
X^{1,0}_{N_2K}(K_{N_2},k'_K) &= Z^{1,0}_{N_2K}(K_{N_2},k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_K\bar{k}_K^2 Z^{1,0}_{N_2K}(K_{N_2},\bar{k}_K)\tau^0_K\left(z-\frac{\bar{k}_K^2}{2\mu_K}\right)X^{0,0}_{KK}(\bar{k}_K,k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_{N_1}\bar{k}_{N_1}^2 Z^{1,0}_{N_2N_1}(k_{N_2},\bar{k}_{N_1})\tau^0_{N_1}\left(z-\frac{\bar{k}_{N_1}^2}{2\mu_{N_1}}\right)X^{0,0}_{N_1K}(\bar{k}_{N_1},k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_{N_1}\bar{k}_{N_1}^2 Z^{1,1}_{N_2N_1}(k_{N_2},\bar{k}_{N_1})\tau^1_{N_1}\left(z-\frac{\bar{k}_{N_1}^2}{2\mu_{N_1}}\right)X^{1,0}_{N_1K}(\bar{k}_{N_1},k'_K).
\end{aligned}
\tag{D-5}
$$

The second set:

$$
\begin{aligned}
X_D(k_K,k'_K) &=\\
&4\pi\int_0^\infty \mathrm{d}\bar{k}_N\bar{k}_N^2 Z^{0,0}_{KN}(k_K,\bar{k}_N)\tau^0_N\left(z-\frac{\bar{k}_N^2}{\mu_N}\right)X_0(\bar{k}_N,k'_K)+\\
&+4\pi\int_0^\infty \mathrm{d}\bar{k}_N\bar{k}_N^2 Z^{0,1}_{KN}(k_K,\bar{k}_N)\tau^1_N\left(z-\frac{\bar{k}_N^2}{\mu_N}\right)X_1(\bar{k}_N,k'_K),
\end{aligned}
\tag{D-6}
$$

$$
\begin{aligned}
X_0(k_N,k'_K) &= 2Z^{0,0}_{NK}(k_N,k'_K)+\\
&+8\pi\int_0^\infty \mathrm{d}\bar{k}_K\bar{k}_K^2 Z^{0,0}_{NK}(k_N,\bar{k}_K)\tau^0_K\left(z-\frac{\bar{k}_K^2}{\mu_K}\right)X_D(\bar{k}_K,k'_K)\\
&-4\pi\int_0^\infty \mathrm{d}\bar{k}_N\bar{k}_N^2 Z^{0,0}_{NN}(k_N,\bar{k}_N)\tau^0_N\left(z-\frac{\bar{k}_N^2}{\mu_N}\right)X_0(\bar{k}_N,k'_K)\\
&-4\pi\int_0^\infty \mathrm{d}\bar{k}_N\bar{k}_N^2 Z^{0,1}_{NN}(k_N,\bar{k}_N)\tau^1_N\left(z-\frac{\bar{k}_N^2}{\mu_N}\right)X_1(\bar{k}_N,k'_K),
\end{aligned}
\tag{D-7}
$$

$$
\begin{aligned}
X_1(k_N,k'_K) &= 2Z^{1,0}_{NK}(k_N,k'_K)+\\
&+8\pi\int_0^\infty \mathrm{d}\bar{k}_K\bar{k}_K^2 Z^{1,0}_{NK}(k_N,\bar{k}_K)\tau^0_K\left(z-\frac{\bar{k}_K^2}{\mu_K}\right)X_D(\bar{k}_K,k'_K)\\
&-4\pi\int_0^\infty \mathrm{d}\bar{k}_N\bar{k}_N^2 Z^{1,0}_{NN}(k_N,\bar{k}_N)\tau^0_N\left(z-\frac{\bar{k}_N^2}{\mu_N}\right)X_0(\bar{k}_N,k'_K)\\
&-4\pi\int_0^\infty \mathrm{d}\bar{k}_N\bar{k}_N^2 Z^{1,1}_{NN}(k_N,\bar{k}_N)\tau^1_N\left(z-\frac{\bar{k}_N^2}{\mu_N}\right)X_1(\bar{k}_N,k'_K).
\end{aligned}
\tag{D-8}
$$

