Comment on the drift mirror instability

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Drift mirror instability is investigated in a slightly inhomogeneous plasma in a low-frequency, long-wavelength limit of the Vlasov-Maxwell equation. It is shown that previously derived threshold conditions for the drift mirror instability in the case of one cold species are incorrect and that it is hard to get analytically a simple threshold condition for the drift mirror instability in this case. It is argued that the same is true for the drift mirror instability in general.

Mirror instability[1,2] is one of the exceptional kinetic instabilities for which it is possible to obtain an analytical threshold condition, at least for some plasma parameters.[3] This instability was studied by Hasegawa[4] in a slightly inhomogeneous plasma,[5,6] assuming a presence of one cold species. In this inhomogeneous case the instability was named a drift mirror instability. Hasegawa[4] showed that the threshold condition of the drift mirror instability is identical to the threshold condition in the homogeneous case and that the real frequency of unstable modes is non zero (compared to zero in the homogeneous case). These results were revisited by Pokhotelov et al.[7] who showed that some important terms were omitted in the original work of Ref. 4. These terms importantly modify the real frequency of unstable modes. Pokhotelov et al.[7] also discussed possible modifications of the threshold condition due to the inhomogeneity in some cases. These results were extended to the case of hot plasma by Pokhotelov et al.[8,9]

In this communication we show that the previously derived threshold conditions for the drift mirror instability in the case of one cold species[4,7] are incorrect and that it is hard to get analytically a simple threshold condition for the drift mirror instability in this case. We argue that the same is true for the drift mirror instability in general.

Let us consider a plasma with one ion population, say protons, and cold electrons. We take the ambient magnetic field $\boldsymbol{B}_0$ as decreasing in the y-direction

$$\boldsymbol{B}_0(y) = B_0(1 - \kappa y)\boldsymbol{e}_z. \tag{1}$$

Then, we chose the unperturbed proton distribution function $f_p(y)$ to be

$$f_p(y) = f_0 \left[1 - \tilde{\kappa}\left(y - \frac{v_x}{\omega_{cp}}\right)\right], \tag{2}$$

where $\omega_{cp} = eB_0/m_p$ is the proton cyclotron frequency (e: proton charge, m_p: proton mass) and f_0 is a normalized bi-Maxwellian distribution function

$$f_0 = \frac{1}{\sqrt{2\pi}^3 v_{p\parallel} v_{p\perp}^2} \exp\left(-\frac{v_\perp^2}{2v_{p\perp}^2} - \frac{v_\parallel^2}{2v_{p\parallel}^2}\right), \tag{3}$$

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$v_{p\perp}$ and $v_{p\parallel}$ being the perpendicular and parallel thermal velocities, respectively. Initially we assume a pressure equilibrium (and no external current) which leads to[7]

$$\tilde{\kappa} = -\kappa\frac{2}{\beta_\perp}. \tag{4}$$

Here we use the following definitions: $\beta_\perp$ and $\beta_\parallel$ are the total perpendicular and parallel betas, which, in the preset case, are equal to the proton perpendicular and parallel betas

$$\beta_{\perp,\parallel} = \beta_{p\perp,\parallel} = 2\mu_0 n_p k_B T_{p\perp,\parallel}/B_0^2, \tag{5}$$

respectively; here μ_0 is the vacuum magnetic permeability, k_B is Boltzmann constant, n_p is the proton density (at $y = 0$), and finally $T_{p\perp,\parallel} = m_p v_{p\perp,\parallel}^2/k_B$.

We assume a perturbation in the form of planar wave, $e^{i(k_\perp x + k_\parallel z - \omega t)}$ at the local value of $y = 0$; $k_\perp$ and $k_\parallel$ are assumed to be positive. In the low frequency, long wavelenth limit

$$\frac{|\omega|}{\omega_{cp}} \ll 1, \qquad \frac{k_\perp v_{p\perp}}{\omega_{cp}} \ll 1, \quad \text{and} \quad \frac{k_\parallel v_{p\parallel}}{\omega_{cp}} \ll 1, \tag{6}$$

and for wavelengths much smaller then the gradient scale

$$\kappa \ll k_\perp \qquad \text{and} \qquad \kappa \ll k_\parallel \tag{7}$$

the dispersion of the drift mirror mode (for one cold population) may be factorized to

$$k^2 - \frac{\omega^2}{c^2}\epsilon_{yy} = 0. \tag{8}$$

The ϵ_{yy} component of the dielectric tensor may given in the form

$$\begin{aligned}
\epsilon_{yy} = {}& 1 + 2\pi\frac{\omega_{pp}^2}{\omega^2}\int_{-\infty}^{\infty} dv_\parallel \int_0^{\infty} dv_\perp v_\perp^2 \\
& \left[\frac{\partial f_0}{\partial v_\perp}\left(\omega - k_\parallel v_\parallel\right) + k_\parallel v_\perp \frac{\partial f_0}{\partial v_\parallel} - \frac{2\kappa}{\beta_\perp} f_0 \frac{k_\perp v_\perp}{\omega_{cp}}\right] \\
& \sum_{n=-\infty}^{\infty} \frac{(J_n')^2}{\omega - k_\parallel v_\parallel - n\omega_{cp} - \kappa\frac{k_\perp v_\perp^2}{2\omega_{cp}}}
\end{aligned} \tag{9}$$

where J_n is the Bessel function of the order n with the argument $k_\perp v_\perp/\omega_{cp}$ and J_n' is its derivative, $\omega_{pp} =$

$(n_p e^2/m_p \epsilon_0)^{1/2}$ is the proton plasma frequency and ϵ_0 is the vacuum electric permittivity.

