PAIRWISE FUZZY IRRESOLUTE MAPPINGS

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Abstract. In this paper the concepts of fuzzy irresolute and fuzzy presemiopen mappings due to Yalvac [12] are generalized to fuzzy bitopological spaces and their basic properties and characterizations are studied.

Keywords: fuzzy bitopological spaces, (i,j)-fuzzy semiopen, (i,j)-fuzzy semiclosed, (i,j)-semineighbourhood, (i,j)-semi-Q-neighbourhood

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1. Introduction

The fundamental concept of a fuzzy set introduced by Zadeh [13] in 1965, provides a natural foundation for building new branches of fuzzy mathematics. In 1968 Chang [2] introduced the concept of fuzzy topological spaces as a generalization of topological spaces. Since then many topologists have contributed to the theory of fuzzy topological spaces. Today fuzzy topology has been firmly established as one of the basic disciplines of fuzzy mathematics. In 1987 Kandil [4] introduced the concept of fuzzy bitopological spaces as an extension of fuzzy topological spaces and a generalization of bitopological spaces. Recently the authors of the paper [10] extended the concepts of fuzzy semiopen sets, fuzzy semicontinuous and fuzzy semiopen mappings due to Azad [1] to fuzzy bitopological spaces. In the present paper we introduce and study the concepts of pairwise fuzzy irresolute and pairwise fuzzy presemiopen mappings in fuzzy bitopological spaces.

Let X be a nonempty set and $I = [0,1]$. A fuzzy set in X is a mapping from X into I. The null fuzzy set 0 is the mapping from X into I which assumes only the value 0, and the fuzzy set 1 is a mapping from X into I which takes value 1 only. The union $\bigcup A_\alpha$ (intersection $\bigcap A_\alpha$) of a family $\{A_\alpha : \alpha \in \Lambda\}$ of fuzzy sets of X is defined to be the mapping $\operatorname{Sup} A_\alpha$ ($\operatorname{Inf} A_\alpha$, respectively). A fuzzy set A of X is contained in a fuzzy set B of X, denoted by $A \leqslant B$, if and only if $A(x) \leqslant B(x)$ for each $x \in X$. A fuzzy point x_β in X defined by

$$x_\beta(y) = \begin{cases} \beta & (\beta \in (0,1]) \text{ for } y = x \\ 0 & \text{otherwise} \end{cases} \qquad (y \in X);$$

x and β are respectively called the support and the value of x_β. A fuzzy point $x_\beta \in A$ iff $\beta \leqslant A(x)$. A fuzzy set A is the union of all fuzzy points which belong to A. A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A, denoted by $x_\beta q A$, if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi-coincident with B, denoted by AqB, if and only if there exists $x \in X$ such that $A(x) + B(x) > 1$. $A \leqslant B$ if and only if (AqB^C) [7].

Let $f \colon X \to Y$ be a mapping. If A is a fuzzy set of X, then $f(A)$ is a fuzzy set of X defined by

$$f(A)\ (y) = \begin{cases} \operatorname{Sup} A(x) & \text{if } \ f^{-1}(y) \neq 0 \\ x \in f^{-1}(y) & \\ 0 & \text{otherwise} \end{cases} \qquad (y \in Y).$$

If B is a fuzzy set of Y, then $f^{-1}(B)$ is a fuzzy set of X defined by $f^{-1}(B)(x) = B(f(x))$ for each $x \in X$.

A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0 and 1 belong to τ and τ is closed with respect to arbitrary union and finite intersection. The members of τ are called τ-fuzzy open sets and their complements are τ-fuzzy closed sets. For a fuzzy set A, the closure of A (denoted by $\tau\text{-cl}(A)$) is the intersection of all τ-fuzzy closed super-sets of A and the interior of A (denoted by $\tau\text{-}\operatorname{int}(A)$) is the union of all τ-fuzzy open subsets of A.

A mapping $f \colon (X,\tau) \to (X^*,\tau^*)$ is said to be fuzzy contiuous (fuzzy open) if the inverse image (image) of every fuzzy open set in X^* (in X) is fuzzy open in X (in X^*).