E Calculation of the kernels

We start with the definition of the kernels (24)

$$K_{\Lambda\Lambda'}(k_\Lambda, k_{int}) = 4\pi\tilde{C}\, k_{int}^2 Z^{I,I'}_{\Lambda\Lambda'}(k_\Lambda, k_{int})\tau^{I'}_{\Lambda'}\left(z - \frac{k_{int}^2}{2\mu_\Lambda}\right). \tag{E-1}$$

Here the $\tau^{I'}_\Lambda$ function can be expressed following equation (7). Next, we will calculate the functions $Z^{I,I'}_{\Lambda\Lambda'}$. We insert into the definion (12)

$$\langle I_\beta, \mathbf{k}_\beta | Z_{\beta\alpha} | \mathbf{k}'_\alpha, I_\alpha\rangle \equiv \langle I_\beta, \mathbf{k}_\beta | \langle g_\beta | G_0 | g_\alpha\rangle | \mathbf{k}'_\alpha, I_\alpha\rangle, \tag{E-2}$$

the closure relations

$$1 = \int \mathrm{d}\bar{\mathbf{p}}_\xi \mathrm{d}\bar{\mathbf{k}}_\xi |\bar{\mathbf{p}}_\xi, \bar{\mathbf{k}}_\xi\rangle\langle\bar{\mathbf{p}}_\xi, \bar{\mathbf{k}}_\xi|, \tag{E-3}$$

where $\xi = 1, 2, 3$ is an arbitrary index, and use the relation $|g_\alpha\rangle|I_\alpha\rangle = |g^{I_\alpha}_\alpha\rangle|I_\alpha\rangle$. We obtain

$$\begin{aligned}\langle \mathbf{k}_\beta | Z^{I_\beta, I_\alpha}_{\beta\alpha} | \mathbf{k}'_\alpha\rangle = \int\int \mathrm{d}\tilde{\mathbf{p}}_\alpha \mathrm{d}\tilde{\mathbf{k}}_\alpha \mathrm{d}\bar{\mathbf{p}}_\beta \mathrm{d}\bar{\mathbf{k}}_\beta \langle \mathbf{k}_\beta | \langle g^{I_\beta}_\beta | \bar{\mathbf{p}}_\beta, \bar{\mathbf{k}}_\beta\rangle \times \\ \times \langle \bar{\mathbf{p}}_\beta, \bar{\mathbf{k}}_\beta | G_0 | \tilde{\mathbf{p}}_\alpha, \tilde{\mathbf{k}}_\alpha\rangle \langle \tilde{\mathbf{p}}_\alpha, \tilde{\mathbf{k}}_\alpha | g^{I_\alpha}_\alpha\rangle | \mathbf{k}'_\alpha\rangle \langle I_\beta | I_\alpha\rangle.\end{aligned} \tag{E-4}$$

Further, we use the relation $\langle \mathbf{k}_\beta | \langle g^{I_\beta}_\beta | \bar{\mathbf{p}}_\beta, \bar{\mathbf{k}}_\beta\rangle = \langle \mathbf{k}_\beta | \bar{\mathbf{k}}_\beta\rangle \langle g^{I_\beta}_\beta | \bar{\mathbf{p}}_\beta\rangle$, act on the vectors $|\tilde{\mathbf{p}}_\alpha, \tilde{\mathbf{k}}_\alpha\rangle$ by the three-body Green's function, and apply the relation (A-8) where is needed:

$$\begin{aligned}\langle \mathbf{k}_\beta | Z^{I_\beta, I_\alpha}_{\beta\alpha} | \mathbf{k}'_\alpha\rangle = \int\int \mathrm{d}\tilde{\mathbf{p}}_\alpha \mathrm{d}\tilde{\mathbf{k}}_\alpha \mathrm{d}\bar{\mathbf{p}}_\beta \mathrm{d}\bar{\mathbf{k}}_\beta \, g^{I_\alpha}_\alpha(\tilde{\mathbf{p}}_\alpha) g^{I_\beta}_\beta(\bar{\mathbf{p}}_\beta) \times \\ \times \frac{1}{z - \frac{\tilde{\mathbf{p}}^2_\alpha}{2M_\alpha} - \frac{\tilde{\mathbf{k}}^2_\alpha}{2\mu_\alpha}} \langle I_\beta | I_\alpha\rangle \delta^{(3)}(\mathbf{k}'_\alpha - \tilde{\mathbf{k}}_\alpha) \delta^{(3)}(\mathbf{k}_\beta - \bar{\mathbf{k}}_\beta) \times \\ \times \delta^{(3)}\left(\bar{\mathbf{p}}_\beta - C^A \tilde{\mathbf{p}}_\alpha - C^B \tilde{\mathbf{k}}_\alpha\right) \delta^{(3)}\left(\bar{\mathbf{k}}_\beta - C^C \tilde{\mathbf{p}}_\alpha - C^D \tilde{\mathbf{k}}_\alpha\right).\end{aligned} \tag{E-5}$$

