Reformation of perpendicular shocks: Hybrid simulations

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We show that in one-dimensional hybrid simulations supercritical perpendicular shocks have a well-defined, slightly oscillating structure (including foot, ramp, overshoot, and undershoot) for hot upstream protons and low Mach numbers. In this case the reflected protons are able to stop the shock steepening at proton scales. However, for colder upstream protons and/or higher Mach numbers simulated shocks show a nonstationary feature, namely a periodic reappearance of a new shock front. These nonstationary shocks simulated by one-dimensional hybrid simulations have shock front gradient scales that are governed only by the spatial resolution used in the code. Consequently, we conclude that the one-dimensional hybrid code is not suitable for description of perpendicular nonstationary shocks.

1. Introduction

Super-critical quasi-perpendicular shocks have typically well-defined structure [*Scudder et al.*, 1986] including foot, ramp, overshoot, and undershoot. These features are connected with the existence of protons reflected off the shock as observed in one-dimensional (1-D) hybrid simulations by *Leroy et al.* [1982]. Theoretical study of *Leroy* [1983] shows that a stationary shock structure exists for wide range of upstream parameters. The model of *Leroy* [1983] is however relatively simple: It is multi fluid, it is assumed to be stationary and it does not resolve self-consistently the reflection process and the interaction between incoming and reflected protons. The model agrees very well with results of numerical simulations by *Leroy et al.* [1982] but the simulated shocks are not exactly stationary: In the shock rest frame various macroscopic quantities $b = b(x,t)$ (for example the density, magnetic field) oscillate around a mean value $\bar{b}(x)$ as $b(x,t) \sim \bar{b}(x) + \delta b(x) sin(\Omega_o t)$ with a small amplitude $\delta b(x) << \bar{b}(x)$ and with a frequency Ω_o, which is approximately the upstream proton gyrofrequency. The most pronounced variations $\delta b(x)$ are in the overshoot [see *Leroy et al.*, 1982, Figure 11]. Henceforth, we call this oscillatory behavior quasi-stationary.

On the other hand, in some cases the shock structure was shown to have a nonstationary feature: a periodic reappearance of a new shock front. 1-D hybrid simulation study by *Quest* [1986] has revealed such a nonstationarity in the case of very strong shocks. *Quest* [1986] has also shown that the nonstationary shocks could be stabilized when a sufficient resistivity is included, however for a too strong resistivity the nonstationarity reappears. Moreover, the 1-D particle-in-cell (PIC) simulations by *Lembège and Dawson* [1987] show a similar nonstationary behavior that *Lembège and Dawson* [1987] describe as a cyclic self-reformation. The shock reflects an important fraction of incident protons that propagate upstream and create a new shock front. *Lembège and Savoini* [1992] confirmed that this self-reformation is not an artifact due to the use of 1-D code since it persists with two-dimensional (2-D) PIC code and it remains in place even when resistive effects due to cross-field currents instabilities are included self-consistently in the simulation (in contrast with hybrid codes where the resistivity is fixed to zero or to a finite constant value). They also show that the shocks are self-reforming (henceforth we shall use the term nonstationary) for a wide range of upstream parameters (even for oblique and medium shocks) in contrast with the results of hybrid simulations [e.g., *Leroy et al.*, 1982] and the model by *Leroy* [1983] that predicts nonstationarity condition for cold upstream protons $\beta_p < 0.1$. This discrepancy was a starting point for *Hada et al.* [2003] who revisited the model of *Leroy* [1983]: They relaxed the stationarity assumed in *Leroy* [1983] and included a self-consistent interaction between reflected and incoming protons. With the use of their improved model *Hada et al.* [2003] were able to show that perpendicular shocks become nonstationary even when there exists a stationary solution of the model by *Leroy* [1983]. *Hada et al.* [2003] have shown that this stationary solution is not dynamically accessible, since during its formation the shock structure breaks down and a new shock front is created before the stationary solution is formed. This nonstationary behavior appears if the number of reflected protons is greater than a critical value that depends on the shock Mach number.