Definition 2.1. A system (X,τ_1,τ_2) consisting of a fuzzy set X with two fuzzy topologies τ_1 and τ_2 on X is called a *fuzzy bitopological space* [4].

Definition 2.2. A fuzzy set A in a fuzzy bitopological space (X, τ_1, τ_2) is called:

(a) (i,j)-*fuzzy semiopen* [10] if there exists a τ_i-fuzzy open set U such that $U \leqslant A \leqslant \tau_j\text{-}\operatorname{cl}(U)$;

(b) (i,j)-*fuzzy semiclosed* [10] if there exists a τ_i-fuzzy closed set F such that $\tau_j\text{-}\operatorname{int}(F) \leqslant A \leqslant F$.

Definition 2.3. A fuzzy set A of a fuzzy bitopological space (X, τ_1, τ_2) is said to be an (i,j)-*semineighbourhood* [10] (an (i,j)-*semi-Q-neighbourhood*) [10] of a fuzzy point x_β of X if there exists a (i,j)-fuzzy semiopen set O such that $x_\beta \in O \leqslant A$ (resp. $x_\beta q O \leqslant A$).

Definition 2.4. Let (X, τ_1, τ_2) be a fuzzy bitopological space. The (i,j)-*semiclosure* [10] (denoted by (i,j)-scl) and the (i,j)-*semiinterior* [10] (denoted by (i,j)-sint) of a fuzzy set A are defined respectively as follows:

$$(i,j)\text{-scl}(A) = \inf\{B \colon B \geqslant A,\ B \text{ is } (i,j)\text{-fuzzy semiclosed}\}$$
$$(i,j)\text{-sint}(A) = \sup\{B \colon B \leqslant A,\ B \text{ is } (i,j)\text{-fuzzy semiopen}\}$$

Definition 2.5. A mapping f from a fuzzy bitoplogical space (X, τ_1, τ_2) to a fuzzy bitopological space $(X^*, \tau_1^*, \tau_2^*)$ is *pairwise fuzzy continuous* [10] (*pairwise fuzzy open* [10]) if $f \colon (X, \tau_1) \to (X^*, \tau_1^*)$ and $f \colon (X, \tau_2) \to (X^*, \tau_2^*)$ are fuzzy continuous (fuzzy open).

Definition 2.6. A mapping f from a fuzzy bitopological space (X, τ_1, τ_2) to a fuzzy bitopological space $(X^*, \tau_1^*, \tau_2^*)$ is *pairwise fuzzy semicontinuous* [10] if the inverse image of every τ^*-fuzzy open set in X^* is (i,j)-fuzzy semiopen in X.

Throughout this paper $i, j = 1, 2$ where $i = j$ and if P is any fuzzy topological property then τ_i-P (τ_j-P) denotes the property P with respect to the fuzzy topology τ_i (τ_j).

3. Pairwise fuzzy irresolute mappings

Definition 3.1. A mapping f from a fuzzy bitopological space (X, τ_1, τ_2) to a fuzzy bitopological space $(X^*, \tau_1^*, \tau_2^*)$ is *pairwise fuzzy irresolute* if the inverse image of every (i,j)-fuzzy semiopen set in X^* is (i,j)-fuzzy semiopen in X.

Remark 3.1. The concepts of pairwise fuzzy irresolute and pairwise fuzzy continuous mappings are independent. Indeed,

Example 3.1. Let $X = \{x, y, z\}, X^* = \{a, b\}$ and $U \subset X, V \subset X^*$ be fuzzy sets defined as follows:

$$U(x) = 0.3, \qquad U(y) = 0.5, \qquad U(z) = 0.4;$$
$$V(a) = 0.4, \qquad V(b) = 0.6.$$

Let $\tau_1 = \{0, U, 1\}$, $\tau_2 = \{0, 1\}$ and $\tau_1^* = \{0, V, 1\}$, $\tau_2^* = \{0, 1\}$. Then the mapping $f\colon (X, \tau_1, \tau_2) \to (X^*, \tau_1^*, \tau_2^*)$ defined by $f(x) = a$, $f(y) = f(z) = b$ is pairwise fuzzy irresolute but not pairwise fuzzy continuous.

