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THE BOOK OF ABSTRACTS (the version of May 6, 2010)

(alphabetical ordering)

Petr Ambrož

Rauzy-like tilings in Ito-Sadahiro and symmetrized $(-\beta)$-numeration systems

Let $\beta > 1$ be a Pisot number. We study forms and (mutual) properties of Rauzy-like tilings arising from classical β-numeration system and from $(-\beta)$-numerations system for both Ito-Sadahiro and symmetrized definition of the $(-\beta)$-transformation.

Lubomíra Balková

Generalization of Sturmian words

Sturmian words are aperiodic words with the lowest possible complexity. No wonder they belong to the most studied infinite words and many equivalent definitions of Sturmian words have been found out. In our contribution we will consider the generalizations of their combinatorial definitions and properties to multiliteral alphabets. We will in particular explain the relations between such properties and we will mention interesting open questions.

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This is a joint work with Štěpán Starosta and Edita Pelantová.

Carl Bender

Complex Correspondence Principle

Quantum mechanics and classical mechanics are two very different theories, but the Ehrenfest theorem states that these two theories approach one another in the limit of high energy. In recent years much research has been done on extending both quantum mechanics and classical mechanics into the complex domain. This talk discusses the corresponding complex extension of the Ehrenfest theorem. Establishing connections between complex quantum mechanics and complex classical mechanics is subtle and requires the use of asymptotics beyond all orders.

Daniel Dombek

Numbers with integer expansion in the numeration system with negative base

We study representations of real numbers in the positional numeration system with negative basis, as introduced by Ito and Sadahiro. We introduce an analogue of the greedy algorithm for obtaining these representations. We describe the distances between consecutive elements of the set $\mathbb{Z}_{-\beta}$ of numbers whose representation uses only non-negative powers of $-\beta$, the so-called $(-\beta)$-integers.

Steven Duplij

On Clairaut-type Hamiltonian procedure for singular theories

We describe singular Lagrangian theories by using a Clairaut-type version of the Hamiltonian formalism. First we generalize the Legendre transform to the case when the Hessian is zero as the mixed (envelope/general) solution of the multidimensional Clairaut equation. The corresponding system of equations of motion is equivalent to the Lagrange equations and has a subsytem for "unresolved" velocities. Then we present it in the Hamiltonian-like form by introducing a new (non-Lie) bracket. This is a "shortened" formalism since finally it does not contain "nondynamical" (degenerate) momenta at all, and therefore there is no notion of constraint: nothing to constrain. We finally show that any classical singular Lagrangian theory in its Clairaut-type Hamiltonian form is equivalent to the many-time classical dynamics.

Francisco M. Fernández

Two-particle harmonic oscillator in a one-dimensional box

We study a harmonic molecule confined to a one-dimensional box with impenetrable walls. We explicitly consider the symmetry of the problem for the cases of different and equal masses. We propose suitable variational functions and compare the approximate energies given by the variation method and perturbation theory with accurate numerical ones for a wide range of values of the box length. We analyze the limits of small and large box size.

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This is a joint work with Paolo Amore (Colima, Mexico).

Andrei Gabrielov

Singular perturbation of polynomial potentials and real spectral loci (joint work with Alex Eremenko)

We study the eigenvalue problems for the Schrodinger equation with complex potential and zero boundary conditions at infinity on two rays in the complex plane. Sufficient conditions for the continuity of the spectrum as the leading coefficient of the potential tends to zero are given. These results are used to understand topology and geometry of the real spectrum of PT-symmetric families with cubic and quartic potentials, and location of zeros of their eigenfunctions.

Eva-Maria Graefe:

Classical limit of non-Hermitian quantum theories: A generalised canonical structure

We investigate the classical limit of non-Hermitian quantum dynamics and show that the resulting classical phase space dynamics can be described by generalised "canonical" equations of motion. The dynamical equations combine a symplectic flow associated with the Hermitian part of the Hamiltonian with a metric gradient flow associated with the anti-Hermitian part of the Hamiltonian. We derive this structure of the classical limit of quantum systems in the case of a Euclidean phase space geometry. As an example we show that the classical dynamics of a damped oscillator can be linked to a non-Hermitian quantum system, and investigate the quantum classical correspondence. Furthermore, we present an example of an angular momentum system whose classical phase space is spherical and show that the generalised canonical structure persists for this nontrivial phase space geometry.

