Universal expectations, generic measure spaces, and Hájek's Probability Logic

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Abstract

Beginning where T. Kroupa's talk left us, we have (i) Hájek's Probability Logic, a tool to reason about expected values of bounded real random variables, and (ii) a two-sorted algebraisation of this logic. I attempt to sketch how (i) and (ii) fit into a broader current research programme aimed at algebraising probability and measure. One of the reasons why I pursue this programme (together with T. Kroupa, further collaborators, and students) is because I would like to know the answers to questions such as the following two. (a) Given a Boolean algebra, is there a universal (=most general) way of assigning a probability degree to its elements? (b) Does there exist a probability space that is generic for the class of all probability spaces, in the same sense that the two-element Boolean algebra is generic for the class of all Boolean algebras? The questions are hand-wavy in this abstract, but will be made precise in the talk. Then the answers are: (a) Yes; (b) Yes. The further implied questions — what do universal expectations and generic probability spaces look like? — can be answered, too: (a) requires considerable involvement with affine representations in the style of Choquet, and is best left to the talk; as to (b), the Cantor space equipped with a specific rational-valued probability measure turns out to be generic. In terms of Hájek's seminal ideas on Probability Logic, the universal expectations of (a) are just the Lindenbaum-Tarski algebras of his logic (relative to a theory in the sort of events); and a generic probability space as in (b) is a single model with respect to which his logic is complete.