Example 3.2. Let $X = \{x, y\}, X^* = \{a, b\}$ and let $U \subset X, V \subset X, W \subset X^*$ be fuzzy sets defined as follows:

$$\begin{aligned} U(x) &= 0.4, & U(y) &= 0.7;\\ V(x) &= 0.2, & V(y) &= 0.3;\\ W(a) &= 0.4, & W(b) &= 0.7. \end{aligned}$$

Let $\tau_1 = \{0, U, 1\}$, $\tau_2 = \{0, V, 1\}$ and $\tau_1^* = \{0, W, 1\}$, $\tau_2^* = \{0, 1\}$. Then the mapping $f\colon (X, \tau_1, \tau_2) \to (X^*, \tau_1^*, \tau_2^*)$ defined by $f(x) = a$, $f(y) = b$ is pairwise fuzzy continuous but not pairwise fuzzy irresolute.

Remark 3.2. Every pairwise fuzzy irresolute mapping is pairwise fuzzy semicontinuous but the converse need not be true since the mapping $f\colon (X, \tau_1, \tau_2) \to (X^*, \tau_1^*, \tau_2^*)$ from Example 3.2 is pairwise fuzzy semicontinuous but not pairwise fuzzy irresolute.

Theorem 3.1. *Let (X, τ_1, τ_2) and $(X^*, \tau_1^*, \tau_2^*)$ be fuzzy bitopological spaces and $f\colon (X, \tau_1, \tau_2) \to (X^*, \tau_1^*, \tau_2^*)$. Then the following conditions are equivalent:*

(a) *f is pairwise fuzzy irresolute;*

(b) *for every fuzzy point x_β of X and every (i, j)-fuzzy semiopen set B in X^* such that $f(x_\beta) \in B$, there is an (i, j)-fuzzy semiopen set A in X such that $x_\beta \in A$ and $f(A) \leqslant B$;*

(c) *for every (i, j)-fuzzy semiclosed set B in X^*, $f^{-1}(B)$ is (i, j)-fuzzy semiclosed in X;*

(d) *for every fuzzy point x_β of X and every (i, j)-semineighbourhood B in X^* of $f(x_\beta)$, $f^{-1}(B)$ is an (i, j)-semineighbourhood of $x_\beta \in X$;*

(e) *for every fuzzy point x_β of X and every (i, j)-semineighbourhood B in X^* of $f(x_\beta)$ there is an (i, j)-semineighbourhood O in X of x_β such that $f(O) \leqslant B$;*

(f) *for every fuzzy point x_β of X and every (i, j)-fuzzy semiopen set B such that $f(x_\beta) q B$ there is an (i, j)-fuzzy semiopen set A in X such that $x_\beta q A$ and $f(A) \leqslant B$;*

(g) *for every fuzzy point x_β of X and every (i, j)-semi-Q-neighbourhood B in X^* of $f(x_\beta)$, $f^{-1}(B)$ is an (i, j)-semi-Q-neighbourhood of x_β in X;*

(h) *for every fuzzy point x_β of X and every (i, j)-semi-Q-neighbourhood B of $f(x_\beta)$ in X^*, there is an (i, j)-semi-Q-neighbourhood O of x_β such that $f(O) \leqslant B$;*

(i) *$f((i, j)\text{-}\mathrm{scl}(A)) \leqslant (i, j)\text{-}\mathrm{scl}\, f(A)$, for every fuzzy set A of X;*

(j) *$(i, j)\text{-}\mathrm{scl}(f^{-1}(B)) \leqslant f^{-1}((i, j)\text{-}\mathrm{scl}(B))$, for every fuzzy set B of X^*;*

(k) $f^{-1}((i,j)\text{-}\operatorname{sint}(B)) \leqslant (i,j)\text{-}\operatorname{sint} f^{-1}(B)$, *for every fuzzy set B of X^*.*

Proof. (a) $\Rightarrow$ (b). Let x_β be a fuzzy point of X and B a fuzzy semiopen set in X^* such that $f(x_\beta) \in B$. Put $A = f^{-1}(B)$. Then by (a) A is an (i,j)-fuzzy semiopen set such that $x_\beta \in A$ and $f(A) \leqslant B$.

