ANALYTIC AND ALGEBRAIC METHODS IN PHYSICS

(special issue, preface)

In the context of theoretical physics the status of the study of various mathematical methods is very specific. Their role is purely auxiliary when evaluated from the mere pragmatic, phenomenological point of view. At the same time, the history of physics offers only too many persuasive examples of an extreme, "unreasonably high" productivity of the intensive mutual interactions between the mathematics and phenomenology.

In the present special issue of Acta Polytechnica the emphasis is put upon the description of various recent analytic and algebraic constructions with applications in several branches of physics. We must admit that our choice of such a topic parallels the range of subjects covered by the series of conferences of the same name which are more or less regularly organized in Prague [1]. It is not too surprising that the list of the authors contributing to our present special issue (see the table of contents) has a large overlap with the list of the participants of the most recent AAMP 2017 conference (see the photo and the Appendix).

Such a parallel between the conference and the texts published in Acta Polytechnica is to be perceived as "traditional" [2]. In our introductory comment the readers will find, therefore, a complement of the table of contents. In a concise recollection we shall review briefly the subjects which were discussed during the fourteenth AAMP conference in the year 2017 (for more details see also the dedicated webpage [3] of the meeting).

This preface intends to provide our readers a better insight in the current developments in the field. Indeed, the recently published background of some of the talks has to be sought elsewhere while, in some other cases, the oral presentations were still only too preliminary to result in a contribution to the present special issue. Before giving a more detailed review of the work of the conference let us also add that even though the attached photo of the participants is incomplete (covering just their 75 % sample), we find there the faces of researchers from 15 states of five continents. This demonstrates that in spite of a certain diversity of the AAMP-related problems the tradition of their presentation seems already well established.

During the meeting, the rough classification of the talks into "analytic" and "algebraic" was refined and separated into dozen sections. An explicit quotation is certainly deserved by sections III (physics of open quantum systems - by the way, the present special issue contains two written reports on the most topical results in this area which were also orally presented by Holger Cartarius), IV (mathematical methods in field theory), VII (scattering theory), VIII (non-Hermitan quantum physics) and XI (physics of quasicrystals, with the nice, picture-rich Petr Ambrož's outline of the cut-and-project method, with emphasis on the Voronoi tilings exhibiting the 12-fold rotational symmetry - with one of the examples having just two prototiles). The latter talk was complemented also by the self-similarities-studying presentation by Jan Mazáč.

The specification of the latter subcategories proved, in places, vague and/or overlapping. In several sections one rather found an inspiring mixture of the approaches to mathematics which ranged from the intuitive and pragmatic studies up to their formal and rigorous opposites. One of the most explicit examples of such a juxtaposition appeared in the last section XII (quantum mechanics in practice) in which the first speaker (viz., Fabio Rinaldi from Rome) used a top-class functional analysis (i.e., the Birman-Schwinger principle and all that) to determine, in an entirely rigorous manner, just the two lowest eigenenergies of a rather elementary one-dimensional Hamiltonian. In contrast, the subsequent, last speaker of the conference claimed, *cum grano salis*, that he "never used a rigorous method in his life". Incidentally, the latter speaker was F. M. Fernández from La Plata, the author of the "benchmark" results which were cited by the preceding "rigorous" speaker, for comparison, as the most reliable numerical values available in the market.

On the rigorous side of the playground an explicit note is to be also given to Oktay Veliev (who offered an unexpected insight in the mechanisms of the breakdown of what is called PT-symmetry

for banded spectra) or to Teoman Turgut (who presented a mathematically unbelievably deep study of the ground state in Lee model) or to Fabio Bagarello (in his high-quality progress report on pseudo-bosons). In a more practically oriented group of theoretical talks the auditorium found the Ray River's observation that "Elevating classical mechanics to quantum mechanics is a non-trivial exercise even for harmonic oscillator!" and the Václav Zatloukal's summary of the Hamiltonian constraint formulation of certain classical field theories.

Several talks were devoted to the applications of the language of the Lie groups and algebras. One of the most interesting results (on the alternative generations of the representations of the first few Poincaré groups) was presented by Jan Kotrbatý. Jiří Hrivnák showed us how the discrete versions of the Weyl orbit functions could be fructified in the discrete cosine-like and sine-like transforms on honeycomb lattice. Finally, Satoshi Ohya from Japan offered an innovative systematic application of the standard Lie group/algebraic language to our understanding of the structure of the so called exactly solvable potentials. The closely related topic of section VI (exactly solvable models) was covered by the Géza Lévai's study (of a finite PT-symmetric square well and of the parallels of its features with the Rosen–Morse case) and by the Artur Ishkhanyan's traditional review of his recent results on the Heun-equation solvability in terms of the so called Hermite functions.

