The 7th international microconference

Analytic and algebraic methods in physics VII

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THE BOOK OF ABSTRACTS

(the version of March 16, 2011)

(alphabetical ordering)

Per Alexandersson

On Eigenvalues of the Schroedinger operator with a complex-valued polynomial potential

We generalize a recent result of A. Eremenko and A. Gabrielov on irreducibility of the spectral discriminant for the Schrdinger equation with quartic potentials. We consider the eigenvalue problem with a complex-valued polynomial potential of arbitrary degree d and show that the spectral determinant of this problem is connected and irreducible. In other words, every eigenvalue can be reached from any other by analytic continuation.

We also prove connectedness of the parameter spaces of the potentials that admit eigenfunctions satisfying k ¿ 2 boundary conditions, except for the case d is even and k = d/2. In the latter case, connected components of the parameter space are distinguished by the number of zeros of the eigenfunctions.

(Joint work with Andrei Gabrielov)

Andreas Fring

PT-symmetry breaking in complex nonlinear wave equations and their deformations

We investigate complex versions of the Korteweg-deVries equations and an Ito type nonlinear system with two coupled nonlinear fields. We systematically construct rational, trigonometric/hyperbolic, elliptic and soliton solutions for these models and focus in particular on physically feasible systems, that is those with real energies. The reality of the energy is usually attributed to different realisations of an antilinear symmetry, as for instance $\mathcal{PT}$-symmetry. It is shown that the symmetry can be spontaneously broken in two alternative ways either by specific choices of the domain or by manipulating the parameters in the solutions of the model, thus leading to complex energies. Surprisingly the reality of the energies can be regained in some cases by a further breaking of the symmetry. In many examples some of the fixed points in the complex solution undergo a Hopf bifurcation in the breaking process. By employing several different variants of the symmetries we propose many classes of new invariant extensions of these models and study their properties.

Eva-Maria Graefe:

Wave packet evolution in non-Hermitian quantum systems

The quantum evolution of the Wigner function for Gaussian wave packets generated by a non-Hermitian Hamiltonian is investigated. In the semiclassical limit $\hbar \to 0$ this yields the non-Hermitian analog of the Ehrenfest theorem for the dynamics of observable expectation values. The lack of Hermiticity reveals the importance of the complex structure on the classical phase space: The resulting equations of motion are coupled to an equation of motion for the phase space metric—a phenomenon having no analogue in Hermitian theories. Furthermore, example studies show that the anti-Hermitian term can improve the accuracy of the classical approximation.

Daniel Hook:

Tunnelling in classical mechanics

A classical particle that is initially in a classically allowed region of a potential is not confined to this region for all time if its energy is complex. Rather, the particle may travel through complex coordinate space and visit other classically allowed regions. Thus, a complex-energy classical particle can exhibit tunneling-like behavior. This tunneling behavior persists as the imaginary part of the energy tends to zero. Hence one may compare complex classical tunneling times with quantum tunneling probabilities. An accurate numerical study of quantum and classical tunneling demonstrates that as the energy increases, the probabilities associated with complex classical tunneling approach the corresponding quantum probabilities.

Hugh F. Jones

Theoretical Aspects of Diffraction from PT-symmetric Periodic Optical Lattices

I discuss some theoretical aspects of the novel features of scattering from PT-symmetric optical lattices, such as birefringence and power oscillations. Below the symmetry-breaking threshold these can be approached by exploiting the similarity transformation to the equivalent Hermitian problem, which involves solutions to the Mathieu equation, and using the method of stationary states. Precisely at the threshold one encounters degenerate eigenfunctions and Jordan blocks. The analogue Schrödinger equation can in this case be transformed to the Bessel equation and the method of stationary states again applied. At first sight one would expect a linear growth of the amplitude, but it turns out that this growth is saturated, in ways that are not fully understood.

Martin Kalina

On centers of atomic Archimedean effect algebras

An effect algebra E is a system with a partial operation $\oplus$. The center C(E) consists of all elements of E which are sharp and compatible with all elements of E. If the center C(E) is an atomic Boolean algebra such that the supremum of the set A in E equals to the top element, i.e., $\bigvee_E A = 1$, then the effect algebra E is subdirectly irreducible. We have already shown that there are atomic Archimedean effect algebras with an atomic center where the equality $\bigvee_E A = 1$ fails to be true. Riecanova and Paseka published an open problem to find necessary and sufficient condition under which the mentioned equality is fulfilled. We give a solution of this problem.

