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THE BOOK OF ABSTRACTS (the version of June 1, 2012)

(alphabetical ordering)

Diana Barseghyan

Spectral analysis of Schrödinger operators with unusual semi-classical behaviour

We analyze two-dimensional Schrödinger operators with the potential

$$|xy|^p - \lambda(x^2+y^2)^{p/(p+2)}$$

where $p \geq 1$ and $\lambda \geq 0$. We show that there is a critical value of λ such that the spectrum for $\lambda < \lambda_{\rm crit}$ is below bounded and purely discrete, while for $\lambda > \lambda_{\rm crit}$ it is unbounded from below. In the subcritical case we prove upper and lower bounds for the eigenvalue sums. We derive estimates of eigenvalue moments for Dirichlet Laplacians and Schrödinger operators in regions having infinite cusps which are geometrically nontrivial being either curved or twisted; we are going to show how those geometric properties enter the eigenvalue bounds. The obtained inequalities reflect the essentially one-dimensional character of the cusps and we give an example showing that in an intermediate energy region they can be much stronger than the usual semiclassical bounds.

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Common work with **Pavel Exner**.

Holger Cartarius

A Bose-Einstein condensate in a $\mathcal{PT}$ symmetric double well

The existence of $\mathcal{PT}$ symmetric wave functions describing Bose-Einstein condensates in a one-dimensional and a fully three-dimensional double-well setup is investigated theoretically. When particles are removed from one well and coherently injected into the other the external potential is $\mathcal{PT}$ symmetric. We solve the underlying Gross-Pitaevskii equation by way of the time-dependent variational principle (TDVP) and show that the $\mathcal{PT}$ symmetry of the external potential is preserved by both the wave functions and the nonlinear Hamiltonian as long as eigenstates with real eigenvalues are obtained. $\mathcal{PT}$ broken solutions of the time-independent Gross-Pitaevskii equation are also found but have no physical relevance. To prove the applicability of the TDVP we compare its results with numerically exact solutions in the one-dimensional case. The linear stability analysis and the temporal evolution of condensate wave functions demonstrate that the $\mathcal{PT}$ symmetric condensates are stable and should be observable in an experiment.

Work done together with:
Dennis Dast, Daniel Haag, Günter Wunner

Pavel Cejnar

Quantum phase transitions in finite algebraic systems

Features of quantum phase transitions affecting collective dynamics of many-body systems will be discussed. Due to a finite number of collective degrees of freedom, the infinite size limit of the corresponding models coincides with the classical limit and quantum critical phenomena are determined by the structure of the classical phase space. It will be shown that collective models in their ground- and excited-state quantum phase transitions exhibit singular features related to relaxation, thermalization and quantum entanglement.

Eva-Maria Graefe:

Breakdown of adiabatic transfer properties in a three level system with decay

We investigate a non-Hermitian generalisation of the STIRAP scheme for population transfer in three-level systems, which can be easily implemented in optical waveguide structures. It is shown that even a small decay rate, which modifies the spectrum or the eigenfunctions only slightly, can lead to a breakdown of the adiabatic population transfer. This breakdown happens at a sharp threshold, which can be calculated to a good approximation by simple analytical arguments.

Amine B. Hammou:

Generalized continuity equation for quasi Hermitian Hamiltonians

Scattering from a quasi Hermitian one dimensional potential is studied. A generalized continuity relation in the physical Hilbert space H(phys) is derived and the probability current density is defined. Using a toy model of discrete quasi Hermitian delta function potential we show that the reflection R and transmission T coefficients computed with this current obey the conventional unitarity relation R+T=1. Some results are also obtained for the continuum limit.

Ladislav Hlavatý:

Poisson-Lie plurality

Review of basic facts of Poisson-Lie T-duality and plurality of sigma-models will be given. They were found as an extension of the Abelian T-duality of nonlinear sigma models to non-Abelian groups of isometries. They are transformations that can relate models with different geometrical backgrounds. They are based on different decompositions of Drinfel'd doubles, i.e. Lie groups whose algebra is provided with an Ad-invariant bilinear forms. Some examples will be shown.

Jiří Janda

Intervals on weakly ordered partial commutative groups of linear operators

The generalized effect algebra was presented as a generalization of effect algebra for an algebraic description of the structure of the set of all positive linear operators densely defined on Hilbert space with the usual sum of operators. A structure of the set of not only positive linear operators can be described with the notion of weakly ordered partial commutative group (wop-group). With a restriction of the usual sum, an important subset of all self-adjoint operators forms a substructure of the set of all linear operators. We investigate the properties of intervals in wop-groups of linear operators and showing that they can be organized into effect algebras with nonempty set of states.

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Petr Jizba

Emergent SUSY as a signature of classicality

This talk briefly reviews the basic ideas of 't Hooft's derivation of quantum from classical physics. In the core part of the talk I describe how 't Hooft's program can be phrased in terms of Gozzi's path integral formulation of classical mechanics. I also emphasize a surprising role that the BRST invariance plays in the whole scheme. Finally, some speculations concerning a possible connection between the the complex phase-space path integrals and pseudo-hermitian dynamics will be presented.