(b) $\Rightarrow$ (a). Let B be an (i,j)-fuzzy semiopen set in X^*. Let $x_\beta \in f^{-1}(B)$. Then $f(x_\beta) \in B$. Now by (b) there is an (i,j)-fuzzy semiopen set A in X such that $x_\beta \in A$ and $f(A) \leqslant B$. Then $x_\beta \in A \leqslant f^{-1}(B)$. Hence by Theorem 3.4 [10] $f^{-1}(B)$ is (i,j)-fuzzy semiopen in X.

(a) $\Leftrightarrow$ (c). Obvious.

(a) $\Rightarrow$ (d). Let x_β be a fuzzy point of X and let B be an (i,j)-neighbourhood of $f(x_\beta)$. Then there is an (i,j)-fuzzy semiopen set O^* such that $f(x_\beta) \in O^* \leqslant B$. Now $f^{-1}(O^*)$ is (i,j)-fuzzy semiopen in X, because f is fuzzy irresolute and $x_\beta \in f^{-1}(O^*) \leqslant f^{-1}(B)$. Thus $f^{-1}(B)$ is an (i,j)-semineighbourhood of x_β in X.

(d) $\Rightarrow$ (e). Let x_β be a fuzzy point of X and let B be an (i,j)-semineighbourhood $f(x_\beta)$. Then $O = f^{-1}(B)$ is an (i,j)-semineighbourhood of x_β and $f(O) = f(f^{-1}(B)) \leqslant B$.

(e) $\Rightarrow$ (b). Let x_β be a fuzzy point of X and let B be an (i,j)-fuzzy semiopen set containing $f(x_\beta)$. Then B is an (i,j)-semineighbourhood of $f(x_\beta)$, so there is an (i,j)-semineighbourhood O of x_β of X such that $x_\beta \in O$ and $f(O) \leqslant B$. Therefore there exists an (i,j)-fuzzy semiopen set A in X such that $x_\beta \in A \leqslant O$. Clearly $f(A) \leqslant f(O) \leqslant B$.

(a) $\Rightarrow$ (f). Let x_β be a fuzzy point of X and B be an (i,j)-fuzzy semiopen set in X^* such that $f(x_\beta)qB$. Let $A = f^{-1}(B)$, then A is (i,j)-fuzzy semiopen in X and $x_\beta qA$ and $f(A) = f(f^{-1}(B)) \leqslant B$.

(f) $\Rightarrow$ (g). Let x_β be a fuzzy point of X and B an (i,j)-semi-Q-neighbourhood of $f(x_\beta)$. Then there exists an (i,j)-fuzzy semiopen set O^* in X^* such that $f(x_\beta)qO^* \leqslant B$. By hypothesis there is an (i,j)-fuzzy semiopen set A in X such that $x_\beta qA$ and $f(A) \leqslant O^*$. Thus $x_\beta qA \leqslant f^{-1}(O^*) \leqslant f^{-1}(B)$. Hence $f^{-1}(B)$ is an (i,j)-semi-Q-neighbourhood of x_β.

(g) $\Rightarrow$ (h). Let x_β be a fuzzy point of X and B an (i,j)-semi-Q-neighbourhood of $f(x_\beta)$ in X^*. Then $O = f^{-1}(B)$ is an (i,j)-semi-Q-neighbourhood of x_β and $f(O) \leqslant f(f^{-1}(B)) \leqslant B$.