Daniel Hook:

The Complex Elliptic Pendulum

A numerical study of the complex classical trajectories of a particle in an elliptic potential is presented. This study of doubly-periodic potentials is a natural sequel to earlier work on complex classical trajectories in trigonometric potentials. For elliptic potentials there is a two-dimensional array of identical cells in the complex plane, and each cell contains a pair of turning points. The particle can travel both horizontally and vertically as it visits these cells, and sometimes the particle is captured temporarily by a pair of turning points. If the particle's energy lies in a conduction band, the particle drifts through the lattice of cells and is never captured by the same pair of turning points more than once. However, if the energy of the particle is not in a conduction band, the particle can return to previously visited cells.

Robin Hudson

Sticky shuffle product algebras and their stochastic representations

Iterated integrals arise in the solution by Picard iteration of stochastic and quantum stochastic differential equations. The space of all such integrals has a rich mathematical structure as a Hopf algebra. This algebra is commutative but non-cccommutative in the classical case and non-commutative in the quantum case, when it contains a nontrivial universal enveloping algebra as a Hopf subalgebra. The talk will describe this structure and how it is related to continuous tensor products, and will apply it to the construction of new classical stochastic processes related to invariants of the general linear group, of which the trace corresponds to Poisson process. A Z^2-graded version of the theory may also be described.

Vít Jakubský

Hidden supersymmetry in Aharonov-Bohm model

We find a hidden supersymmetry in the (spinless) Aharonov-Bohm problem. Similar superalgebraic structure is observed in two different self-adjoint extensions of the formal Aharonov-Bohm Hamiltonian. We show that the hidden supersymmetry is compatible with the conformal invariance of the system and we find an additional non-local dynamical symmetry.

Martin Kalina

Bifullness of centers of lattice effect algebras

If element z of a lattice effect algebra $(E, \oplus, \mathbf{0}, \mathbf{1})$ is central, then the interval $[\mathbf{0}, z]$ is a lattice effect algebra with the new top element z and with inherited partial binary operation $\oplus$.

It is a known fact that if the set $C(E)$ of central elements of E is an atomic Boolean algebra and the supremum of all atoms of $C(E)$ in E equals to the top element of E, then E is subdirectly irreducible ([2]). In [1] Paseka and Riečanová published as open problem whether $C(E)$ is a bifull sublattice of an Archimedean atomic lattice effect algebra E. We show that there are lattice effect algebras $(E, \oplus, \mathbf{0}, \mathbf{1})$ with atomic $C(E)$ which are not bifull in E. Moreover, we show that also $B(E)$, the center of compatibility, may not be bifull in E.

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References

[1] Paseka, J., Riečanová, Z., The inheritance of BDE-property in sharply dominating lattice effect algebras and (o)-continuous states. Preprint.

[2] Riečanová, Z., Subdirect decompositions of lattice effect algebras, *Inter. J. Theor. Phys.* **42** (2003), 1425-1433.

Maurice Kibler:

Phase operator, temporally stable phase states, mutually unbiased bases and exactly solvable quantum systems

This talk presents some new developments on the work published, under the same title, in collaboration with M. Daoud, in J. Phys. A: Math. Theor. 43 (2010) 115303. In particular, the phase operators and their corresponding phase eigenstates relative to a generalized oscillator algebra are described in a group theoretical approach involving SU(1,1) and SU(2).

Oleg N. Kirillov:

The role of exceptional points in describing the relation of standard and helical magnetorotational instability

The magnetorotational instability (MRI) plays a crucial role for cosmic structure formation by enabling turbulence in Keplerian disks which would be otherwise hydrodynamically stable. With particular focus on MRI experiments with liquid metals, which have small magnetic Prandtl numbers, it has been shown that the helical version of this instability (HMRI) has a scaling behaviour that is quite different from that of the standard MRI (SMRI). We discuss the relation of HMRI to SMRI by exploring various parameter dependencies. We identify the mechanism of transfer of instability between modes through a spectral exceptional point that explains both the transition from a stationary instability (SMRI) to an unstable travelling wave (HMRI) and the excitation of HMRI in the inductionless limit. For certain parameter regions we find new islands of the HMRI.

Joint work with Frank Stefani.

Milan Krbálek:

Analytical derivation of spectral rigidity for thermodynamical traffic gas.

We introduce an one-dimensional thermodynamical particle model which is efficient in prediction about a microscopical structure of vehicular traffic streams. For such a model we show analytical calculations leading to formulae for time clearance distribution as well as for time spectral rigidity. Furthermore, the results obtained are reformulated in terms of traffic theory and consecutively compared to experimental traffic data.

M. Howard Lee:

Analytical study of 3-cycles in a chaotic map.

