The 12th international microconference

Analytic and algebraic methods in physics

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the meeting dedicated to the 75th birthday

of professor Miloslav Havlíček

→ THE BOOK OF ABSTRACTS ←

(the version of September 10, 2013)

(alphabetical ordering)

Čestmír Burdík

Extremal vectors for Verma type representations of semisimple algebras and quantum groups

We will formulate method how to rewrite the Verma type representations as boson realizations. It will help us to find a full set of extremal vectors for this kind of representations, so we can solve explicitly the problem of irreducibility of these representations. All will be presented on simple examples.

Common work with **Ondřej Navrátil**.

Pavel Exner

Resonances in quantum graphs and their generalizations

Resonance effects in transport on quantum networks are important both theoretically and practically. In this talk I am going to discuss several recent results concerning this topic. The first concerns the notion of resonance and the dynamical mechanisms involved. Next we will focus at the resonance behaviour at high energies and show that their count may not follow the usual Weyl law; a criterion will be presented demonstrating when such an effect occurs. In addition, it will be shown that a magnetic field can change the effective size of the graph which enters the semiclassical asymptotics. Finally, we will mention that similar effects can be observed in generalized graphs with components of different dimensions.

The results discussed in the talk come from common work with **Brian Davies and Jiří Lipovský**.

Igor Jex

Iterated quantum operations

In the talk we shall present and comment critically recent results about properties of iterated quantum operations, used both in quantum information theory as well as in statistical physics.

Edward Kapuscik:

On a fatal error in tachyonic physics

A fatal error in the famous paper on tachyons by Gerald Feinberg is pointed out. The correct expressions for energy and momentum of tachyons are derived.

Jerzy Lukierski

Noncommutative space-time from quantized twistors

We consider the deformed and enlarged relativistic phase space coordinates as the functions of a pair of primary twistor coordinates. If twistors are canonically quantized the space-time coordinates become noncommutative. In order to obtain closed deformed relativistic Heisenberg algebra one should add the Pauli-Lubanski fourvector components. We comment on star-product quantization of such algebraic structure which we call spin- extended deformed Heisenberg algebra.

This research I am doing together with **dr. Mariusz Woronowicz**.

Zuzana Masáková

From quasicrystals to infinite words and numeration systems

Most of the quasicrystal models can be recast in the frame of the so-called cut-and-project scheme which enables one - by projection of higher-dimensional lattices - to obtain discrete point sets with rotational symmetries forbidden in periodic structures. We shall explain the cut-and-project method and demonstrate the connection to symbolic dynamical systems, infinite words and number systems. We shall present several results from combinatorics on words and non-standard numeration obtained with the use of the properties of cut-and-project sets.

Patrick Moylan:

Applications of Localization in Physics

We present some important physical applications of noncommutative localization in algebras centering around the pioneering efforts of *Miloslav Havlcek* and his students on the subject.

Severin Pošta:

Verma bases for representations of simple Lie algebras

Complete bases are constructed for all finite-dimensional irreducible representations over $\mathbb{C}$ of Lie algebras $sl(n)$.

Common work with **M. Havlíček and J. Patera**.

Peter Prešnajder:

Coulomb scatering in noncommutative quantum mechanics

Recently in [1] and [2] we formulated Coulomb problem in a rotationally invariant NC configuration space specified by NC coordinates x_i, $i = 1, 2, 3$, satisfying commutation relations $[x_i, x_j] = 2i\lambda\varepsilon_{ijk}$ (λ being our NC parameter). We found that the problem is exactly solvable, and in [1] we give an exact simple formula for energies of negative bound states $E_n^\lambda < 0$ (n - the principal quantum number); the full solution of the NC Coulomb problem was given in [2]. In this contribution we present an exact calculation of the NC Coulomb scattering matrix $S_j^\lambda(E)$ in j-th partial wave. As the calculations are exact, we can recognize remarkable non-perturbative aspects of the model: 1) energy cut-off: the scattering is restricted to the interval $E \in (0, E_{crit})$, $E_{crit} = 2/\lambda^2$; 2) presence of two sets of poles of the S-matrix in the complex energy plane: as expected, the poles at negative energy $E_{\lambda n}^I = E_n^\lambda$ for Coulomb *attractive potential*, and poles at ultra-high energies $E_{\lambda n}^{II} = E_{crit} - E_n^\lambda$ for Coulomb *repulsive potential*, that disappear in the commutative limit $\lambda \to 0$.

[1] V. Gáliková, P. Prešnajder, *Hydrogen atom in fuzzy spaces - Exact solution*, J. Phys.: Conf. Ser. **343** (2012) 012096.
[2] V. Gáliková, P. Prešnajder, *Coulomb problem in NC quantum mechanics: Exact solution and non-perturbative aspects* , (2013) arXiv:1302.4623 [math-ph].

Common work with **Veronika Gáliková**.

Silvia Pulmannová:

Effect algebras with state operator and conditional expectations

State operators on convex effect algebras, in particular effect algebras of unital JC-algebras, JW-algebras and convex σ-MV algebras are studied and their relations with conditional expectations in algebraic sense as well as in the sense of probability on MV-algebras are shown.

