# Abstracts

## Uwe Guenther:

### The 4D Naimark dilated PT brachistochrone as 2D Hermitian brachistochrone.

After a brief exposition of the Naimark dilated PT brachistochrone [PRL 101, 230404 (2008)] evidence is provided that the dilation (doubling of the Hilbert space dimension) preserves the brachistochrone features of the model. The dilated PT brachistochrone in 4D-Hilbert space behaves as an effective Hermitian brachistochrone in the 2D subspace spanned by the 4D initial and final states.

## Andreas Fring:

### The Ising quantum spin chain in an imaginary field.

We study a lattice version of the Yang-Lee model which is characterized by a non-Hermitian quantum spin chain Hamiltonian. We analyse the role played by PT-symmetry in order to guarantee the reality of the spectrum in certain regions of values of the coupling constants and find the Hermitian counterpart of the Hamiltonian for small values of the number of sites, both exactly and perturbatively. Finally we compute the magnetization of the chain.

## Emanuela Caliceti:

### PT-symmetric Schroedinger operators: spectral and perturbation theory.

In the framework of perturbation theory criteria for the reality and non-reality of the spectrum of PT-symmetric Schroedinger operators have been recently established. After describing the main criteria and their applications, including cases of discrete spectra and of continuous ones as well, the mathematical techniques supporting the proofs of the results are outlined.

## Boris Shapiro:

### Exact and asymptotic results on root distribution of eigenfunctions of a univariate Schroedinger equation with a polynomial potential.

I present some recent results on the root distribution of eigenfunctions in the univariate case. In particular, it will be explained that for the classical quartic oscillator all these roots are either real or pure imaginary. I will also describe that for an arbitrary polynomial potential these roots (after an appropriate scaling) asymptotically fill an interesting part of the Stokes line for a standard potential depending only on the leading term of the original potential when the absolute value of the eigenvalue tends to infinity. Recommended preparatory reading: papers with A. Gabrielov and A. Eremenko, e.g., "High energy eigenfunctions of one-dimensional Schroedinger operators with polynomial potentials" [Comput. Methods Funct Theory 8(2), (2008), 513-529.)] or "Zeros of eigenfunctions of some anharmonic oscillators" [Annales de l'institut Fourier, 58(2), (2008), 603-624].

## Vincenzo Grecchi:

### Stieltjes property of cubic oscillator.

The prove the conjecture of Bender and Weniger about the Pade' summability of the perturbation series of each eigenvalue of the cubic oscillator, is given and discussed.

## Geza Levai:

### On the asymptotic properties of exactly solvable PT-symmetric potentials.

The asymptotic region of potentials have strong impact on their general properties. This problem is especially interesting for PT-symmetric potentials, the real and imaginary components of which allow for a wider variety of asymptotic properties than in the case of purely real potentials. We consider exactly solvable potentials defined on an infinite domain and investigate their scattering and bound states with special attention to the boundary conditions determined by the asymptotic regions. The examples include potentials with asymptotically vanishing and non-vanishing real and imaginary potential components (Scarf II, Rosen-Morse II, Coulomb, etc.).

## Stefan Rauch-Wojciechowski:

### Structure and separability of driven and triangular systems of Newton equations.

(abstract printed separately)

## Hugh Jones:

### Which Green functions does the path integral represent?

In the context of quasi-Hermitian theories we address the problem of how functional integrals and Feynman diagrams "know" about the metric $\eta$. The resolution is

that, although $\eta$ does not appear explicitly, the derivation of the path integral and Feynman rules is based on the Heisenberg equations of motion, and these only take their standard form when matrix elements are evaluated using $\eta$.

# Daniel Hook:

### Numerical study of PT quantum mechanical systems.

We postulate the form of the probability amplitude $\rho(z)$ for a PT quantum mechanical system. As an illustrative example, we calculate $\rho(z)$ for a number of the eigenstates of the harmonic oscillator system and present a numerical study surrounding these results.

# Roberto Tateo:

### PT symmetry breaking and exceptional points.

We discuss a three-parameter family of PT -symmetric Hamiltonians, show that real eigenvalues merge and become complex at quadratic and cubic exceptional points. The mapping of the phase diagram is completed using a combination of numerical, analytical and perturbative approaches. (With P.Dorey, C.Dunning and A.Lishman)

# Steven Duplij:

### A novel Hamiltonian procedure for constraint theories.

We consider an analog of Legendre transform for non-convex functions with vanishing Hessian and propose to mix the envelope and general solutions of the Clairaut equation. Then we show that the procedure of finding a Hamiltonian for a singular Lagrangian is just that of solving a corresponding Clairaut equation with a subsequent application of the proposed Legendre-Clairaut transformation. We do not use the Lagrange multiplier method and show the origin of the Dirac primary constraints in the presented framework.

## Petr Siegl:

### Surprising spectra of PT -symmetric point interactions.

Spectra of the second derivative operators corresponding to the PT -symmetric point interactions on a line are studied. The particular PT -symmetric point interactions causing unusual spectral effects are investigated for the systems defined on finite interval as well. The spectrum of this type of interactions is very far from the self-adjoint case despite of PT -symmetry, P-pseudo-Hermiticity and T -self-adjointness.

## Giuseppe Scolarici:

### Bi-hamiltonian descriptions for composite quantum systems.

We discuss bi-hamiltonian quantum descriptions when composite systems and interaction among them are considered. Some examples are also exhibited.

## Takuya Mine:

### Norm resolvent convergence to Schroedinger operators with infinitesimally thin toroidal megnetic fields.

We consider the Schrödinger operators in the three-dimensional space with magnetic fields supported in concentric tori, which are generated by toroidal solenoids. We prove that the operators converge to an operator in the norm resolvent sense as the thicknesses of the tori tend to 0, if we choose the gauge of the vector potentials appropriately. The limit operator is the Schrödinger operator with a singular magnetic field supported on a circle. This is a collaborated work with A. Iwatsuka and S. Shimada.

# Hynek Bila:

## Scattering in $i\phi^3$ pseudo-scalar theory.

Elementary analysis of scattering in non-Hermitian field theories will be presented on the toy model with imaginary cubic interaction. Necessary modifications of the standard perturbative approach demanded by the crypto-hermiticity of the theory will be discussed.

# Miloslav Znojil:

## All metrics for a toy Hamiltonian.

Complete list of eligible metrics (i.e., of physical inner products in Hilbert space of states) is derived for the one-parametric family of cryptohermitian toy Hamiltonians of paper I (M. Znojil, Phys. Rev. D 78 (2008) 025026). A natural classification of these metrics is found and interpreted as a fundamental length $\theta$. The asymptotically local inner product of paper I recurs at minimal $\theta = 0$ while the popular $\mathcal{CPT}-$symmetric option appears to corresponds to the maximal $\theta \to \infty$.