In the logistic map, the simplest of stable odd-numbered cycles is a 3-cycle. It comes into existence after the stable 2k cycles cease to exist. The 3-cycle is thus important as a route to chaos. In 1977 Guckenheimer et al., J. Math. Biol. 4, 101 (1977) first estimated the value of the control parameter a at which it can be superstable. It implies that a stable 3-cycle can be formed and deformed at some values of a straddling that of the superstable 3-cycle. In this work we present an exact value of a for the superstable 3-cycle by reducing a polynomial of degree 7 to that of 3. The relevance of this work to Sharkovskiis theorem is also discussed [cf. J. Math. Phys. 50, 122702 (2009)].

Zuzana Masáková

Balanced numbers systems with negative base

We study non-standard number system with negative base $-\beta$. We present several disadvantages of the Ito-Sadahiro definition, based on the transformation $T_{-\beta}$ of the interval $I_\beta = [-\frac{\beta}{\beta+1}, \frac{1}{\beta+1})$ into itself. We suggest a

generalization using an interval $[c, c+1)$ and show that the corresponding number system is well defined and has properties analogic to the Ito-Sadahiro version. We show how the choice of c influences the set of digits. We propose the choice $c = -\frac{1}{2}$ which leads to a balanced number system which avoids the disadvantages of the Ito-Sadahiro definition.

Vladimír Olejček (joint work with Martin Kalina)

Atomic Archimedean MV-effect algebra need not be sharply dominating

Atomicity and sharp domination are among properties playing important role in the problem of existence of states on lattice effect algebras. The study of the two properties has brought the question, published in [J. Paseka, Z. Riečanová, Soft Computing 2010], of whether the Archimedean property and atomicity implies the sharp domination property of a special effect algebra called MV-effect algebra. In this contribution a negative answer to this question is given.

Jan Paseka

States on Archimedean atomic lattice effect algebras

Effect algebras (introduced by D.J. Foulis and M.K. Bennett in for modelling unsharp measurements in a Hilbert space) may be carriers of states or probabilities when events are noncompatible or unsharp resp. fuzzy. In this case the set E(H) of effects is the set of all self-adjoint operators A on a Hilbert space H between the null operator 0 and the identity operator 1 and endowed with the partial operation + defined iff A+B is in E(H), where + is the usual operator sum.

We study several important classes of Archimedean atomic lattice effect algebras (modular, sharply dominating etc.) to ensure the existence of a state (probability, modular measure) on them.

Silvia Pulmannová:

Type-decompositions of effect algebras and similar algebraic structures I

Effect algebras (EAs), play a significant role in quantum logic, are featured in the theory of partially ordered abelian groups, and generalize orthoalgebras, MV-algebras, orthomodular posets, orthomodular lattices, modular ortholattices, and boolean algebras. We study centrally orthocomplete effect algebras (COEAs), i.e., EAs satisfying the condition that every family of elements that is dominated by an orthogonal family of central elements has a supremum. For COEAs, we introduce a general notion of decomposition into types; prove that a COEA factors uniquely as a direct sum of types I, II, and III; and obtain a generalization for COEAs of Ramsay's fourfold decomposition of a complete orthomodular lattice.

Stefan Rauch-Wojciechowski:

Qualitative analysis of Newton equations for rolling and gliding rigid body dynamics

Rigid bodies rolling and gliding in the plane such as the children toy top, tippe top and the rattleback display diverse and often counterintuitive behavior that is difficult to understand from the basic Newton equations of motion for the rigid body dynamics. The complexity of the problem is enhanced by the large number of dynamical variables varying from 4 up to 10 unknowns. In this talk I shall discuss how we can combine the analysis of the vector form and of the Euler angle form of Newton equations to derive rigorous qualitative statements about properties of solutions in several cases of interest. This will provides us with basic intuition about why these bodies move as they do. I will give some answers and will make us aware about mathematical challenges that stay ahead. In particular, I shall present a deformation approach to analysis of the tippe top motion. It reduces the problem to study of a single equation, called the Main Equation of the tippe top. Analysis of this equation captures the main features of dynamics of inversion.

Zdenka Riečanová

Subfamilies of lattice effect algebras with special properties of their important sub-lattice effect algebras

Lattice ordered effect algebras (introduced by D.J. Foulis and M.K.Bennett) generalize orthomodular lattices (that include non-compatible pairs of elements) and MV-algebras (that may include unsharp elements). Thus they are very natural algebraic structures for to be carriers of states or probability measures when elements of these structures represent properties, questions or events with fuzziness, uncertainty or unsharpness that may be mutually non-compatible.