The results of *Hada et al.* [2003] remove the discrepancy between the model and PIC simulations, however the important differences between PIC and hybrid simulations remain. In this paper we examine perpendicular shocks using 1-D hybrid simulations. We study whether shock steepening could be stopped by the proton reflection on the proton scales, which makes the shock quasi-stationary [*Leroy*, 1983; *Hada et al.*, 2003]. Since we expect that the stationarity depends on the shock front thickness [*Quest*, 1986; *Hada et al.*, 2003], we first study the influence of a spatial resolution used in 1-D hybrid simulations on the shock properties. We find that the shock properties depend on the resolution for shocks that turn out to be nonstationary. Finally, we determine for which upstream parameters the shock is quasi-stationary and for which ones the shock is nonstationary.

The paper is organized as follows: Section 2 describes the hybrid code and the simulation method. In the section 3 we show results of 1-D perpendicular shock simulations for different upstream proton betas and various spatial resolutions. In section 4 we perform simulation parametric study to determine the region of upstream plasma parameters that leads to quasi-stationary or nonstationary shocks. In section 5 we discuss the results.

2. Hybrid code

For the numerical simulation we use a 1-D hybrid code developed by *Matthews* [1994]. In this code electrons are considered as a massless fluid, with a constant temperature; ions are treated as particles and are advanced by a leapfrog scheme that requires the fields to be known at half time steps ahead of the particle velocities. This is obtained by advancing the current density to this time step with only one computational pass through the particle data at each time step. The particle contribution to the current density at the relevant nodes is evaluated with bilinear weighting followed by smoothing over three points, smoothing is also performed on the electric field, and the resistivity is set to zero (if not stated otherwise). The magnetic field is advanced in time with a modified midpoint method, which allows time sub-stepping for the advance of the field.

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The units and parameters of the simulation are the following: units of space and time are c/ω_{pi} and Ω_i, respectively, where $\omega_{pi} = (n_p e^2/m_p \epsilon_0)^{1/2}$ is the upstream proton plasma frequency and $\Omega_i = eB_0/m_p$ is the upstream proton gyrofrequency. In these expressions, n_p and B_0 are the density of the plasma protons and the magnitude of the initial magnetic field upstream, respectively, while e and m_p are the proton electric charge and mass, respectively; and, finally c, ϵ_0, and μ_0 are the speed of light, and the dielectric permitivity and magnetic permeability of vacuum, respectively. The fields and moments are defined on a 1-D grid. The time step for the particle advance is $dt = 0.01\Omega_i^{-1}$ while the magnetic field $\boldsymbol{B}$ is advanced with a smaller time step, $dt_B = dt/4$. Velocities are given in units of v_A. The same units are used in all subsequent figures. Plasma betas, ratios between upstream particle pressures and the upstream magnetic pressure $p_B = B_0^2/(2\mu_0)$, are the electron beta $\beta_e = n_p k_B T_e/p_B$, and the proton beta $\beta_p = n_p k_B T_p/p_B$. Here k_B is the Boltzmann constant, and T_e and T_p is the upstream electron and proton temperatures, respectively. Magnetic field $\boldsymbol{B}_0$ is initially constant, directed along the y axis, $\boldsymbol{B}_0 = (0,\ B_0,\ 0)$.

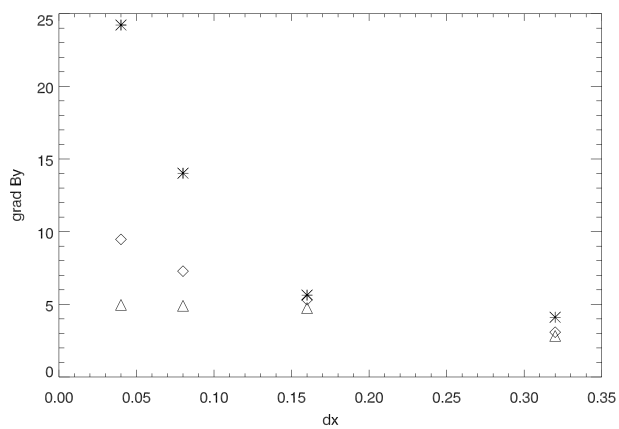

Figure 1. Dependence of the maximum gradient $dB_y/dx/B_0$ in the shock front on the spatial resolution dx for the three different upstream proton betas: for $\beta_p =$ (stars) 0.2, (diamonds) 0.5, and (triangles) 1.0.