(h) $\Rightarrow$ (f). Let x_β be a fuzzy point of X and B an (i,j)-fuzzy semiopen set such that $f(x_\beta)qB$. Then B is an (i,j)-semi-Q-neighbourhood of $f(x_\beta)$. So there is an (i,j)-semi-Q-neighbourhood O of x_β such that $f(O) \leqslant B$. Therefore there exists an (i,j)-fuzzy semiopen set A in X such that $x_\beta qA \leqslant O$. Hence $x_\beta qA \leqslant O$. Hence $x_\beta qA$ and $f(A) \leqslant f(O) \leqslant B$.

(f) $\Rightarrow$ (a). Let O^* be an (i,j)-fuzzy semiopen set in X^* and $x_\beta \in f^{-1}(O^*)$. Clearly $f(x_\beta) \in O^*$. Choose the fuzzy point $x_\beta^c = 1 - x_\beta$. Then $f(x_\beta^c)qO^*$. And so by (f) there exists an (i,j)-fuzzy semiopen set A such that $x_\beta^c qA$ and $f(A) \leqslant O^*$.

Now $x_\beta^c qA \Rightarrow x_\beta^c + A(x) = 1 - x_\beta + A(x) > 1 \Rightarrow A(x) > x_\beta \Rightarrow x_\beta \in A$. Thus $x_\beta \in A \leqslant f^{-1}(O^*)$. Hence by Theorem 3.4 [10], $f^{-1}(O^*)$ is (i,j)-fuzzy semiopen in X.

(c) $\Rightarrow$ (i). Suppose that (c) holds. Let A be a subset of X. Since $A \leqslant f^{-1}(f(A))$, then $A \leqslant f^{-1}((i,j)\text{-}\operatorname{scl} f(A))$. Now $(i,j)\text{-}\operatorname{scl}(f(A))$ is (i,j)-fuzzy semiclosed so $f^{-1}((i,j)\text{-}\operatorname{scl} f(A))$ is (i,j)-fuzzy semiclosed and contains A. Consequently $(i,j)\text{-}\operatorname{scl}(A) \leqslant f^{-1}((i,j)\text{-}\operatorname{scl}(A))$, and so $f(i,j)\text{-}\operatorname{scl}(A) \leqslant (i,j)\text{-}\operatorname{scl}(f(A))$.

(i) $\Rightarrow$ (c). Suppose that (i) holds for any subset A of X. Let B be an (i,j)-fuzzy semiclosed subset of x^*. Then $f(i,j)\text{-}\operatorname{scl}(f^{-1}(B)) \leqslant (i,j)\text{-}\operatorname{scl} f(f^{-1}(B)) \leqslant (i,j)\text{-}\operatorname{scl}(B) = B$. Hence $(i,j)\text{-}\operatorname{scl}(f^{-1}(B)) \leqslant (B)$ and $f^{-1}(B)$ is (i,j)-fuzzy semiclosed in X.

(i) $\Rightarrow$ (j). Let B be a fuzzy set of X^*. Then $f^{-1}(B)$ is a fuzzy set of X. Therefore by (i), $f((i,j)\text{-}\operatorname{scl}(f^{-1}(B))) \leqslant (i,j)\text{-}\operatorname{scl}(f(f^{-1}(B))) \leqslant (i,j)\text{-}\operatorname{scl}(B)$. Hence $(i,j)\text{-}\operatorname{scl}(f^{-1}(B)) \leqslant f^{-1}((i,j)\text{-}\operatorname{scl}(B))$.

(j) $\Rightarrow$ (i). Let $B = f(A)$ where A is a subset of X, and we know that $A \leqslant B \Rightarrow (i,j)\text{-}\operatorname{scl}(A) \leqslant (i,j)\text{-}\operatorname{scl}(B)$ thus $(i,j)\text{-}\operatorname{scl}(A) \leqslant (i,j)\text{-}\operatorname{scl} f^{-1}(B) \leqslant f^{-1}((i,j)\text{-}\operatorname{scl}(B)) \leqslant f^{-1}((i,j)\text{-}\operatorname{scl}(f(A))$. Therefore

$f\big((i,j)\text{-}\operatorname{scl}(A)\big) \leqslant (i,j)\text{-}\operatorname{scl}(f(A))$.