In the limit (7) one gets drift mirror waves with non-zero real frequency[7]

$$\omega_r = \left[\frac{3}{2}\left(\frac{T_{p\perp}}{T_{p\parallel}} - 1\right) - \frac{2}{\beta_\perp}\right]\frac{\kappa k_\perp v_{p\parallel}^2}{\omega_{cp}} \qquad (10)$$

and a growth/damping rate, including finite Larmor radius corrections ($r_{Lp} = v_{p\perp}/\omega_{cp}$ being the proton Larmor radius)

$$\gamma = \sqrt{\frac{2}{\pi}}|k_\parallel|v_{p\parallel}\frac{T_{p\parallel}}{T_{p\perp}\beta_{p\perp}}\left(\Gamma - \frac{3}{2}r_{Lp}^2 k_\perp^2 - \frac{k_\parallel^2}{k_\perp^2}\Pi\right), \quad (11)$$

where

$$\Pi = 1 + \frac{\beta_\perp - \beta_\parallel}{2} \quad \text{and} \quad \Gamma = \beta_{p\perp}\left(\frac{T_{p\perp}}{T_{p\parallel}} - 1\right) - 1. \quad (12)$$

During the derivation of (10,11) we have assumed

$$\left|\frac{\omega - \kappa\frac{k_\perp v_\perp^2}{2\omega_{cp}}}{\sqrt{2}k_\parallel v_{p\parallel}}\right| \ll 1 \quad \text{and} \quad \frac{\omega^2}{k^2 v_A^2} \ll 1. \qquad (13)$$

These results may be easily generalized to a multi-species plasma.

Expression (11) for the growth rate is identical to that in a homogeneous plasma, a fact which may lead to the conclusion that the homogeneous threshold condition

$$\Gamma = \beta_{p\perp}\left(\frac{T_{p\perp}}{T_{p\parallel}} - 1\right) - 1 = 0 \qquad (14)$$

remains valid also in the inhomogeneous case. However, when Γ goes to zero, the wave vectors $k_\parallel$ and $k_\perp$ go to zero as well which, at some point, violates condition (7). When condition (7) is not satisfied, the drift mirror mode becomes strongly coupled to the shear Alfvén mode and factorization (8) is no longer valid. Consequently, the condition (14) is not the threshold condition for the drift mirror instability in the case of one cold species and the threshold conditions derived in Refs. 4,7 are not correct. Further investigation of the drift mirror instability when (7) is not satisfied seems to require a numerical treatment which is beyond the scope of this communication.

In a hot, homogeneous plasma the behaviour of the mirror instability near threshold requires (most of) the whole dispersion matrix.[3,10] This property remains valid for the drift mirror instability. We conclude that in general there is no simple threshold condition for the drift mirror instability and a numerical solution for the full kinetic dispersion is in order. We also expect that some other results on the drift mirror instability, e.g. Refs. 8,9, need to be revisited.

[1] A. A. Vedenov and R. Z. Sagdeev, in *Plasma Physics and the Problem of Controlled Thermonuclear Reactions* (Pergamon, New York, 1958), vol. 3, pp. 332–339.

[2] S. A. Chandrasekhar, A. N. Kaufman, and K. M. Watson, Proc. R. Soc. London, Ser. A **245**, 435 (1958).

[3] P. Hellinger, Phys. Plasmas **14**, 082105 (2007).

[4] A. Hasegawa, Phys. Fluids **12**, 2642 (1969).

[5] N. A. Krall and M. N. Rosenbluth, Phys. Fluids **4**, 163 (1961).

[6] N. A. Krall and M. N. Rosenbluth, Phys. Fluids **5**, 1435 (1962).

[7] O. A. Pokhotelov, M. A. Balikhin, R. A. Treumann, and V. P. Pavlenko, J. Geophys. Res. **106**, 8455 (2001).

[8] O. A. Pokhotelov, O. G. Onishchenko, M. A. Balikhin, R. A. Treumann, and V. P. Pavlenko, J. Geophys. Res. **106**, 13237 (2001).

[9] O. A. Pokhotelov, I. Sandberg, R. Z. Sagdeev, R. A. Treumann, O. G. Onishchenko, M. A. Balikhin, and V. P. Pavlenko, J. Geophys. Res. **108**, 1098 (2003).

[10] A. N. Hall, J. Plasma Phys. **21**, 431 (1979).