Vladimir V. Kisil

Representations of the Heisenberg group and all sorts of mechanics

In the spirit of geometric quantisation we consider representations of the Heisenberg(–Weyl) group induced by hypercomplex characters of its centre. This allows to gather under the same framework, called p-mechanics, the three principal cases: quantum mechanics (elliptic character), hyperbolic mechanics (associated to Krein spaces with indefinite Minkowsky-type metric) and classical mechanics (parabolic character). In each case we recover the corresponding dynamic equation as well as rules for addition of probabilities. Notably, we are able to obtain whole classical mechanics without any kind of semiclassical limit $\hbar \to 0$.

M. Howard Lee:

Cyclic solutions in chaos, Sharkovskii's theorem and isomorphism.

At the fully developed chaos in the logistic map it is possible to solve analytically the 3-cycle problem. There are found two 3-cycles each with a simple structure. By Sharkovskii's theorem, we can assert that there are all other multi-cycles of the same structure given by a set of all rational numbers in the interval of (0,1). We shall show that the resultant invariant density of cycles is isomorphic to the frequency density of a harmonic oscillator chain at the thermodynamic limit. This isomorphism shows the same underpinnings for ergodicity in two entirely unrelated systems.

J. Lipovský:

Resonance asymptotics for quantum graphs in magnetic field

We investigate the asymptotics of the number of resonances in quantum graphs. It has been previously shown by Davies and Pushnitski that the constant in the main term of asymptotics is in some cases smaller than the expected one (the sum of lengths of all internal edges). First, we recall the results for the case of non-magnetic graphs. The main question we address is whether the asymptotics may change under the influence of magnetic field. We show that the Weyl or non-Weyl character of the asymptotics is not changed in the presence of magnetic field. On the other hand, if corresponding non-magnetic quantum graph is already non-Weyl, then its "effective size" may depend on the value of magnetic potential.

(Joint work with P. Exner)

Corresponding article: Phys. Lett. A375 (2011), 805–807

G. Malenová:

Asymptotic behaviour of the heat equation in twisted waveguides"

We consider the heat equation in a straight strip, subject to a combination of Dirichlet and Neumann boundary conditions. It it known that the switch of the respective boundary conditions leads to an improvement of the decay rate of the heat semigroup. We support the result by numerical studies which we developed in the MATLAB environment.

This is a joint work with David Krejčiřík and Miloš Tater.

B. Mityagin:

Eigensystem of an L^2-perturbed harmonic oscillator is an unconditional basis

We prove the following. For any complex valued Lp-function b(x), 2 p ¡ 1 or L1-function with the norm kb—L1k ¡ 1, the spectrum of a perturbed harmonic oscillator operator $L = -d^2/dx^2 + x^2 + b(x)$ in $L^2(R^1)$ is discrete and eventually simple. Its SEAF (system of eigen- and associated functions) is an unconditional basis in $L^2(R^1)$.

available also at: arXiv:0912.2722v2 [math.SP],

authors: James Adduci and Boris Mityagin,

R. Novák:

The Pauli equation with non-Hermitian PT-symmetric boundary conditions

The spectrum of non-Hermitian Hamiltonian in Pauli equation is explored. Many cases of possible boundary conditions can be proven to be Hermitian and thus lead to the reality of the spectrum. This also applies in one case where both components of spinor are interdependent. Furthermore, the reality of the spectrum holds also for another PT-symmetric boundary conditions.

Jan Paseka

$\mathcal{PT}-$Symmetry in (Generalized) Effect Algebras

We show that an η_+-pseudo-Hermitian operator for some metric operator η_+ of a quantum system described by a Hilbert space $\mathcal{H}$ yields an isomorphism between the partially ordered commutative group of linear maps on $\mathcal{H}$ and the partially ordered commutative group of linear maps on $\mathcal{H}_{\rho_+}$. The same applies to the generalized effect algebras of positive operators and to the effect algebras of c-bounded positive operators on the respective Hilbert spaces $\mathcal{H}$ and $\mathcal{H}_{\rho_+}$. Hence, from the standpoint of (generalized) effect algebra theory both representations of our quantum system coincide. The existence of states on intervals of studied (generalized) effect algebras is discussed.