Hugh F. Jones

WKB for PT-symmetric Sturm-Liouville Problems

The talk would cover the paper Carl and I wrote on the potential $V = ix$, plus another paper in preparation, which deals with $V = ix^3$ etc. The question raised in our first paper is answered in the affirmative, namely that in those cases a two-turning-point WKB approximation works perfectly.

Frieder Kleefeld

Complex covariance

According to some generalized correspondence principle the classical limit of a non-Hermitian Quantum theory describing quantum degrees of freedom is expected to be well known classical mechanics of classical degrees of freedom in the complex phase space, i.e., some phase space spanned by complex-valued space and momentum coordinates. As special relativity has been developed by Einstein merely for real-valued space-time and four-momentum we will try to understand how special relativity and covariance can be extended to complex-valued spacetime and four-momentum. Our considerations will lead us not only to some unconventional derivation of Lorentz transformations for complex-valued velocities, yet also to the non- Hermitian Klein-Gordon and Dirac equations which are to lay the foundations of a non-Hermitian quantum theory.

David Krejčiřík:

On the similarity of Sturm-Liouville operators with non-Hermitian boundary conditions to self-adjoint and normal operators

We consider one-dimensional Schroedinger-type operators in a bounded interval with non-self-adjoint Robin-type boundary conditions. It is well known that such operators are generically conjugate to normal operators via a similarity transformation. Motivated by recent interests in quasi-Hermitian Hamiltonians in quantum mechanics, we study properties of the transformations and similar operators in details. In the case of PT-symmetric boundary conditions, we establish closed integral-type formulae for the similarity transformations, derive the similar self-adjoint operator and also find the associated "charge conjugation" C operator, which plays the role of fundamental symmetry in a Krein-space reformulation of the problem.

Peter G. L. Leach:

Emmy Noether and Linear Evolution Equations.

Noethers Theorem relates the Action Integral of a Lagrangian with symmetries which leave it invariant and the first integrals consequent upon the variational principle and the existence of the symmetries. These each have an equivalent in the Schrödinger Equation corresponding to the Lagrangian and by extension to linear evolution equations in general. The implications of these connections are investigated.

M. Howard Lee:

Making repulsive fixed points attractive: A reforming method.

For obtaining the roots of a nonlinear equation, the fixed point analysis is an effective method. The roots are made the fixed points of a function related to an original equation. The standard iterative process by which fixed points are calculated requires that they be attractive. A fixed point of a function f is said to be attractive or repulsive if —df(x*ii)/dx—¡1 or ¿1, xi* a fixed point of f(x), i = 1,2,... The slope is termed the character. In this work we show how repulsive fixed points are made attractive by reforming their characters, thereby made amenable to the iterative analysis, just as the attractive ones. The reforming method holds fixed points fundamental, not their characters. It recalls the theorems of Sharkovskii and Li-Yorke, which refer only to the existence of fixed points, not to their character. This similarity suggests that this method may rest on the same ground from which these theorems have sprung.

Takuya Mine:

Explicit eigenfunctions for the Schrödinger operators with two-solenoidal Aharonov-Bohm fields

In 1988, Gu-Qian obtained the explicit eigenfunctions for the Schrödinger operators on the Euclidean plane with two pointlike magnetic fields of the same magnitude. However, their solutions necessarily have some discontinuity on the slit between two pointlike fields, unless both the fluxes are 0 or π. We shall explain this fact in terms of the representation theory of the covering group. This idea also leads a new solvable Schrödinger type operator with non-commutative gauge $U(2)$.

Ali Mostafazadeh:

Ghost-Free Quantization of the Pais-Uhlenbeck Oscillator, Self-Dual Spectral Singularities, and PT-Symmetry

This talk consists of two parts. In part 1, I outline the imaginary scaling quantization of the 4th order Pais-Uhlenbeck Oscillator that yields a stable and unitary quantum system and has the correct classical limit. In part 2, I report on some recent results on the self-dual spectral singularities and their optical realizations. In particular, I introduce and examine non-PT-symmetric Coherent Perfect Absorbing (CPA) Lasers.

Radek Novák:

Pauli equation with complex boundary conditions

We consider one-dimensional Pauli Hamiltonians in a bounded interval with possibly non-self-adjoint Robin-type boundary conditions. We study the influence of the spin-magnetic interaction on the interplay between the type of boundary conditions and the spectrum. A special attention is paid to PT -symmetric boundary conditions with the physical choice of the time-reversal operator T.

Jan Paseka

Realization of generalized effect algebras

The set V(H) of all positive linear operators on a Hilbert space H (including unbounded operators) with the usual sum of operators forms a generalized effect algebra. Hence the effect algebraic partial order coincides with the usual partial order of positive operators. We give a partial answer to the question which properties of generalized effect algebras ensure their realization as a sub-effect algebra of V(H).

