Lecture 4: Polarization of light

- Monochromatic wave
  - Concept of polarized light
  - Jones matrix formalism
    - Polarization states of light
    - Optical polarization elements
    - Application: Lyot filter
- General polychromatic wave
  - Concept of partially polarized and non polarized light

Monochromatic waves

$$E_x = A_x \cos(\omega t - kz)$$
$$E_y = A_y \cos(\omega t - kz + \delta)$$

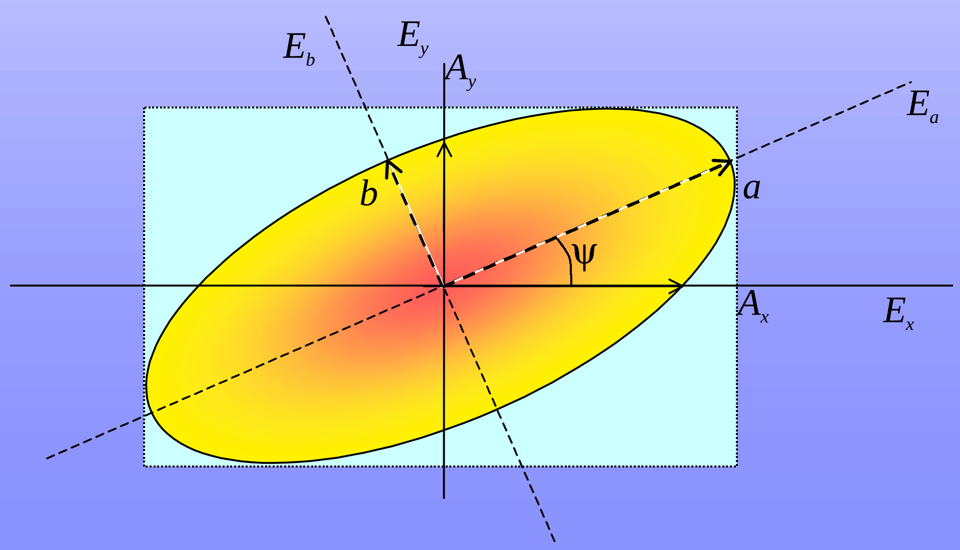
$$E_a = a \cos(\omega t - kz)$$
$$E_b = \pm b \sin(\omega t - kz)$$

$$\left(\frac{E_x}{A_x}\right)^2 - 2\frac{E_x E_y}{A_x A_y}\cos\delta + \left(\frac{E_y}{A_y}\right)^2 = \sin^2\delta$$

$$\left(\frac{E_a}{a}\right)^2 + \left(\frac{E_b}{b}\right)^2 = 1$$

$$\operatorname{tg}\chi = \pm\frac{b}{a}$$

$A_x/A_y, \delta$



ψ, χ

Monochromatic waves

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$$\left(\frac{E_a}{a}\right)^2 + \left(\frac{E_b}{b}\right)^2 = 1$$

Linear polarization

$$\delta = 0, \pi$$

$$\frac{E_y}{E_x} = \pm\frac{A_y}{A_x}$$

$$b = 0$$

$$\psi$$

Monochromatic waves

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$$\left(\frac{E_a}{a}\right)^2 + \left(\frac{E_b}{b}\right)^2 = 1$$

Circular polarization

$$\delta = \pm\frac{\pi}{2}$$
$$A_x = A_y$$

$$\chi = \pm\frac{\pi}{4}$$
$$a = b$$

"–" left-, "+" right-handed

"–" right-, "+" left-handed

Monochromatic waves

$$E_x = A_x \cos(\omega t - kz)$$
$$E_y = A_y \cos(\omega t - kz + \delta)$$

$$E_a = a\cos(\omega t - kz)$$
$$E_b = \pm b\sin(\omega t - kz)$$

$$\left(\frac{E_x}{A_x}\right)^2 - 2\frac{E_x E_y}{A_x A_y}\cos\delta + \left(\frac{E_y}{A_y}\right)^2 = \sin^2\delta$$