(a) $\Rightarrow$ (k). Let B be an (i,j)-fuzzy semiopen set in X^*. Clearly $f^{-1}((i,j)\text{-}\operatorname{sint}(B))$ is (i,j)-fuzzy semiopen and we have

$f^{-1}((i,j)\text{-}\operatorname{sint}(B)) \leqslant (i,j)\text{-}\operatorname{sint}(f^{-1}((i,j)\text{-}\operatorname{sint}(B)) \leqslant (i,j)\text{-}\operatorname{sint}(f^{-1}(B))$.

(k) $\Rightarrow$ (a). Let B be any (i,j)-fuzzy semiopen set in X^*. Then $(i,j)\text{-}\operatorname{sint}(B) = B$ and $f^{-1}(B) - f^{-1}(i,j)\text{-}\operatorname{sint}(B)) \leqslant (i,j)\text{-}\operatorname{sint}(f^{-1}(B))$. Hence we have $f^{-1}(B) = (i,j)\text{-}\operatorname{sint}(f^{-1}(B))$. This shows that $f^{-1}(B)$ is (i,j)-fuzzy semiopen. □

Theorem 3.2. *Let $f\colon (X,\tau_1,\tau_2) \to (X^*,\tau_1^*,\tau_2^*)$ and $g\colon (X^*,\tau_1^*,\tau_2^*) \to (X^{**},\tau_1^{**},\tau_2^{**})$ be two mappings. Then $g\circ f$ is*

(a) *pairwise fuzzy irresolute if f and g are pairwise fuzzy irresolute,*

(b) *pairwise fuzzy semicontinuous if f is pairwise fuzzy irresolute and g is pairwise fuzzy semicontinuous.*

Proof. Obvious. □

Theorem 3.3. *Let $f\colon (X,\tau_1,\tau_2) \to (X^*,\tau_1^*,\tau_2^*)$ be a pairwise fuzzy semicontinuous and pairwise fuzzy open mapping. Then f is pairwise fuzzy irresolute.*

Proof. Let B be an (i,j)-fuzzy semiopen set in X^*. Then there exists a τ_1^*-fuzzy open set U such that $U \leqslant B \leqslant \tau_j^*\text{-}\operatorname{cl}(U)$. Therefore $f^{-1}(U) \leqslant f^{-1}(B) \leqslant f^{-1}(\tau_j^*\text{-}\operatorname{cl}(U)) \leqslant \tau_j\text{-}\operatorname{cl} f^{-1}(U)$ because f is pairwise fuzzy open. Since f is pairwise fuzzy semicontinuous, $f^{-1}(U)$ is (i,j)-fuzzy semiopen X. Hence by Theorem 3.5 [10], $f^{-1}(B)$ is (i,j)-fuzzy semiopen in X. □

4. Pairwise fuzzy presemiopen mappings

Definition 4.1. A mapping f from a fuzzy bitopological space (X, τ_1, τ_2) to a fuzzy bitopological space $(X^*, \tau_1^*, \tau_2^*)$ is *pairwise fuzzy presemiopen* if the image of every (i,j)-fuzzy semiopen set of X is (i,j)-fuzzy semiopen in X^*.

Remark 4.1. Every pairwise fuzzy presemiopen mapping is pairwise fuzzy semiopen, but the converse may be false. Indeed,

Example 4.1. Let $X = \{x, y\}$, $X^* = \{a, b\}$ and $U \subset X$, $V \subset X^*$ and $W \subset X^*$ be fuzzy sets defined as follows:

$$\begin{aligned} U(x) &= 0.5, & U(y) &= 0.6, \\ V(a) &= 0.5, & V(b) &= 0.6, \\ W(a) &= 0.2, & W(b) &= 0.3. \end{aligned}$$

Let $\tau_1 = \{0, U, 1\}$, $\tau_2 = \{0, 1\}$, $\tau_1^* = \{0, V, 1\}$, $\tau_2^* = \{0, W, 1\}$. Then the mapping $f\colon X \to X^*$ defined by $f(x) = a$ and $f(y) = b$ is pairwise fuzzy open and hence pairwise fuzzy semiopen but not pairwise fuzzy presemiopen.