In a formally separate category of reports on the broader areas of research and results lying at a transition boundary between mathematics and physics we listened to Joshua Feinberg (random matrix theory review), to Daya and to Usha Kulshreshtas (strings and gravity), to Ali Mostafazadeh and to Hugh Jones (transfer matrices and all that, including the hot topics like invisibility in classical optics), or to Vít Jakubský (Dirac Hamiltonians and the confluent Crum-Darboux transformations) and to Miloslav Znojil (about the unitary evolution in the innovative non-Hermitian interaction picture).

Inside the realm of physics in the laboratory the talk by Daniel Reitzner informed us about the news in the construction of the quantum-walk searches in certain discrete graph structures while Stanislav Skoupý shifted attention to the perfect state transfer distinguishing between the discrete- and continuous-time quantum walk search algorithms on highly symmetric graphs. Among the further, mainly phenomenologically motivated talks we enjoyed listening to the carefully prepared Axel Pérez-Obiol's outline of the physical appeal of the carbon nanotubes. Albert Kotvytskiy explained to us the applicability of the algebraic geometry methods in the analysis of the sets of polynomial equations describing gravitational lenses in observational astronomy.

What remains to mention in the conclusion is the very fashionable field of PT symmetry in classical optics. Marginally, let us point out that this topic was recently interpreted as belonging among the ten "hottest" directions of research in contemporary physics [4]. In this framework, an innovative time-delayed gain and loss version (with the experimental realization via coupled semiconductor lasers) was proposed and described by Yogesh Joglekar. In the same context, last though not least, Mustapha Maamache showed that for certain time-dependent and manifestly non Hermitian SU(1,1) and SU(2) Hamiltonians the use of certain judicious transformations may make the related Schrödinger-type evolution equations exactly solvable.

Reference

[1] http://gemma.ujf.cas.cz/∼znojil/conf/mikroindex.html

[2] http://gemma.ujf.cas.cz/∼znojil/conf/proceedaamp.html;
V. Jakubský and M. Znojil, Acta Polytechnica 51 (2011) 7 - 8.

[3] http://gemma.ujf.cas.cz/∼znojil/conf/micromeetingctrnact.html

[4] J. Cham, Nature Phys. 11 (2015) 799.

Appendix: The list of the names and affiliations of the AAMP XIV participants in the photo taken on the 13th of September 2017 at 12:20 hours in lecture hall 301. The list goes from the right side to the left.

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Daya Shankar Kulshreshtha (University of Delhi, India)
Hugh F. Jones (Imperial College, London, UK)
Artur Ishkhanyan (Inst. Phys. Res. AAS, Ashtarak, Armenia)
Joshua Feinberg (University of Haifa, Israel)
Usha Kulshreshtha (Delhi University, India)
Teoman Turgut (Bogazici University, Istanbul, Turkey)
Géza Lévai (ATOMKI Debrecen, Hungary)
Fabio Rinaldi (Marconi University, Rome, Italy)
Holger Cartarius (Universität Stuttgart, Germany)
Ali Mostafazadeh (Koç University, Istanbul, Turkey)
Václav Zatloukal (FNSPE of CTU, dept. of phys., Prague, CR)
Fabio Bagarello (Università di Palermo, Italy)
Jaroslav Smejkal (UTEF CTU, Prague)
Francisco M. Fernández (INIFTA, La Plata, Argentina)
Petr Jizba (FNSPE of CTU, dept. of phys., Prague, CR)
Ray Rivers (Imperial College London, UK)
Uwe Günther (HZ Dresden-Rossendorf, Germany)
Jan Mazáč (FNSPE of CTU, dept. of phys., Prague, CR)
Axel Pérez-Obiol (NPI Řež, Czech Republic)
Albert T. Kotvytskiy (Kharkov National University, Ukraine)
Oktay Veliev (Dogus University, Istanbul, Turkey)
Miloslav Znojil (Nuclear Physics Institute, Řež, CR)
Satoshi Ohya (Nihon University, Tokyo, Japan)
Daniel Reitzner (Institute of Physics SAS, Bratislava, Slovakia)
Iveta Semorádová (NPI Řež, Czech Republic)
Vít Jakubský (Nuclear Physics Institute, Řež, CR) (invisible, taking the photo)

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