$$\left(\frac{E_a}{a}\right)^2 + \left(\frac{E_b}{b}\right)^2 = 1$$

Elliptic polarization

$\delta \; (-\pi, \pi)$

A_x / A_y

$\psi \; (0, 2\pi)$

$\chi \; (-\pi/4, \pi/4)$

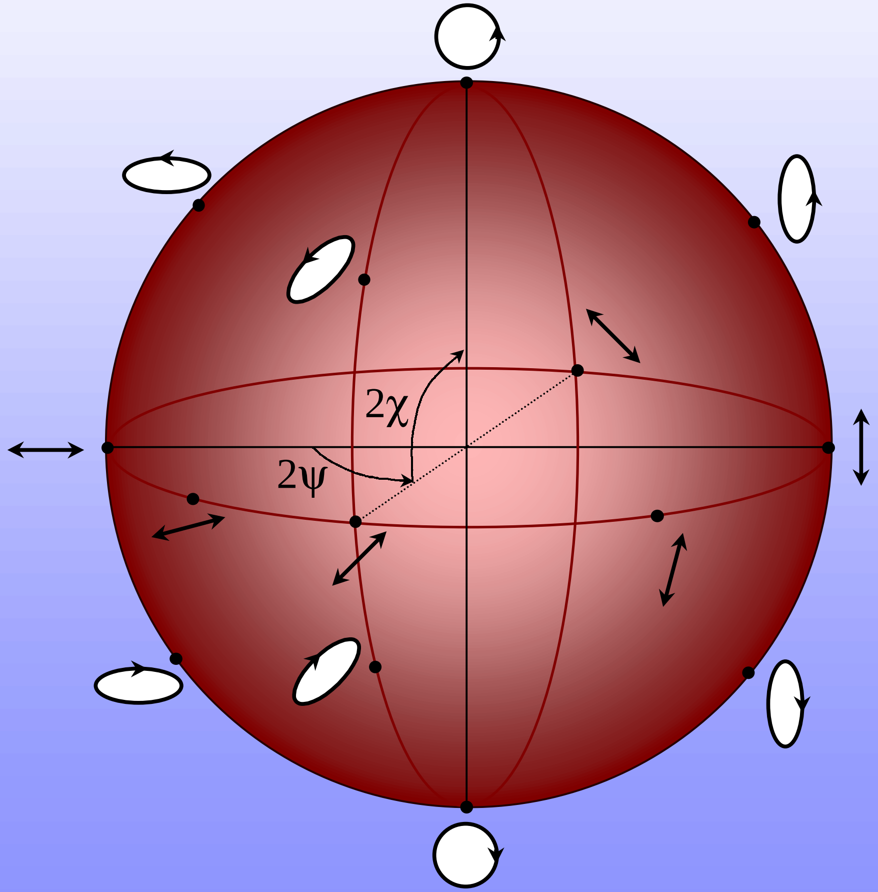
$\delta < 0 \ldots$ left

$\delta > 0 \ldots$ right

$\chi > 0 \ldots$ left

$\chi < 0 \ldots$ right

Graphic representation: Poincaré sphere



Mathematical representation: Jones matrices

$$E_x = A_x \cos(\omega t - kz + \delta_x)$$
$$E_y = A_y \cos(\omega t - kz + \delta_y)$$

Representation of a polarization state by a complex Jones vector:

$$\boldsymbol{J} = \begin{pmatrix} A_x e^{i\delta_x} \\ A_y e^{i\delta_y} \end{pmatrix}$$

$$\boldsymbol{J}^* \cdot \boldsymbol{J} = J_x^* J_x + J_y^* J_y = 1$$

$$\boldsymbol{J} \equiv \boldsymbol{J}\ e^{i\delta}$$

Polarization states: Jones vectors

Linear:

$$\boldsymbol{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \boldsymbol{y} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \boldsymbol{J}(\psi) = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix}$$

Circular:

$$\boldsymbol{R} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \qquad \boldsymbol{L} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

