Lecture 9: Birefringent optical elements

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- Polarizers
- Phase retarders
- Applications: Jones matrices

- Optical activity

Basics

Birefringent optical elements:

Plates or prisms which can change the polarization state of a beam in a well-defined way

Calcite ($CaCO_3$):

$n_o = 1.662, \qquad n_e = 1.488$

large birefringence ($n_e - n_o = -0.174$): polarizers

Quartz (SiO_2):

$n_o = 1.546, \qquad n_e = 1.555$

smaller birefringence ($n_e - n_o = +0.009$): phase retarders

Polarizers

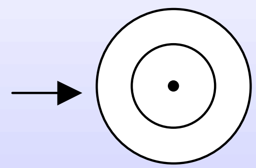
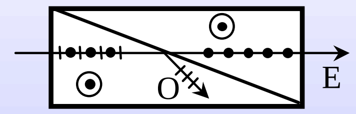
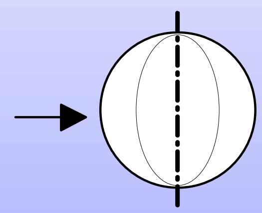
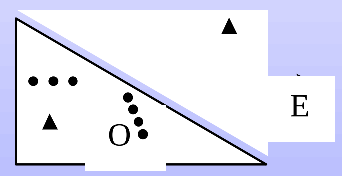
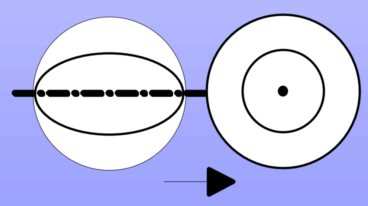
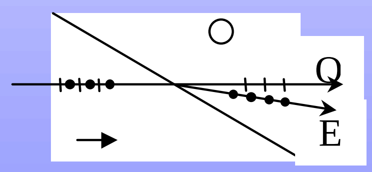
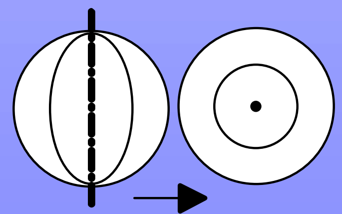
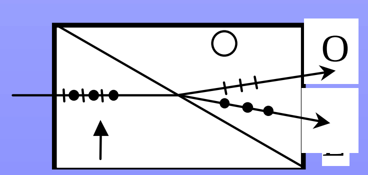
couple of prisms (working on the total reflection principle) separated by

- Canadian balm (n = 1.54)
- Air layer

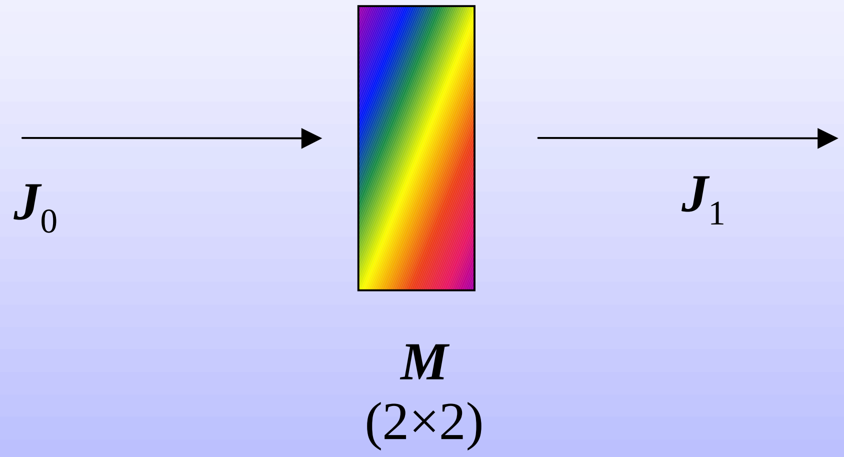
Dichroic (sheet) polarizers

- Different absoption coefficient for both polarization

Examples of polarizers

Name	normal surface	polarizer layout	comment
Glan-Thompson			❑ Large range of incidence angles (≈ 20°)
Glan-Taylor			❑ Small range of incidence angles (≈ 5°) ❑ Brewster angle incidence ❑ Possible use: UV, high power
Rochon			❑ Separation of the O and E beams (typically 10°)
Wollaston			❑ Symmetric separation of the O et E beams (typically 20°)

Optical elements: Jones matrices



$$\boldsymbol{J}_1 = \boldsymbol{M} \cdot \boldsymbol{J}_0$$

Jones matrices of polarizers

$$\boldsymbol{P}_x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad \mathbf{P}_y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{P}_\psi = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} = \begin{pmatrix} \cos^2\psi & \sin\psi\cos\psi \\ \sin\psi\cos\psi & \sin^2\psi \end{pmatrix}$$

Parallel and perpendicular polarizations:

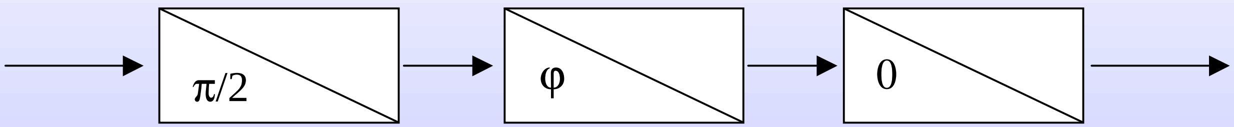
$$\begin{pmatrix} c^2 & sc \\ sc & s^2 \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} = \begin{pmatrix} c \\ s \end{pmatrix} \qquad \begin{pmatrix} c^2 & sc \\ sc & s^2 \end{pmatrix} \begin{pmatrix} -s \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Crossed polarizers (angles φ et φ + π/2):

$$\begin{pmatrix} c^2 & sc \\ sc & s^2 \end{pmatrix} \begin{pmatrix} s^2 & -sc \\ -sc & c^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Jones matrices of polarizers