Common work with **Anna Jenčová**.

Zdena Riečanová

Blocks in pairwise summable generalized effect algebras

In the important study of positive linear operators, densely defined on an infinite dimensional complex Hilbert space $\mathcal{H}$, their sum is defined only if the intersection of their domains is a dense linear sub-space of $\mathcal{H}$. Consequently, the summation on this set $\mathcal{V}_{\mathcal{D}}(\mathcal{H})$ of linear operators is a partial binary operation. Nevertheless, this partial algebraic structure (with zero element being the null operator $\mathbf{0}$ on $\mathcal{H}$) is a generalized effect algebra. Moreover, the maximal sub-sets of pairwise summable operators are its sub-generalized effect algebras. Note that the similar situation occurs in any abstract generalized effect algebras.

We show some conditions for generalized effect algebras, resp. their sub-generalized effect algebras under which pairwise summable generalized effect algebras, resp. their sub-generalized effect algebras, become generalized MV-effect algebras in its own right.

Common work with **Jiří Janda**.

References

[1] Blank J., Exner P., Havlíček M., *Hilbert Space Operators in Quantum Physics*, 2nd edn. Springer, Berlin (2008).

[2] Riečanová Z., Janda J., *Maximal subsets of pairwise summable elements in Generalized Effect Algebras*, Acta Polytechnica, accepted.

[3] Riečanová Z., Janda J., *Blocks in Pairwise Summable Generalized effect algebras*, preprint.

[4] Riečanová Z.,Zajac M., Pulmannová S., *Effect algebras of positive linear operators densely defined on Hilbert spaces*, Rep. Math. Phys. **68**, (2011), 261–270.

Libor Šnobl

Classification of solvable & Levi decomposable Lie algebras

We will review what is known about the classification of solvable and Levi decomposable Lie algebras from the perspective of extensions of a given nilradical.

This approach to the classification of solvable algebras goes back to G. M. Mubarakzyanov who successfully employed it in his classification of 5- and 6-dimensional solvable Lie algebras. During past two decades it was applied by P. Winternitz and his collaborators, including myself, to classifications of solvable algebras of arbitrary finite dimension with a given structure of their nilradical, e.g. Abelian, Heisenberg, Borel and certain classes of filiform ones. We outline the general procedure together with its recent refinements and show some of its results.

Next, Lie algebras which possess nontrivial Levi decomposition, i.e. into a nontrivial semidirect sum of a semisimple subalgebra and the radical, are considered. We assume that the nilradical is given and investigate restrictions on possible Levi factors which are implied by the structure of characteristic ideals of the nilradical. We present a new perspective on Turkowski's classification of Levi decomposable algebras up to dimension 9. In particular we explain, mostly by simple dimensional analysis, why majority of 5- and 6-dimensional nilpotent and solvable algebras do not possess any nontrivial Levi extension.

References:

[1] L. Šnobl and P. Winternitz, Solvable Lie algebras with Borel nilradicals, J. Phys. A: Math. Theor. 45 (2012) 095202 [arXiv:1110.5492].

[2] L. Šnobl, On the structure of maximal solvable extensions and of Levi extensions of nilpotent Lie algebras, J. Phys. A: Math. Theor. 43 (2010) 505202 [arXiv:1003.4223].

[3] L. Šnobl and D. Karásek, Classification of solvable Lie algebras with a given nilradical by means of solvable extensions of its subalgebras, Linear Algebra and its Applications 432 (2010) 1836-1850, [arXiv:0908.0271].

[4] L. Šnobl and P. Winternitz, All solvable extensions of a class of nilpotent Lie algebras of dimension n and degree of nilpotency n-1, J. Phys. A: Math. Theor. 42 (2009) 105201 [arXiv:0809.3259].

Agnieszka Tereszkiewicz:

Permanents – past and present

The History of permanents began in 1812, when Binet and Cauchy introduced them simultaneously in their papers. Then in next century Minc wrote monograph about permanents and summarized all knowledge from the beginning.

The theory of permanents is used in combinatorics, probability, graph theory, quantum field theory and chemistry. Also group theory is using permanents for special type of matrix as one of orbit functions of Weyl group.

The permanent $\mathrm{per}(A)$ is defined as symmetric function for the matrix A of dimension $m \times n$, where $m = n$. Some of their properties are analogous to properties of determinant. Formulae for the permanent are specially useful to applying in special case of the matrix, i.e. $(0, 1)$–matrix, stochastic or double stochastic matrix. One type of orbit functions of the Weyl group could be written as permanent of matrix with exponents entries. Using the permanent theory helps to describe these functions.

Valeriy N. Tolstoy:

$\mathbb{Z}_2 \times \mathbb{Z}_2$-graded Supersymmetries

A mathematical background of $\mathbb{Z}_2 \times \mathbb{Z}_2$-graded supersymmetries is given and some their applications are discussed.