Consider the following example.

Example 4.2. Let $X = \{x, y\}$, $x^* = \{a, b\}$ and $S \subset X$, $T \subset X$, $U \subset X^*$ and $V \subset X^*$ be fuzzy sets defined as follows:

$$\begin{aligned} S(x) &= 0.2, & S(y) &= 0.3; \\ T(x) &= 0.3, & T(y) &= 0.2; \\ U(a) &= 0.1, & U(b) &= 0.2; \\ V(a) &= 0.2, & V(b) &= 0.1. \end{aligned}$$

Let $\tau_1 = \{0, S, 1\}$, $\tau_2 = \{0, T, 1\}$ and $\tau_1^* = \{0, U, 1\}$, $\tau_2^* = \{0, V, 1\}$. Then the mapping $f\colon X \to Y$ defined by $f(x) = a$ and $f(y) = b$ is pairwise fuzzy presemiopen but not pairwise fuzzy open.

Remark 4.2. Examples 4.1 and 4.2 show that the concepts of pairwise fuzzy open and pairwise fuzzy presemiopen mappings are independent.

Theorem 4.1. *Let $f\colon (X, \tau_1, \tau_2) \to (X^*, \tau_1^*, \tau_2^*)$ be a pairwise fuzzy presemiopen mapping. If B is a fuzzy set of X^* and A is an (i,j)-fuzzy semiclosed set of X containing $f^{-1}(B)$ then there exists an (i,j)-fuzzy semiclosed set F of X^* containing B such that $f^{-1}(F) \leqslant A$.*

Proof. Let $F = 1 - f(1 - A)$. Since $f^{-1}(B) \leqslant A$, we have $f(1 - A) \leqslant 1 - B$. Since f is pairwise fuzzy presemiopen then F is an (i,j)-fuzzy semiclosed set of X^* and $f^{-1}(F) = 1 - f^{-1}(f(1 - A)) = (1 - A) = A$. Thus $f^{-1}(F) \leqslant A$. □

Theorem 4.2. *If $f\colon (X,\tau_1,\tau_2)\to(X^*,\tau_1^*,\tau_2^*)$ is pairwise fuzzy presemiopen then $f^{-1}((i,j)\text{-}\operatorname{scl}(B))\leqslant(i,j)\text{-}\operatorname{scl}f^{-1}(B)$ for every fuzzy set B of X^*.*

Theorem 4.3. *A mapping $f\colon (X,\tau_1,\tau_2)\to(X^*,\tau_1^*,\tau_2^*)$ is pairwise fuzzy presemiopen if and only if $f((i,j)\text{-}\operatorname{sint}(A))\leqslant(i,j)\text{-}\operatorname{sint}f(A)$ for every fuzzy set A of X.*

Proof. Necessity: Suppose f is fuzzy presemiopen then $f((i,j)\text{-}\operatorname{sint}(A))$ is (i,j)-fuzzy semiopen in Y. Hence $f((i,j)\text{-}\operatorname{sint}(A))=(i,j)\text{-}\operatorname{sint}f((i,j)\text{-}\operatorname{sint}(A))\leqslant(i,j)\text{-}\operatorname{sint}f(A)$.

Sufficiency: Let A be an (i,j)-fuzzy semiopen set of X, then by hypothesis $f((i,j)\text{-}\operatorname{sint}(A))\leqslant(i,j)\text{-}\operatorname{sint}f(A)$. Hence $f(A)$ is (i,j)-fuzzy semiopen in X^*. □

Theorem 4.4. *Let two mappings*

$$f\colon (X,\tau_1,\tau_2)\to(X^*,\tau_1^*,\tau_2^*)\quad\text{and}\quad g\colon (X^*,\tau_1^*,\tau_2^*)\to(X^{**},\tau_1^{**},\tau_2^{**})$$

be pairwise fuzzy presemiopen. Then $g\circ f$ is pairwise fuzzy presemiopen.

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