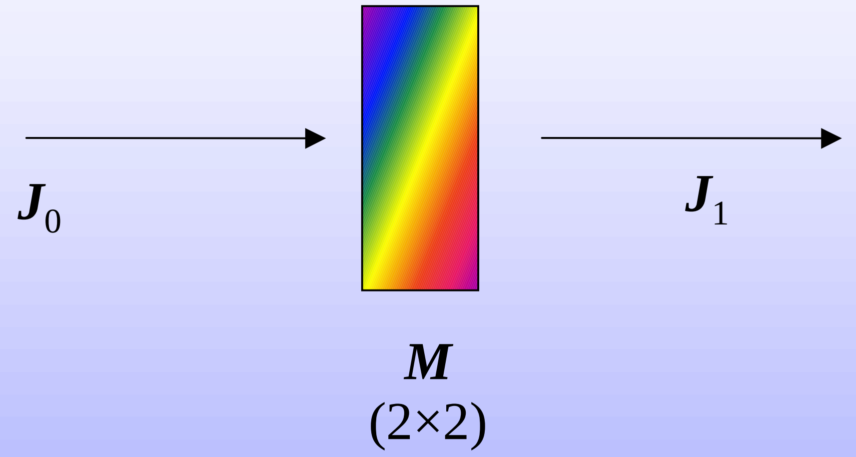
Elliptic:

$$\boldsymbol{J}(\psi = 0, \chi) = \begin{pmatrix} \cos\chi \\ \sin\chi \, e^{\pm i\frac{\pi}{2}} \end{pmatrix}$$

$$E_a = a\cos(\omega t - kz)$$
$$E_b = \pm b\sin(\omega t - kz)$$

$$\boldsymbol{J}(\psi, \chi) = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \cos\chi \\ \sin\chi \, e^{\pm i\frac{\pi}{2}} \end{pmatrix} = \begin{pmatrix} \cos\psi\cos\chi \mp i\sin\psi\sin\chi \\ \sin\psi\cos\chi \pm i\cos\psi\sin\chi \end{pmatrix}$$

Optical elements: Jones matrices



$$J_1 = M \cdot J_0$$

Optical elements: polarizers

$$\boldsymbol{P}_x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad \mathbf{P}_y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

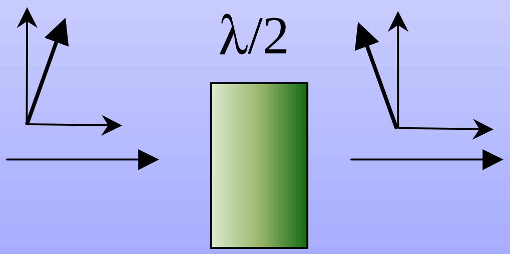
$$\mathbf{P}_\psi = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} = \begin{pmatrix} \cos^2\psi & \sin\psi\cos\psi \\ \sin\psi\cos\psi & \sin^2\psi \end{pmatrix}$$

Optical elements: phase retardators

$$\mathbf{C}_{\delta,x} = \begin{pmatrix} e^{i\delta} & 0 \\ 0 & 1 \end{pmatrix} \qquad \mathbf{C}_{\delta,y} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix}$$

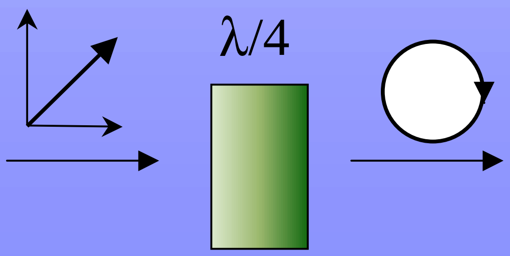
Half-wave plate: $\delta = \pi$: (polarization rotation)

$$\mathbf{C}_{\pi,y} \cdot \mathbf{J}(\psi) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix} = \begin{pmatrix} \cos\psi \\ -\sin\psi \end{pmatrix}$$



Quarter-wave plate: $\delta = \pi/2$: (circular polarization preparation)

$$\mathbf{C}_{\pi/2,y} \cdot \mathbf{J}(45^\circ) = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$



