Lesson 6: Item response theory models

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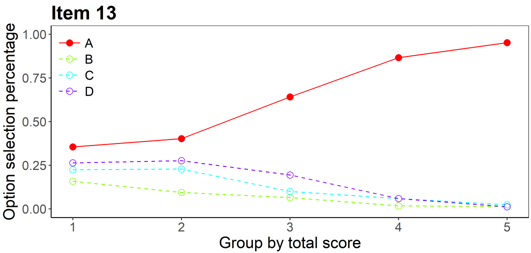
Outline

Review: Traditional Item Analysis

Traditional item analysis describes item properties by

- percentages of correct response
- proportions of those who selected given distractor
- differences of percentages for groups by total score
- correlations of item score with total score



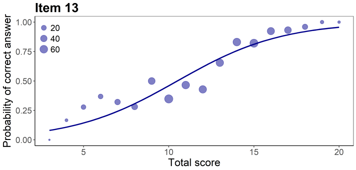
Review: Logistic Regression

Item properties described by parameters β_{0j} and β_{1j} of logistic function

$$\pi_{ij} = P(Y_{ij} = 1 | X_i, \beta_{0j}, \beta_{1j}) = \frac{\exp(\beta_{0j} + \beta_{1j} X_i)}{1 + \exp(\beta_{0j} + \beta_{1j} X_i)}$$

Also can be written as:

$$\text{logit}(\pi_{ij}) = \log\left(\frac{\pi_{ij}}{1 - \pi_{ij}}\right) = \beta_{0j} + \beta_{1j} X_i$$



Notes:

- Linear model is related to response variable via a link function (GLM)
- Link functions: logit, probit (inverse of the cumulative distribution function)

Logistic Regression - IRT parametrization, Z scores

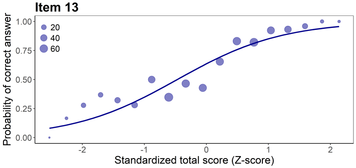
Logistic regression with IRT parametrization

$$\pi_{ij} = P(Y_{ij} = 1 | Z_i, a_j, b_j) = \frac{\exp[a_j(Z_i - b_j)]}{1 + \exp[a_j(Z_i - b_j)]}$$

b_j difficulty of item j
a_j discrimination of item j
Z_i standardized total score of person i

Also can be written as:

$$\text{logit}(\pi_{ij}) = \log\left(\frac{\pi_{ij}}{1 - \pi_{ij}}\right) = a_j(Z_i - b_j)$$



Nonlinear Regression

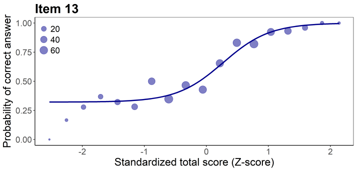
$$\pi_{ij} = P(Y_{ij} = 1 | Z_i, a_j, b_j, c_i) = (1 - c_i) \frac{\exp[a_j(Z_i - b_j)]}{1 + \exp[a_j(Z_i - b_j)]}$$

b_j difficulty of item j

a_j discrimination of item j

c_j probability of guessing of item j

Z_i standardized total score of person i



Notes:

- Not a GLM

Introduction to Item Response Theory

Framework for

- estimating *latent traits* (ability levels) θ

  by means of *manifest* (observable) variables (item responses)

  and appropriate *psychometric* (statistical) model

Notes:

- Ability θ now treated as random variable
- Items: dichotomous, polytomous, multiple-choice, ...
- IRT model: describes probability of (correct) answer as function of
  - ability level and
  - item parameters

  This function is called:
  - *Item response function (IRF)*
  - *Item characteristic curve (ICC)*

Introduction to IRT models

Aim of IRT models:

- To calibrate items (estimate difficulty, discrimination, guessing,...)
- To assess respondents' latent trait (ability, satisfaction, anxiety,...)
- To describe test properties (standard error, test information,...)

Other applications of IRT models:

- Test linking and equating
- Differential item functioning
- Computerized adaptive testing
- etc.

