Emission of nonlinear whistler waves at the front of perpendicular supercritical shocks: Hybrid versus full particle simulations

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New behavior of strictly perpendicular shocks in supercritical regime is analyzed with the help of both two-dimensional (2-D) hybrid and full particle electromagnetic simulations. Surprisingly, in both simulation cases, the shock front region appears to be dominated by emission of coherent large amplitude whistler waves for some plasma conditions and shock regimes. These whistler waves are oblique with respect to the shock normal as well as to the upstream magnetic field and are phase-standing in the shock rest frame. A parametric study shows that these whistler waves are emitted in 2-D perpendicular shocks and, simultaneously, the self-reformation of the shock front associated with reflected ions disappears; the 2-D shock front is almost quasi-stationary. In contrast, both corresponding one-dimensional (1-D) hybrid and full particle simulations performed in similar plasma and Mach regime conditions show that the self-reformation takes place for 1-D perpendicular shock. These results indicate that the emission of these 2-D whistler waves can inhibit the self-reformation in 2-D shocks. Possible generating mechanisms of these waves emissions and comparison with previous works are discussed.

1. Introduction

Numerical simulations [*Lembège and Dawson*, 1987; *Lembège and Savoini*, 1992] revealed a strongly nonstationary behavior of quasi-perpendicular shocks in supercritical regime. Similar behavior is also evidenced with parameters relevant to the heliospheric and astrophysical contexts [*Shimada and Hoshino*, 2000; *Schmitz et al.*, 2002]. This nonstationarity is characterized by a periodic self-reformation of the shock front over ion time scale. In the case of strictly perpendicular shocks, the self-reformation process reveals to be very sensitive to Alfvén Mach number M_A and ion upstream β_i [*Hellinger et al.*, 2002; *Hada et al.*, 2003]. Recent reviews [*Hellinger*, 2003; *Lembège et al.*, 2004] stress the importance of the self-reformation process for the shock properties. By using one-dimensional (1-D) full particle-in-cell (PIC) simulation code, *Scholer et al.* [2003] pointed out that the nonstationary self-reformation process switches off some acceleration mechanisms, for example the shock surfing mechanism, which has been proposed for time-stationary shock solutions [*Sagdeev*, 1966; *Lipatov and Zank*, 1999]. On the other hand, 1-D PIC simulations [*Schmitz et al.*, 2002; *Lee et al.*, 2004] show that the shock reformation leads to a strong energization of a portion of reflected ions during their subsequent interaction with the nonstationary shock.

In this letter, we present results from two-dimensional (2-D) hybrid and PIC simulations of strictly perpendicular shocks, where large amplitude coherent whistler waves are emitted in the foot region and dominate the whole shock front dynamics. The letter is organized as follows: First, the simulation method is described in section 2. Second, results of typical 2-D simulations are shown in section 3. A parametric study showing the upstream plasma conditions and shock regimes in which these oblique whistler waves are emitted, is summarized in section 4. Finally, discussion and conclusions are drawn in section 5.