Optical activity

$$\mathbf{A} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Polychromatic waves concept of non-polarized light

$$E_x(t) = A_x(t) e^{i(\omega t - kz)}$$
$$E_y(t) = A_y(t) e^{i(\omega t - kz + \delta)}$$

Projection of ***E*** to a direction θ (polarizer):

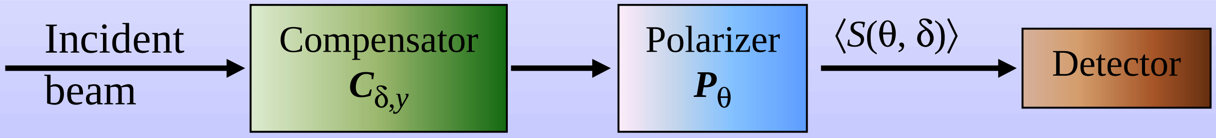
$$E(t,\theta) = \left[A_x(t)\cos\theta + A_y(t)\sin\theta\, e^{i\delta}\right] e^{i(\omega t - kz)}$$

Light intensity:

$$\langle S(\theta,\delta)\rangle \propto \langle E(t,\theta)E^*(t,\theta)\rangle = \langle A_x(t)A_x^*(t)\rangle \cos^2\theta + \langle A_y(t)A_y^*(t)\rangle \sin^2\theta +$$
$$\left[\langle A_x(t)A_y^*(t)\rangle e^{-i\delta} + \langle A_y(t)A_x^*(t)\rangle e^{i\delta}\right]\sin\theta\cos\theta$$

Correlation between components

$$\langle S(\theta,\delta)\rangle \propto \langle E(t,\theta)E^*(t,\theta)\rangle = \langle A_x(t)A_x^*(t)\rangle \cos^2\theta + \langle A_y(t)A_y^*(t)\rangle \sin^2\theta +$$
$$\left[\langle A_x(t)A_y^*(t)\rangle e^{-i\delta} + \langle A_y(t)A_x^*(t)\rangle e^{i\delta}\right]\sin\theta\cos\theta$$



- No correlation between A_x and A_y:
  - cross-correlation terms vanish
  - polarization is only “locally” elliptic — jumps of phase and amplitude
  - Non polarized or partially polarized light
- Circular polarization
  - sum of the cross-correlation terms vanish
  - non-vanishing correlation is found when δ is being varied

Degree of polarization

$$P = \frac{S_{\max} - S_{\min}}{S_{\max} + S_{\min}}$$

S … incident intensity

S_{max} … absolute maximum found in our experiment

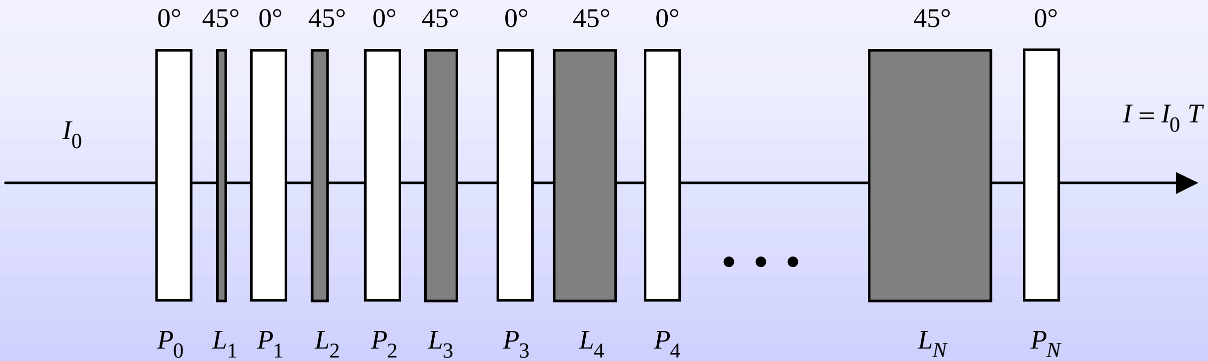
S_{min} … absolute minimum found in our experiment

