

Intuitionistic Provability Logic

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Abstract

One of the most striking features of Solovay's arithmetical completeness theorem for Löb's Logic is its great stability. As a first approximation: It holds for any classical Σ_1^0 -sound theory that extends Elementary Arithmetic. (We stipulate that theories are equipped with a designated elementary axiom set.)

The situation is dramatically different when we consider constructive theories like Heyting Arithmetic. Different theories satisfy very different sets of principles. Solovay style completeness theorems are only known for very few constructive theories.

My talk will offer a scenic tour across the landscape of possible principles. I will discuss what is known about closed fragments. (We will see a surprising appearance of Gödel-Dummett Logic here.) Moreover, I will discuss the characterization of the Σ_1^0 -provability logic of Heyting Arithmetic due to Mojtabedi and Ardeshir. I will also sketch the recent work of Jetze Zoethout that gives an arithmetical completeness theorem for a reasonably natural theory. Jetzes work also provides an alternative route to the Σ_1^0 -provability logic of Heyting Arithmetic.

Tremendous beauty is hidden in this subject, but we have to work hard to make it visible.