Every lattice effect algebra E is a union (pasting) of MV-effect algebras (maximal subsets of their pairwise compatible elements) called blocks of E. Thus the intersection B(E) of all blocks of E (called a center of compatibility) is again a sub-MV-effect algebra of E. The set S(E) of all sharp elements of E (such that x and "non" x are disjoint) is an orthomodular lattice. In this sense, E is a "smeared" orthomodular lattice, while MV-effect algebra is a "smeared" Boolean algebra, as in this case S(E) is a Boolean algebra. Finally, the center C(E) of sharp elements that are compatible with all elements of E is a Boolean algebra. On the other hand a lattice effect algebra E is an orthomodular lattice iff S(E) = E, E is an MV-effect algebra iff B(E) = E and E is a Boolean algebra iff C(E) = E.

We show that some further important subfamilies of the family of lattice effect algebras E have other special properties of B(E), S(E), C(E) and blocks of E. For instance, if E is an Archimedean atomic lattice effect algebra then every block of E and S(E) are bifull sub-lattices of E. If S(E) is a complete lattice then E is sharply dominating (equivalently, elements of E have basic decomposition property). Further E is sharply dominating iff every block of E is sharply dominating. We can also prove that if an Archimedean atomic lattice effect algebra E can be densely embedded into a complete lattice effect algebra then the center C(E) of E is an atomic Boolean algebra.

Petr Siegl

Krein Spaces in de Sitter Quantum Theories

Experimental evidences and theoretical motivations lead to consider the curved space-time relativity based on the de Sitter group SO(1,4) or Sp(2,2) as an appealing substitute to the flat space-time Poincar relativity. Quantum elementary systems are then associated to unitary irreducible representations of that simple Lie group. At the lowest limit of the discrete series lies a remarkable family of scalar representations involving Krein structures and related indecomposable representation cohomology which deserves to be thoroughly studied in view of quantization of the corresponding carrier fields. The purpose of this presentation is to indicate possible extensions of an exemplary case, namely the so-called de Sitterian massless minimally coupled field.

This is a joint work with J.-P.Gazeau and A. Youssef.

Štěpán Starosta

Θ-palindromes in infinite words

We will define a Θ-palindrome - a generalization of the notion of palindrome which is based on the reversal mapping. Differences of this generalization with standard palindromes will be highlighted. A class of infinite words having maximum number of Θ-palindromes will be presented together with some examples.

Tomáš Vávra

Arithmetics in number systems with negative base

We study the numeration system with negative base, introduced by Ito and Sadahiro. We focus on arithmetic operations in the set $\mathrm{Fin}(-\beta)$ and $\mathbb{Z}_{-\beta}$ of numbers having finite resp. integer $(-\beta)$-expansions. We show that $\mathrm{Fin}(-\beta)$ is trivial if β is smaller than the golden ratio $\frac{1}{2}(1+\sqrt{5})$. For $\beta \geq \frac{1}{2}(1+\sqrt{5})$ we prove that $\mathrm{Fin}(-\beta)$ is a ring, only if β is a Pisot or Salem number with no negative conjugates. We prove the conjecture of Ito and Sadahiro that $\mathrm{Fin}(-\beta)$ is a ring if β is a quadratic Pisot number with positive conjugate. For quadratic Pisot units we determine the number of fractional digits that may appear when adding or multiplying two $(-\beta)$-integers.

Alberto Ventura

Algebraic approach to non-central potentials in the n-dimensional Schrödinger equation

Central potentials whose three-dimensional Schrödinger Hamiltonian has an underlying Lie-algebraic symmetry admit non-central extensions with symmetries in larger dimensions, n ¿ 3, compact for bound states and non-compact for scattering states, which lend themselves to fully analytic solutions. The scattering matrix, in particular, can be worked out as an intertwining operator between Weyl-equivalent unitary irreducible representations of the corresponding potential algebra [1]. An example of bound state calculation is a non-central extension of the three-dimensional harmonic oscillator with u(4) symmetry [2] and an example of scattering calculation is provided by a non-central extension of the null potential with e(4) symmetry [3]. In the present work we study in particular non-central extensions of the n-dimensional Coulomb potential, with underlying so(n+4) symmetry in the bound-state problem and so(n+3,1) symmetry in the scattering problem, with n ¿2.

This is a joint work with Gul-Mirza Kerimov (Trakya University, Edirne, Turkey).

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Jakub Železný

Closed integral formula for the metric of a PT-symmetric model

This talk deals with a simple PT-symmetric non-Hermitian model previously introduced by Krejcirik, Bila and Znojil. They derived a closed formula for the metric operator which relates the problem in the Hilbert space to a Hermitian one. We present a new, integral form of the metric operator, which is more suitable for practical computations and enables us to thoroughly analyse its spectrum.