Common Assumptions of IRT models

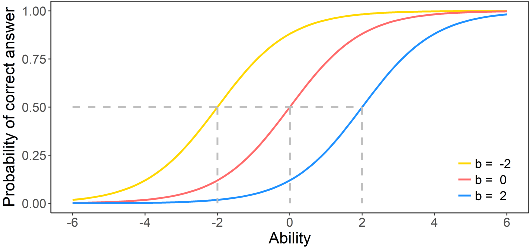
1. Unidimensionality of latent variable
   - all items measure only one construct
   - can be tested
   - examples when violated?
2. Local independence
   - also called conditional independence
   - given latent ability, the responses to items are independent
   - examples when violated?
3. Monotonicity
   - the ICC is monotonically increasing or decreasing with the ability level
4. Invariance of parameters
   - Estimates of item parameters are the same over samples of examinees
   - Estimates of ability parameters are the same over samples of items
   - examples when violated?
5. Independence of respondents
   - examples when violated?

Rasch Model

$$\pi_{ij} = P(Y_{ij} = 1|\theta_i, b_j) = \frac{\exp(\theta_i - b_j)}{1 + \exp(\theta_i - b_j)}$$

θ_i ability of person i
b_j difficulty of item j (location of inflection point)

Item Characteristic Curve (ICC)
also called Item Response Function (IRF)

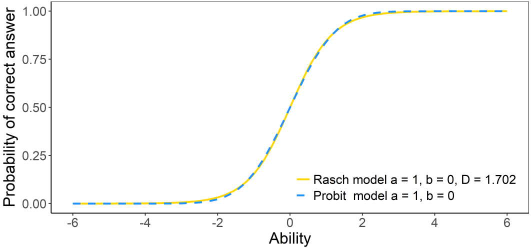


Logistic vs. Probit model (Note on Scaling parameter D)

Rasch model is sometimes defined as:

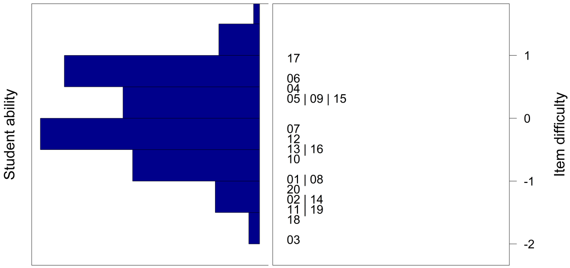
$$\pi_{ij} = P(Y_{ij} = 1|\theta_i, b_j) = \frac{\exp(D[\theta_i - b_j])}{1 + \exp(D[\theta_i - b_j])}$$

$D = 1.702$ is scaling parameter introduced in order to match logistic and probit metrics very closely (Lord and Novick, 1968)



Item-Person Map (Wright Map)

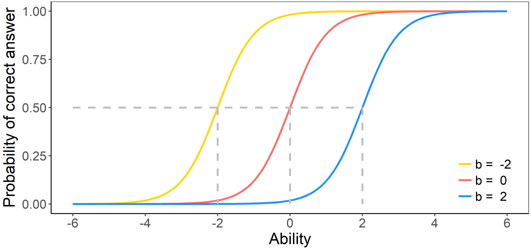
IRT models allow us to put *items* and *persons* on the same scale



1PL IRT Model

$$\pi_{ij} = P(Y_{ij} = 1 | \theta_i, b_j) = \frac{\exp[a(\theta_i - b_j)]}{1 + \exp[a(\theta_i - b_j)]}$$

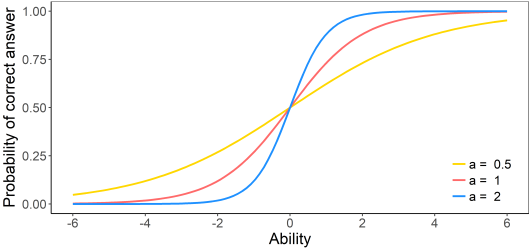
θ_i ability of person i
b_j difficulty of item j (location of inflection point)
a discrimination common for all items (slope at inflection point)



2PL IRT Model

$$\pi_{ij} = P(Y_{ij} = 1 | \theta_i, a_j, b_j) = \frac{\exp[a_j(\theta_i - b_j)]}{1 + \exp[a_j(\theta_i - b_j)]}$$

θ_i ability of person i
b_j difficulty of item j (location of inflection point)
a_j discrimination of item j (slope at inflection point)



3PL IRT Model

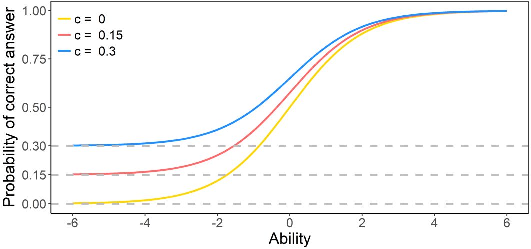
$$\pi_{ij} = P(Y_{ij} = 1 | \theta_i, a_j, b_j, c_j) = c_j + (1 - c_j) \frac{\exp[a_j(\theta_i - b_j)]}{1 + \exp[a_j(\theta_i - b_j)]}$$