2. Simulation conditions

A 2-D hybrid simulation code [*Matthews*, 1994] is used, where electrons are considered as a massless, charge neutralizing adiabatic fluid, whereas ions are treated as macro-particles. The hybrid simulation box is 512×128 points with a spatial resolution $\Delta x = 0.1c/\omega_{pi}$ and $\Delta y = 0.2c/\omega_{pi}$, and there are 120 particles per cell in the upstream region. The time step for the particle advance is $\Delta t = 0.01/\omega_{ci}$ whereas the magnetic field is advanced with $\Delta t_B = \Delta t/10$. In these definitions, c is the speed of light, ω_{pi} is the upstream ion plasma frequency, and ω_{ci} is the upstream ion cyclotron frequency. Protons and electrons have initial (upstream) ratios between the particle and magnetic pressures $\beta_i = 0.2$ and $\beta_e = 0.5$, respectively. A small resistivity $\eta = 10^{-4}\mu_0 v_A^2/\omega_{ci}$ is used; here μ_0 is the magnetic permitivity of vacuum and v_A is the upstream Alfvén velocity. The plasma is streaming along x axis with $v_0 = 2v_A$ and interacts with a piston (an infinitely conducting wall). The Mach number of the resulting shock is $M_A \sim 3.6$. Results from the hybrid simulation will also be compared with those obtained from a 2-D PIC electromagnetic simulation [*Lembège and Savoini*, 1992]. For the PIC simulation the plasma box has 6144×256 grids with a spatial resolution $\Delta x = \Delta y = 1/60\,(c/\omega_{pi})$. Number of particles per cell is 4 for each specie and the time step for the particle advance is $\Delta t = 7.5 * 10^{-5}/\omega_{ci}$. The used mass ratio is $m_p/m_e = 400$ and the ratio between the upstream electron plasma and cyclotron frequencies is $\omega_{pe}/\omega_{ce} = 2$. The upstream beta values are $\beta_e = 0.24$ and $\beta_i = 0.15$ for electrons and protons, respectively. The shock is excited by using the magnetic piston method which leads to a Mach number $M_A \sim 5.5$. The plasma parameters and the shock regime are approaching those used in the 2-D hybrid run. For both simulations, the upstream magnetostatic field is directed along y-axis.

3. Simulation Results

Present results issued from 2-D hybrid simulations show that the shock front is quasi-stationary, in the sense that no (expected) periodic self-reformation is evidenced. Instead, the front is mainly dominated by large amplitude whistler waves which are emitted and persist during the whole simulation until the end at $t = 28\omega_{ci}^{-1}$. These waves occur rapidly during the shock build up, over a time range much shorter than a characteristic self-reformation period [*Lembège and Savoini*, 1992]. Typical profile of the main magnetic field component B_y versus x and y is shown in Figure 1a (top). Henceforth the magnetic field is normalized to its upstream value; the simulation x-coordinates for both hybrid and PIC simulations are centered at the approximate shock front location and only the

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region $-3c/\omega_{pi} < x < 3c/\omega_{pi}$ is shown and investigated herein. Main relevant features are: (i) these whistlers are most pronounced in the shock front, (ii) their wave vectors are oblique with respect to both the upstream magnetic field and the shock normal; (iii) they have a strong amplitude reaching a maximum of $\delta B/B \sim 0.7$ such that the foot pattern is not clearly apparent, and (iv) these are approximately phase standing (have zero frequency) in the shock rest frame (the shock front is quasi-stationary). In order to verify whether these unexpected whistlers do not result from any numerical artifact related to the use of hybrid simulation, we have performed corresponding 2-D PIC simulation of shocks in similar plasma and Mach regime conditions. The striking point is that the four main features of these waves mentioned above are retrieved, and again no periodic self-reformation is evidenced. Again, these waves are emitted quickly within a time range much shorter than any characteristic cyclic self-reformation period and persist during the whole simulation time until the end at $t = 7.55\omega_{ci}^{-1}$. The corresponding gray scale plot of B_y component is also shown as a function of x and y in Figure 1b (top); the PIC simulation may be regarded as a blow up of the hybrid one (Figure 1a, dotted lines).

The oblique whistler waves strongly interact with the protons. Figure 1 (bottom) shows the corresponding proton phase space (x, v_x) in the shock rest frame, both for the 2-D hybrid (a) and PIC (b) simulations. Figure 1 (bottom) indeed indicates that the reflected protons are strongly scattered in both the simulations. Some slight differences appear in the phase space due to some additional fluctuations (micro-instabilities) in 2-D PIC simulations which will be analyzed in a further paper.