In analogy with equations (C-2–C-7), we can write

$$\langle \bar{\mathbf{p}}_\alpha | V^{I_\alpha}_\alpha | \tilde{\mathbf{p}}_\alpha\rangle = \lambda_{I_\alpha} g^{I_\alpha}_\alpha(\tilde{\mathbf{p}}_\alpha) g^{I_\alpha}_\alpha(\bar{\mathbf{p}}_\alpha) = \lambda_{I_\alpha} \sum_{l=0}^{\infty} \mathrm{P}_l(\hat{\tilde{\mathbf{p}}}_\alpha \cdot \hat{\bar{\mathbf{p}}}_\alpha) \langle \tilde{p}_\alpha lm | g^{I_\alpha}_\alpha\rangle \langle g^{I_\alpha}_\alpha | \bar{p}_\alpha lm\rangle, \tag{E-6}$$

and we will consider only the s-wave ($l = 0$) term. By using the formfactors for s-waves, $g^{I_\Lambda}_\Lambda(p_\Lambda) = (|\mathbf{p}^2_\lambda| + (\beta^{I_\Lambda}_\Lambda)^2)^{-1}$, and after applying the delta functions, the

equation (E-5) transforms in the form

$$\begin{aligned}\langle \mathbf{k}_\beta | Z_{\beta\alpha}^{I_\beta,I_\alpha} | \mathbf{k}'_\alpha \rangle = \\ \frac{1}{\left| \frac{C^A}{C^C}\mathbf{k}_\beta + \left(C^B - \frac{C^A C^D}{C^C} \right) \mathbf{k}'_\alpha \right|^2 + (\beta_\beta^{I_\beta})^2} \times \\ \times \frac{1}{z - \frac{1}{2M_\alpha} \left| \frac{\mathbf{k}_\beta}{C^C} - \frac{C^D}{C^C}\mathbf{k}'_\alpha \right|^2 - \frac{|\mathbf{k}'_\alpha|^2}{2\mu_\alpha}} \times \\ \times \frac{1}{\left| \frac{\mathbf{k}_\beta}{C^C} - \frac{C^D}{C^C}\mathbf{k}'_\alpha \right|^2 + (\beta_\alpha^{I_\alpha})^2} \langle I_\beta | I_\alpha \rangle .\end{aligned} \tag{E-7}$$

After operations with the Legendre polynomials we get the formula

$$\langle k_\beta | Z_{\beta\alpha}^{I_\beta,I_\alpha} | k'_\alpha \rangle = \frac{1}{2} \int_{-1}^{1} \mathrm{d}\Omega \langle \mathbf{k}_\beta | Z_{\beta\alpha}^{I_\beta,I_\alpha} | \mathbf{k}'_\alpha \rangle . \tag{E-8}$$

In each denominator in (E-7) there is a scalar product of the vectors $\mathbf{k}_\beta$ and $\mathbf{k}'_\alpha$ which can be expressed using $\mathbf{k}_\beta \cdot \mathbf{k}'_\alpha = \Omega k_\beta k'_\alpha$, where Ω is the angle between both momenta. Hence we rewrite (E-7) in the general form

$$\langle k_\beta | Z_{\beta\alpha}^{I_\beta,I_\alpha} | k'_\alpha \rangle = \frac{1}{2} \sum_{j=1}^{3} \int_{-1}^{1} \mathrm{d}\Omega \frac{1}{A_j \Omega + B_j} \langle I_\beta | I_\alpha \rangle , \tag{E-9}$$

where $A_j, B_j, j = 1, 2, 3$, are functions of masses, two-body potential parameters and the three-body energy. After integration, we get the final relation

$$Z_{\beta\alpha}^{I_\beta,I_\alpha}(k_\beta, k'_\alpha) = \frac{1}{2} \sum_{j=1}^{3} \frac{D_j}{A_j} \ln \frac{|B_j + A_j|}{|B_j - A_j|} \langle I_\beta | I_\alpha \rangle , \tag{E-10}$$

where $D_j, j = 1, 2, 3$, is a function of A_j and B_j.

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