Polarized light: $S_{min} = 0$, $S_{max} = S$. $P = 1$

Non-polarized light: $S_{min} = S_{max} = S/2$. $P = 0$

$$P = \frac{S_P}{S_P + S_{NP}} = \frac{S_P}{S}$$

Lyot filter



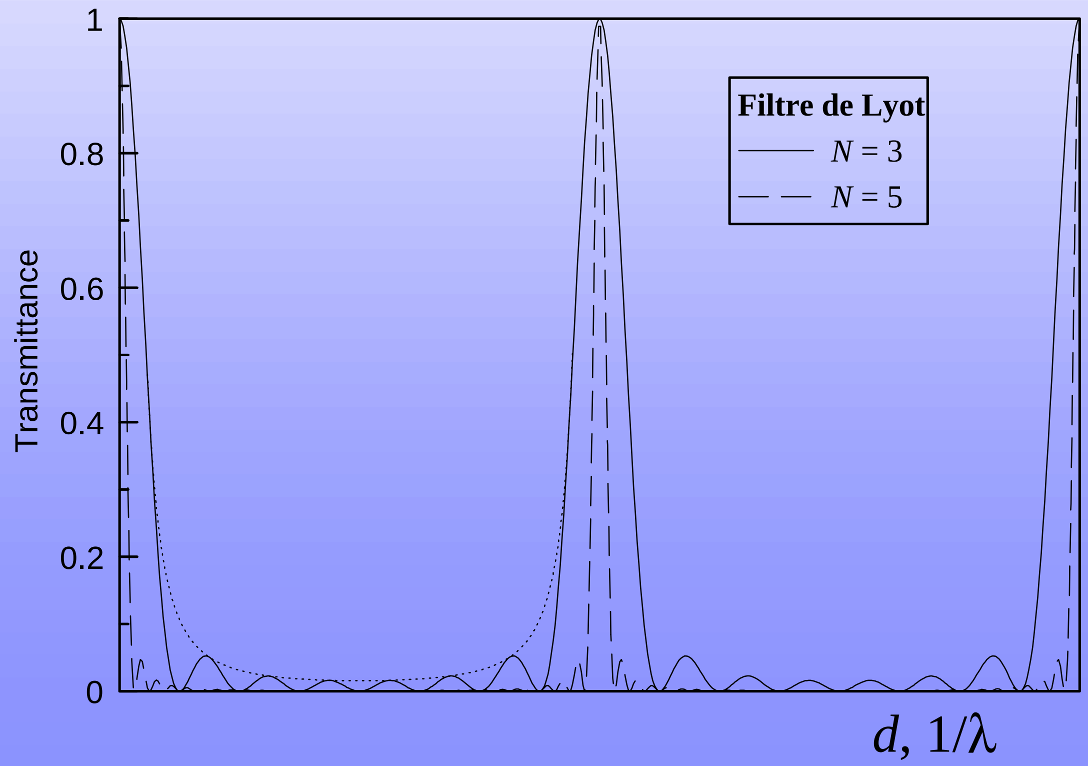
$$\delta_j = \frac{2\pi}{\lambda} \Delta n\, 2^{j-1} d = 2^{j-1}\delta$$

$$L_n = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} e^{i\delta_n} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} c & -s \\ s & c \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(1+e^{i\delta_n}) & \frac{1}{2}(1-e^{i\delta_n}) \\ \frac{1}{2}(1-e^{i\delta_n}) & \frac{1}{2}(1+e^{i\delta_n}) \end{pmatrix} \qquad P_n = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$P_N L_N \dots P_1 L_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \dfrac{(1+e^{i\delta})(1+e^{i2\delta})\dots\left(1+e^{i2^{N-1}\delta}\right)}{2^N} \\ 0 \end{pmatrix}$$

Lyot filter

$$T = \frac{1}{2^{2N}} \frac{\sin^2\left(2^N \frac{\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)}$$



Finesse $\ldots 2^N$

Contrast $\ldots 1/2^{2N}$