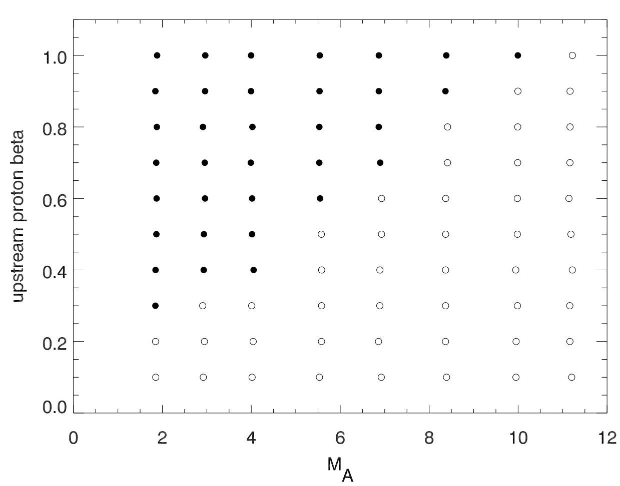

Figure 2. 1-D hybrid simulations: (full circles) quasi-stationary and (empty circles) nonstationary perpendicular shocks for different shock parameters: Alfvén Mach number and upstream proton beta.

3. Role of spatial resolution

In this section we study the dependence of shock structure on spatial resolution, dx, used in the 1-D hybrid code. A set of twelve 1-D hybrid simulations was run for different dx: $dx = 0.04$, 0.08, 0.16, and 0.32 and different upstream proton betas: $\beta_p = 0.2$, 0.5, and 1.0. For all the cases the simulation box has size $X = 52.16$, and there are 1024 particles per cell. We initialize the simulations with a homogeneous Maxwellian protons in all cases streaming with uniform velocity $\boldsymbol{v} = (4,\ 0,\ 0)$. The interaction of the streaming plasma with an infinitely conducting wall located at $x = X$ launches a shock wave propagating in the negative x directions. During the simulation the plasma is injected at $x = 0$. For all the spatial resolution dx and upstream proton betas indicated above the resulting shock wave has approximately the Alfvén Mach number $M_A \sim 6.6$. However, the shock structure and stationarity differ. For $dx = 0.16$ and 0.32 the shock structure is quasi-stationary for all betas: in this case the simulated shocks have the usual well defined structure (foot, overshoot, undershoot), and this structure oscillates periodically with about the upstream proton gyrofrequency.

For better spatial resolutions $dx = 0.04$ and 0.08 the simulated shocks with colder upstream protons $\beta_p = 0.2$ and 0.5 are nonstationary. These nonstationary shocks exhibit the features of self-reformation seen in the PIC simulations [*Lembège and Dawson*, 1987; *Lembège and Savoini*, 1992]. On the other hand, for hot upstream protons $\beta_p = 1.0$ the simulated shock is quasi-stationary; actually the simulated shock structure and its evolution for the upstream $\beta_p = 1.0$ is almost identical for the spatial resolutions $dx = 0.04$, 0.08 and 0.16. Figure 1 shows quantitative results that justify the above statements. Figure 1 displays the dependence of the maximum gradient $dB_y/dx/B_0$ in the shock front (calculated during several upstream proton gyroperiods when the shock was well separated from the wall) on the used spatial resolution dx for the three different upstream proton betas: for $\beta_p =$ (stars) 0.2, (diamonds) 0.5, and (triangles) 1.0. Figure 1 clearly shows that the maximum gradient $dB_y/dx/B_0$ increases when dx decreases for $\beta_p = 0.2$ and 0.5. On the other hand, in the case of $\beta_p = 1.0$ the maximum gradient of $dB_y/dx/B_0$ is constant for $dx = 0.04$, 0.08, and 0.16. Therefore, in the case $\beta_p = 1.0$, the shock steepening is stopped by the process of proton reflection [cf. *Leroy et al.*, 1982; *Leroy*, 1983]. However, for $\beta_p = 0.2$ and 0.5 the shock steepening goes on and the shocks are nonstationary. We obtain similar results also for a finite (anomalous) resistivity η in the range 0.001–$0.1\mu_0 v_A^2/\Omega_i$. When the resistivity increases (within the studied range) the maximum gradient within the shock front decreases but the overall properties of the shock structure, its evolution, and its dependence on the spatial resolution dx remain the same. Let us now study a range of upstream parameters for which the shock is quasi-stationary or nonstationary.