Zdena Riečanová

Effect algebras of positive linear operators densely defined in Hilbert spaces

Recently was proved that a set of all positive linear operators, densely defined in an infinite-dimensional Hilbert space,can be equiped with partial sum of operators, making it a generalized effect algebra. This sum coincides with the usual sum of operators, whenever it exists. All intervals in this set become effect algebras which are Archimedean, convex interval effect algebras , for which the set of vector states is order determining. We show that these intervals [0,Q], for positive linear operators Q ,can be embedded into the Hilbert space effect algebra E(H) of all positive self-adjoint operators dominated by identity operator I in H.Thus every effect algebra [0,Q] is isomorphic to a sub-efect algebra of E(H). Moreover, an abstract effect algebra E can be represented by positive linear operatoras in some Hilbert space H iff E possesses an order determinig set of states. This is equivalen to the condition that E is isomorphic to a sub-effect algebra of E(H).

References

[1] D.J. Foulis and M. K. Bennet , Effect algebras and unsharp quantum logics, Found. Phys. 26(1994) , pp.1331-1352.

[2] Z.Riecanová, M. Zajac, S. Pulmannová, Effect algebras of positive linear operators densely defined on Hilbert spaces, Reports on Mathematical Physics, 68(2011), pp.261-270.

[3] Z. Riecanová, M. Zajac, Hilbert space effect-representations of effect algebras, preprint.

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Ingrid Rotter (with H. Eleuch)

Avoided level crossings in open quantum systems

At high level density, two states avoid usually crossing at the critical value $a_{\rm cr}$ of the parameter a by which the system is controlled. The wavefunctions of the two states are mixed in a finite parameter range around $a_{\rm cr}$. This holds true for discrete states as well as for narrow resonance states which are coupled via the environment of scattering wavefunctions. We study the influence of avoided level crossings onto four overlapping complex eigenvalues of a symmetric non-Hermitian operator. The mixing of the two wavefunctions around $a_{\rm cr}$ is simulated, in each case, by assuming a Gaussian distribution around $a_{\rm cr}$. At high level density, the Gaussian distributions related to avoided crossings of different levels may overlap. Here, new effects arise, especially from the imaginary part of the coupling term via the environment. The results show, moreover, the influence of symmetries onto the multi-level avoided crossing phenomenon.

Teoman Turgut:

Condensation of bosons with contact interaction (on manifolds)

In this talk, we will discuss a kind of condensation for bosons in two dimensions. We consider non-relativistic bosons interacting via an attractive contact interaction on a two dimensional Riemannian manifold. Following an approach proposed by Rajeev for the flat case, we obtain a finite formulation of the system, and give an estimate for large number of bosons in the mean field approximation. We show that our approach agrees with the mean field results in one dimension (which itself agrees with the exact solution).

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[This talk is based on joint work with F. Erman.]

Matěj Tušek:

A New Estimate on the Two-Dimensional Indirect Coulomb Energy

We prove a new lower bound on the indirect Coulomb energy in two dimensional quantum mechanics in terms of the single particle density of the system. The new universal lower bound is an alternative to the Lieb–Solovej–Yngvason bound with a smaller constant, $C = (4/3)^{3/2}\sqrt{5\pi - 1} \approx 5.90 < C_{LSY} = 192\sqrt{2\pi} \approx 481.27$, which also involves an additive gradient energy term of the single particle density.

The talk is based on the joint work with Rafael Benguria.

Miloslav Znojil:

PT-symmetric solvable models in quantum cosmology

In a cryptohermitian toy model, the phenomenon of quantum Big Bang is shown compatible with the principles of quantum physics and, in addition, mathematically tractable as a singularity in the evolution of the inner-product-defining Hilbert-space-metric $\Theta = \Theta(t)$. First of all, the model circumvents the standard key obstruction of the quantization of the Universe (viz., the avoided-crossing phenomenon). Secondly, the quantitative description of the evolution near BB is made feasible by the use of non-covariant time-dependent 1D operators in adiabatic and discrete approximations. Among eligible strategies the kinematical approach is finally chosen, with the $N-$point-grid toy-model operator Q(t) selected in advance, and with all of the another (dynamical) observables reconstructed. The main mathematical surprise provided by the model is that the symbolic manipulations made the BB-degeneracy phenomenon described in a compact, non-numerical form.

Miloslav Znojil:

Non-Hermitian star-shaped quantum graphs

A new exactly solvable model of a quantum system is proposed, living on an equilateral q-pointed star graph (q is arbitrary). The model exhibits a weak and spontaneously broken form of $\mathcal{PT}-$symmetry, offering a straightforward generalization of one of the standard solvable square wells with $q = 2$ and unbroken $\mathcal{PT}-$symmetry. The kinematics is trivial, Kirchhoff in the central vertex. The dynamics is one-parametric (viz., $\alpha-$dependent), prescribed via complex Robin boundary conditions (i.e., the interactions are non-Hermitian and localized at the outer vertices of the star). The (complicated, trigonometric) secular equation is shown reducible to an elementary and compact form. This renders the model (partially) exactly solvable at any $q \geq 2$ – an infinite subset of the real roots of the secular equation proves q-independent and known (i.e., inherited from the square-well $q = 2$ special case). The systems with $q = 4m - 2$ are found anomalous, supporting infinitely many (or, at m=1, one) additional real m-dependent and $\alpha-$dependent roots.