Sequence of 3 polarizers (φ-polarizer between two crossed polarizers):



$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} c^2 & sc \\ sc & s^2 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & cs \\ 0 & 0 \end{pmatrix}$$

If this sequence is applied on a linearly polarized beam:

$$\begin{pmatrix} 0 & cs \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} cs \\ 0 \end{pmatrix}$$

Maximum transmission for $\varphi = \pi/4$, no transmission for $\varphi = 0, \pi/2$

Phase retardation plates

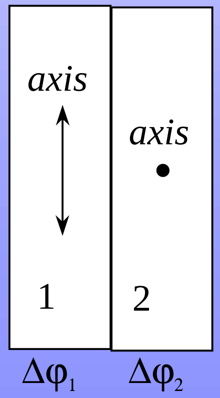
Phase retardation (due to different optical path) for 2 orthogonal linear polarizations:

$$\Delta\varphi = \frac{2\pi\,(n_e - n_o)d}{\lambda}$$

Example: for a quarter-wave plate ($\Delta\varphi = \pi/2$) we need:

$d = 15.2$ µm pour $\lambda = 546.1$ nm

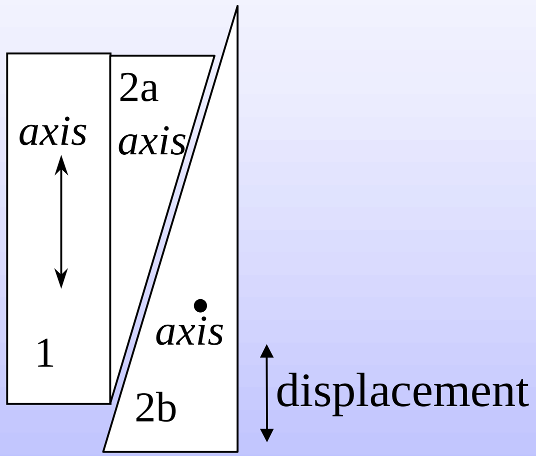
- higher order thick plates ($\Delta\varphi = 5\pi/2, 9\pi/2\ldots$)
- 2 plates in optical contact with mutually perpendicular orientation and with a slightly different thickness



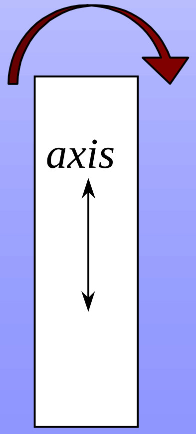
$$\Delta\varphi = \Delta\varphi_1 - \Delta\varphi_2 = \frac{2\pi\,(n_e - n_o)(d_1 - d_2)}{\lambda}$$

Compensator

Babinet-Soleil:



Berek:



Jones matrices of phase retarders

$$\begin{pmatrix} e^{i\delta} & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} c & s \\ -s & c \end{pmatrix}\begin{pmatrix} e^{i\delta} & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} c & -s \\ s & c \end{pmatrix} = \begin{pmatrix} c^2 e^{i\delta} + s^2 & cs\left(1 - e^{i\delta}\right) \\ cs\left(1 - e^{i\delta}\right) & s^2 e^{i\delta} + c^2 \end{pmatrix}$$

Phase retardation plate between 2 polarizers:

$$\begin{pmatrix} s^2 & -sc \\ -sc & c^2 \end{pmatrix}\begin{pmatrix} e^{i\delta} & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} c^2 & sc \\ sc & s^2 \end{pmatrix}$$

The light does not pass for $\varphi = 0, \pi/2$

Maximum transmission for $\varphi = \pi/4$ (optical axis of the plate shows 45° with respect to the polarizing directions)

Modulator: phase retarder between 2 polarizers

Optical axis of the plate shows 45° with respect to the polarizing directions

$$\begin{pmatrix} s^2 & -sc \\ -sc & c^2 \end{pmatrix}\begin{pmatrix} e^{i\delta} & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} c^2 & sc \\ sc & s^2 \end{pmatrix}\begin{pmatrix} c \\ s \end{pmatrix} = cs\begin{pmatrix} s(e^{i\delta}-1) \\ c(1-e^{i\delta}) \end{pmatrix} = \frac{\sqrt{2}}{4}\begin{pmatrix} e^{i\delta}-1 \\ 1-e^{i\delta} \end{pmatrix}$$

Transmitted light intensity:

$$I = \frac{1}{8}\left(\left(e^{i\delta}-1\right)\left(e^{-i\delta}-1\right)+\left(1-e^{i\delta}\right)\left(1-e^{-i\delta}\right)\right) = \frac{1}{2}\left(1-\cos\delta\right)$$

The dephasing δ can have 2 parts

- a large one which is constant (close to $\delta_0 = \pi/2$ — quarter-wave plate)
- a small one which is variable ($\delta_1 = \Gamma \sin \omega_m t$, $\Gamma << 1$):

$$I = \frac{1}{2}\left(1-\cos(\delta_0+\delta_1)\right) = \frac{1}{2}\left(1+\sin(\Gamma \sin \omega_m t)\right) \approx \frac{1}{2}\left(1+\Gamma \sin \omega_m t\right)$$