Zdenka Riečanová*

Operator effect algebras in Hilbert spaces

We show that the set of all positive linear operators densely defined in an infinite-dimensional complex Hilbert space can be equiped with partial sum of operators making it a generalized effect algebra. This sum coincides with the usual sum of two operators whenewer for them exists. Moreover, blocks of these generalized effect algebras are maximal proper sub-generalized effect algebras. All intervals in this generalizedeffect algebra become effect algebras which are Archimedean , convex , interval effect algebras, for which the set of vector states are order determining. Further , this interval operator effect algebras posess also faithful states.

(*) coauthors : S. Pulmannová and M. Zajac

Ray Rivers

Path integrals for classical and quantum physics

We explore path integrals for extended phase space models of classical and quantum physics. We suggest that the reason why classical physics in complex phase space shows some quantum characteristics lies in the way the complex degrees of freedom encode $\hbar$.

Ingrid Rotter

Phase rigidity of the eigenfunctions of the Hamiltonian of a realistic quantum system

The phases of the eigenfunctions of a non-Hermitian Hamilton operator are rigid only far from exceptional points. In the case of a two-level system, the phase rigidity of the eigenfunctions approaches zero in the neighborhood of an exceptional point. This causes a nonlinear behavior of the eigenvalue trajectories: an avoided crossing of, respectively, discrete and narrow resonance states can be seen. In the case of a many-level system, the phase rigidity approaches zero in a region with many exceptional points. Here, width bifurcation occurs and, finally, an environmentally induced dynamical phase transition can be observed.

Boris Shapiro

Heine-Stieltjes theory, exact and asymptotic results

In the 1860's Eduard Heine initiated the study of polynomial solutions of a general 2nd order linear differential equation with polynomial coefficients. This area was later developed by Stieltjes, Klein, Bocher, Bochner and many others. We will survey a number of classical and recent results with the specific emphasis on the classical Heun equation.

Helena Šedováková

Effective Hamiltonian in curved quantum waveguides as a consequence of norm resolvent convergence

The Dirichlet Laplacian in a curved two-dimensional strip built along a plane curve is investigated in the limit when the uniform cross-section of the strip diminishes. We show that the Laplacian converges in a norm resolvent sense to the well known one-dimensional Schrödinger operator whose potential is expressed solely in terms of the curvature of the reference curve. In comparison with previous results we allow curves which are unbounded and whose curvature is not differentiable. Also the introduction into the problem of three-dimensional quantum waveguides under mild regularity conditions is contained. This is a joint work with David Krejcirik.

Günter Wunner

Exceptional points for nonlinear Schrödinger equations describing Bose-Einstein condensates

The coalescence of two eigenfunctions with the same energy eigenvalue is not possible in Hermitian Hamiltonians. It is, however, a phenomenon well known from non-Hermitian quantum mechanics. It can appear, e.g., for resonances in open systems, with complex energy eigenvalues. If two eigenvalues of a quantum mechanical system which depends on two or more parameters pass through such a branch point singularity at a critical set of parameters, the point in the parameter space is called an exceptional point. I will demonstrate that exceptional points not only occur for non-Hermitean Hamiltonians but also in the nonlinear Schrödinger equations which describe Bose-Einstein condensates, i.e., the Gross-Pitaevskii equation for condensates with a short-range contact interaction, and with additional long-range interactions. Typically, in these condensates the exceptional points are also found to be bifurcation points in parameter space. For condensates with a gravity-like interaction between the atoms, these findings can be confirmed in an analytical way.

Jakub Železný

Formula for a general metric operator and for the C operator in a simple PT-symmetric model

In PT-symmetric quantum mechanics one interprets a non-self adjoint operators as self-adjoint ones in a different Hilbert space with scalar product modified using the so-called metric operator. We present a simple PT-symmetric model and derive a formula for its general metric operator. We then use this formula to find the special case of a metric operator related to the so called C operator.