θ_i ability of person i

b_j difficulty of item j (location of inflection point)

a_j discrimination of item j (slope at inflection point)

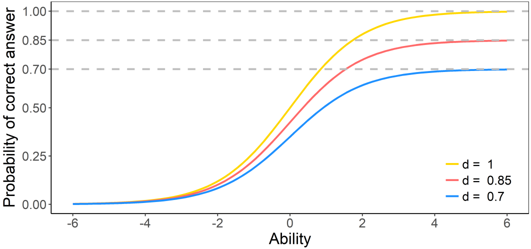
c_j pseudo-guessing parameter of item j (lower/left asymptote)



4PL IRT Model

$$\pi_{ij} = P(Y_{ij} = 1 | \theta_i, a_j, b_j, c_j, d_j) = c_j + (d_j - c_j) \frac{\exp[a_j(\theta_i - b_j)]}{1 + \exp[a_j(\theta_i - b_j)]}$$

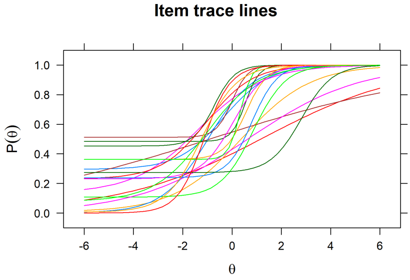
θ_i ability of person i
b_j difficulty of item j (location of inflection point)
a_j discrimination of item j (slope at inflection point)
c_j pseudo-guessing parameter of item j (lower/left asymptote)
d_j innatention parameter of item j (upper/right asymptote)

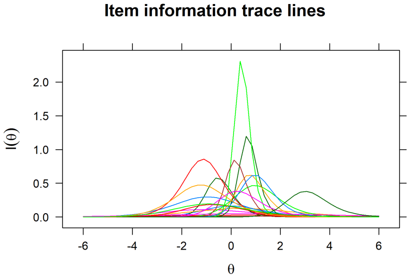


Information Function

$$P(\theta, a_j, b_j, c_j, d_j) = c_j + (d_j - c_j)\frac{\exp[a_j(\theta - b_j)]}{1 + \exp[a_j(\theta - b_j)]},$$

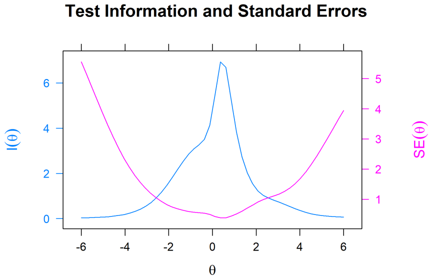
$$\mathrm{I}_j(\theta, a_j, b_j, c_j, d_j) = \frac{\delta P}{\delta \theta} = a_j(d_j - c_j)\frac{\exp[a_j(\theta - b_j)]}{\{1 + \exp[a_j(\theta - b_j)]\}^2}$$

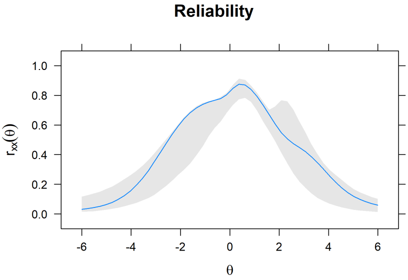




Test Information and Reliability

$$\mathrm{I}(\theta) = \sum_j \mathrm{I}_j(\theta, a_j, b_j, c_j, d_j)$$





$$SEM = \sigma\sqrt{(1 - r_{xx})}$$

Further Topics

Further issues

- Estimation of item parameters
- Estimation of student abilities
- Item and Person Fit Assessment, etc.

Further models

- Polytomous IRT models (ordinal/nominal)
- Multidimensional IRT models
- Hierarchical IRT models, etc.
- Accounting for Differential item functioning, etc.

Applications

- Test equating
- Computerized adaptive testing, etc.

Vocabulary

- Item Characteristic Curve (ICC)
- Item Response Fuction (IRF)
- Item Information Function (IIF)
- Test Information Function (TIF)
- Likelihood function
- Rasch model, 1PL, 2PL, 3PL, 4PL IRT models