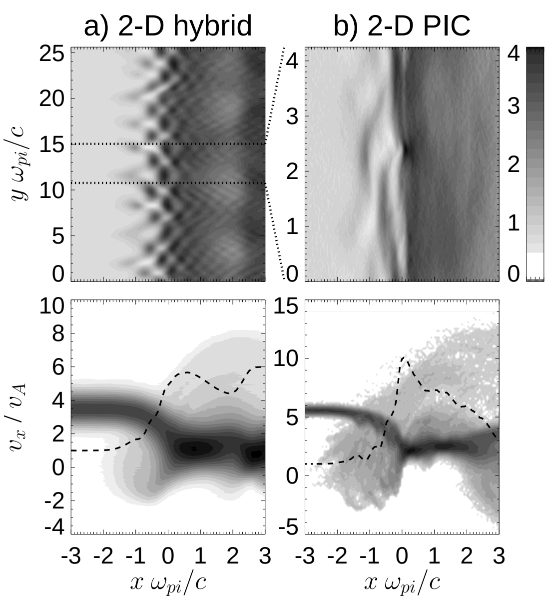

Figure 1. Hybrid (a) and PIC (b) simulation results: (top) Gray scale plots of the compressional component B_y as a function of x and y. The corresponding gray scale is shown in the right;(bottom) Gray scale plots of the proton phase space (x, v_x). The gray scales are arbitrary and independent. Profiles of the corresponding (y-averaged) B_y component are superimposed (dashed curves).

In order to compare more closely the hybrid and PIC simulation results and to analyze in more details the whistler waves, the spatial energy spectrum of fluctuating total field δB in k_x-x and k_y-x diagrams is calculated within the whole simulation box and results are shown only around the shock front region in Figure 2. The spectra have been obtained by performing first the Fast Fourier Transform (FFT) along the (periodic) y-direction and removing $k_y = 0$ modes (Figure 2, bottom panels). Then, a wavelet transform was applied in x-direction to these FFT spectra and averaged over k_y (Figure 2, top panels). Note that the gray scales are arbitrary and independent for each plot; limited dynamics of these gray scales is used herein in order to resolve only the dominant wave activity. The spectra $\delta B^2(x, k_y)$ exhibit a periodicity in x (Figure 2, bottom panels). This feature is owing to a superposition of the two whistler wave packets with opposite signs in k_y: assuming a superposition of two waves $\delta B \propto \cos(k_x x + k_y y) + \cos(k_x x - k_y y)$ we have a modulation of $\langle \delta B^2 \rangle$, which is defined as δB^2 averaged over y-direction, $\langle \delta B^2 \rangle \propto \cos(2k_x x)$.

Combining the previous results, the hybrid and PIC simulations exhibit the same whistler waves with the following main features: (i) the maximum wave energy is centered in the foot region, (ii) the observed spectrum has a maximum around a propagation angle $\theta_{\mathbf{kB}} \sim 75^\circ$ (hybrid) and $\theta_{\mathbf{kB}} \sim 78^\circ$ (PIC) with respect to the upstream magnetostatic field, (iii) these waves have comparable wavelengths $\lambda = 1.1c/\omega_{pi}$ (hybrid) and $0.87c/\omega_{pi}$ (PIC), and are symmetrically emitted with respect to a mirroring around the shock normal, and (iv) k_x wave vectors decrease when moving upstream from the front (a possible signature of a refraction). Additional analysis shows that these waves cannot escape far upstream and may be present in the foot where the reflected protons slow down the mean plasma velocity [and change the whistler dispersion, *Hellinger and Mangeney*, 1997].

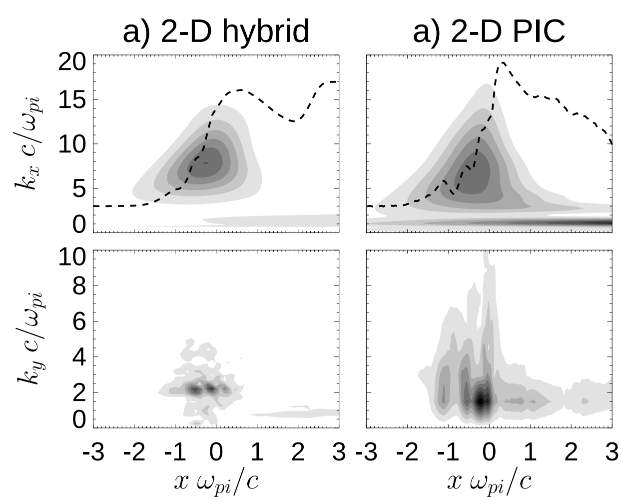

Figure 2. Hybrid (a) and PIC (b) simulation results: Gray scale plots of the spatial energy spectrum of the fluctuating total magnetic field δB^2 as a function of x and k_x (top panels) and x and k_y (bottom panels). The gray scales are arbitrary and independent for each plot. Profiles of the corresponding (y-averaged) B_y component are superimposed (dashed curves).