4. Parametric study: M_A and β_p

In this section we perform a set of 1-D hybrid simulation of perpendicular shocks for different upstream parameters: upstream streaming velocity v_0 and upstream proton beta β_p. Electron beta is for all the cases $\beta_e = 0.5$. For these 1-D simulations we use the spatial resolution $dx = 0.04$, and the box size $X = 100$ and there are $N = 1,024$ particles per cell upstream. The time step for the particle advance is $dt = 0.005/\Omega_i$ while the magnetic field $\boldsymbol{B}$ is advanced with a smaller time step $dt_B = dt/4$. For all the simulations we have determined the Alvén Mach number and their behavior: The shocks are nonstationary for cold upstream protons and/or strong shocks. A quantitative view is shown on Figure 2. On Figure 2 we have plotted full circles denoting quasi-stationary and empty circles denoting nonstationary perpendicular shocks, respectively, as a function of the shock parameters: Alfvén Mach number M_A and upstream proton beta β_p. Figure 2 clearly shows that the perpendicular shocks are nonstationary for a wide range of parameters.

5. Conclusion

In 1-D hybrid simulations the supercritical perpendicular shocks are quasi-stationary or nonstationary. The simulated quasi-stationary shocks are similar to the shocks usually observed in hybrid simulations [*Leroy et al.*, 1982] while the simulated nonstationary shocks are similar to the self-reforming shocks observed in PIC simulations [*Lembège and Dawson*, 1987; *Lembège and Savoini*, 1992]. We investigated conditions under which a shock is nonstationary. We performed a set of 1-D hybrid simulations of perpendicular shocks with a different set of upstream parameters: β_p within the range $0.1-1$, M_A within the range $2-11$. The simulations show that the nonstationarity appears for colder upstream protons and/or higher Mach numbers (see Figure 2).

These results show that there is a good qualitative agreement between the shock simulations of hybrid and PIC codes and the model by *Hada et al.* [2003]. However, we have also shown that in the case of nonstationary shocks the maximum gradient of the compressional magnetic component B_y is determined only by the spatial resolution used in the simulation: The maximum of $dB_y/dx/B_0$ strongly increases when dx decreases. This is true even when the resolution approaches the electron inertial length c/ω_{pe}, which means that in the case of nonstationary shocks the proton reflection is not able to stop the steepening. These results suggest that 1-D hybrid codes are not suitable to describe the nonstationary shocks. Of course, one could include a sufficiently strong resistivity that is able to stabilize the shock structure [*Quest*, 1986] but in any case PIC simulations are needed to describe correctly the nonstationarity with a self-consistent anomalous resistivity [see *Lembège and Savoini*, 1992].

The present study was restricted to one dimension and strictly perpendicular shocks. *Lembège and Savoini* [1992] have shown that the self-reformation is not an artifact due to the use of 1-D code since it persists with 2-D PIC code even when resistive effects due cross-fields currents instabilities are included self-consistently in the simulation. However, it is an open question, whether higher dimensionality effects on proton scales, such as strong shock ripples [*Winske and Quest*, 1988], have an impact on the shock structure and stationarity. We also expect a similar behavior for quasi-perpendicular shocks. Indeed, results of PIC simulation by *Lembège and Savoini* [1992] show that also oblique shocks are self-reforming. Their study moreover indicates that there is a critical angle θ_c such that the nonstationarity disappears when the angle between the shock normal and the upstream magnetic field is smaller than θ_c. Also for an oblique propagation the dispersive effects become important [*Krasnoselskikh et al.*, 2002]. These problems will be subject for future works.

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