

Proof complexity and games

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Proof complexity studies two kinds of concepts: propositional proof systems and axiomatizations of weak fragments of arithmetic. They can be viewed as a nonuniform and uniform versions of the same concepts. The particular proof systems and theories are associated with complexity classes. Like in complexity theory the main problem is to prove separations of complexity classes, in proof complexity we want to prove separations of theories or proof systems. In principle, the corresponding problems in proof complexity should be easier, but in reality we are as powerless as in complexity theory. What we can do, however, is to find combinatorial characterizations of provable sentences of particular logical complexity. Then the increasing formal complexity of these characterizations is some evidence of increasing strengths of the theories. These characterizations are often presented in a form of some game. Interestingly, games also appear in the contexts where there are essentially no quantifier alternation. In the lecture I will present some games that have been introduced for this purpose.