4. Parametric study

In order to analyse the occurrence conditions of these whistler waves at the front, we have performed a set of 2-D hybrid simulations for different Alfvén Mach numbers M_A and upstream β_i with $N_x = 500$, $N_y = 80$ $\Delta x = 0.06$ and $\Delta y = 0.2$. The duration of these simulations is the same ($t = 10\omega_{ci}^{-1}$) and the resistivity in the generalized Ohm's law is fixed to $\eta = 10^{-3}$. Results are reported in Figure 3, where each circle corresponds to a 2-D hybrid simulation. A frontier clearly appears separating full and empty circles regions. Full circles denote quasi-stationary shocks (no periodic self-reformation) dominated by the emission of oblique whistler waves (Figure 1a). It is important to note that the region where the whistler waves are observed in the 2-D runs is very similar to the region where the shock exhibits the shock reformation in 1-D hybrid simulations [cf. *Hellinger et al.*, 2002, Figure 2]. Empty circles correspond to quasi-stationary shocks (no self-reformation) but without any whistler waves emission: the corresponding region also corresponds to that where no self-reformation is observed in 1-D simulations. Results issued from a few 2-D PIC simulations (not shown here) confirm these features.

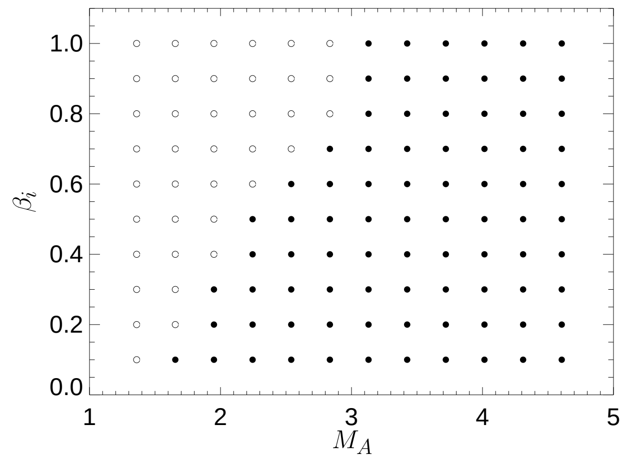

Figure 3. Results issued from 2-D hybrid simulations versus Alfvén Mach numbers M_A and ion upstream β_i, summarizing the occurence (full circles) and absence (empty circles) of oblique whistler waves within the shock front (Figure 1a).

5. Discussion

In this letter, results issued from both 2-D hybrid and 2-D PIC simulations evidence a new behavior of strictly perpendicular shocks propagating in supercritical regime not observed in previous works. Within a wide range of parameters β_i and M_A (Figure 3), the front of 2-D shocks appears to be quasi-stationary (no self-reformation) and are dominated by the emission of large amplitude oblique whistler waves. These waves have maximum intensity in the foot region and are phase-standing in the shock rest frame; they are not able to escape far upstream, stay almost in the foot region and efficiently diffuse the reflected protons (Figure 1).

These waves could not have been observed in most of previous hybrid simulations (with a coarse spatial resolution $\sim c/\omega_{pi}$, since the short-wavelength whistlers require a fine spatial resolution of a small fraction of c/ω_{pi} [similarly to the self-reformation process, cf., *Hellinger et al.*, 2002]. On the other hand, some previous 2-D hybrid simulations show that a structure of strong quasi-perpendicular shocks is dominated by large scale rippling [*Winske and Quest*, 1988; *Lowe and Burgess*, 2003]. Such ripples have not been observed in the present simulations probably because they appear for stronger shocks $M_A \gtrsim 5$ [*Hellinger and Mangeney*, 1997]; moreover, the oblique whistlers occur rapidly, over time scales much shorter than those necessary for these ripples.

The present results depart from those of 2-D PIC simulations by *Lembège and Savoini* [1992] where the self-reformation of perpendicular shock front has been observed, and the resulting shock is nonstationary. This apparent discrepancy is clarified by noting that, in contrast with present results ($m_p/m_e = 400$), a low $m_p/m_e = 42$ has been used at that time. Such a low mass ratio strongly modifies the group velocity of the whistlers [*Krauss-Varban et al.*, 1995] which is undoubtedly very important property for a their generation. Also, when a more realistic mass ratio is used, scale differences between ions and electrons are better differentiated and accessibility to electron scales is more easily identified.

The generating mechanism of the oblique foot whistler waves is not determined yet. One possibility could be their generation by the beam of reflected protons [*Hellinger and Mangeney*, 1997]. However, this mechanism tends to generate waves with a non-zero phase velocity in the shock rest frame through Landau resonance [cf. *Hellinger et al.*, 1996, Figure 1]. Another candidate mechanism which comes in mind could be the lower hybrid instability triggered by cross-field currents supporting the large field gradient at the ramp as proposed in results of *Lembège and Savoini* [1992]. However, the oblique waves are difficult to interpret as the lower hybrid mode in hybrid simulations with the zero electron mass.

Another more attractive possibility is a generation by a three-wave nonlinear process: a decay of the steepened shock wave into daughter whistler waves. The shock may be regarded as a nonlinear composition of a wide range of wave vectors along the shock normal x. Because of the strictly perpendicular propagation of the shock front, linear whistler waves cannot propagate from the front and accumulate inside. These locally reach a nonlinear level above a certain threshold where nonlinear wave decay can take place. The features of the observed whistler waves are qualitatively compatible with a decay of a mother wave with $\mathbf{k}_1$ and ω_1 contained in the shock front ($k_{1y} = 0$, $\omega_1 = 0$ in the shock rest frame) into two daughter waves with $\mathbf{k}_2$ and ω_2, and $\mathbf{k}_3$ and ω_3. The condition of the three-wave interaction appears to be satisfied: $k_{2y} \sim -k_{3y}$ and the two daughter waves have the zero frequency in the shock rest frame so that the condition $\omega_1 = \omega_2 + \omega_3 = 0$ is satisfied. This process is well supported by the stronger steepening at the shock ramp noted above accessible thanks to a finer spatial resolution (hybrid) or to the use of a more realistic mass ratio (PIC). However, this proposed decay process is not a standard three-wave interaction and requires a further investigation.

A last possibility is that these waves are generalized, oblique nonlinear dispersive wave-trains. Recently, *Krasnoselskikh et al.* [2002] proposed a mechanism for generating nonlinear whistler waves from the front of a quasi-perpendicular shock. Under certain conditions, the shock front is highly nonstationary and characterized by an emission of these nonlinear whistler waves and by a build up of a new ramp at the edge of the emitted whistler waves. This is in strong contrast with the present quasi-stationary shock structure. However, that calculation was limited to an obliquely propagating shock and does not include any contribution reflected protons; in addition, 2-D effects allowing the emission of the oblique whistler waves are excluded. Therefore, no direct comparison between our simulations and the work of *Krasnoselskikh et al.* [2002] can be performed. Considering the present status of these works, we expect that the direction of the shock normal is another important parameter that determines when a shock front is quasi-stationary or nonstationary.

On the observational side, strong whistler waves are observed in the shock foot region [*Krasnoselskikh et al.*, 1991; *Balikhin et al.*, 1997], sometimes with signatures compatible with the shock self-reformation [*Walker et al.*, 1999; *Lobzin et al.*, 2007]. At the present time, a direct comparison is not possible yet since these experimental observations concern quasi-perpendicular shocks, whereas the present simulation results are obtained for strictly perpendicular shocks.

In summary, present 2-D hybrid and 2-D PIC simulations surprisingly show that the large amplitude whistler waves emitted by the perpendicular shock front stabilize the front itself, persist in time and inhibit any periodic shock self-reformation. Further theoretical and simulation work is under active investigation in order to understand the generation and properties of the oblique whistler waves, and to compare in detail experimental observations and numerical simulation results.

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